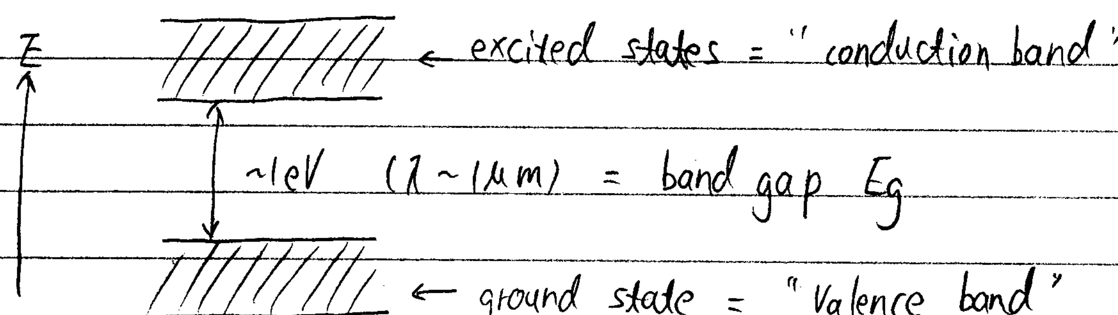
03/28/05 lecture 23

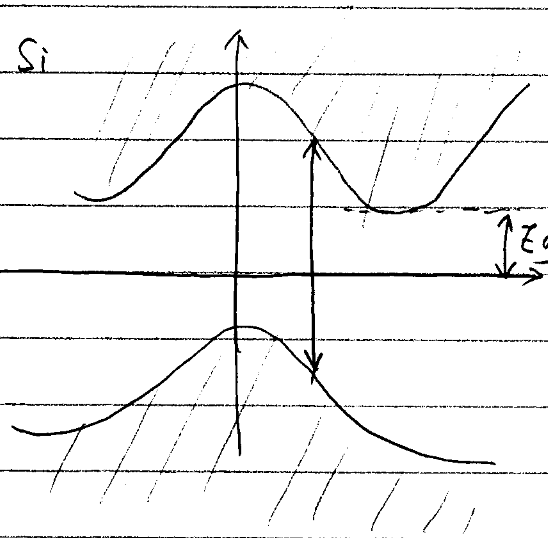
Diode lasers (ch 15 - 16)

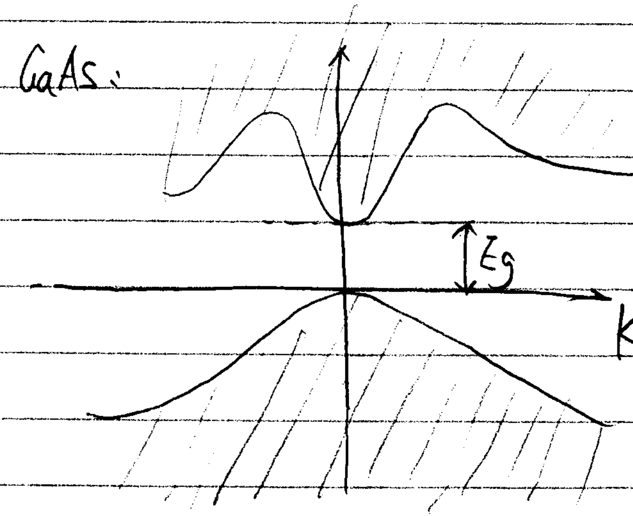
Diode lasers are a little different from other lasers, really have energy bands, not levels



(These are energy states for electrons in a semiconductor)

Can label states by "crystal momentum", $p = \hbar k$
like ordinary momentum, but modified a little by fact that e^-s move in crystal lattice.
In particular, $E \neq \frac{p^2}{2m}$





In equilibrium at low temperatures, states in valence band are all occupied, states in conduction band are all empty.

Can drive e^-'s between bands by radiative transitions.
– Leaves "holes" in valence band, excited electrons can decay and refill holes.

Usually change point of view and think of holes as as actual particles ($\sim$ like positrons), positive charge against background.

Then excitation produces e^--hole pairs, decay destroys pairs: recombination.

Two important notes:
1) To good approximation, k is conserved during transition (photon momentum is small)
2) k is NOT conserved during free evolution: electrons thermalize with lattice.

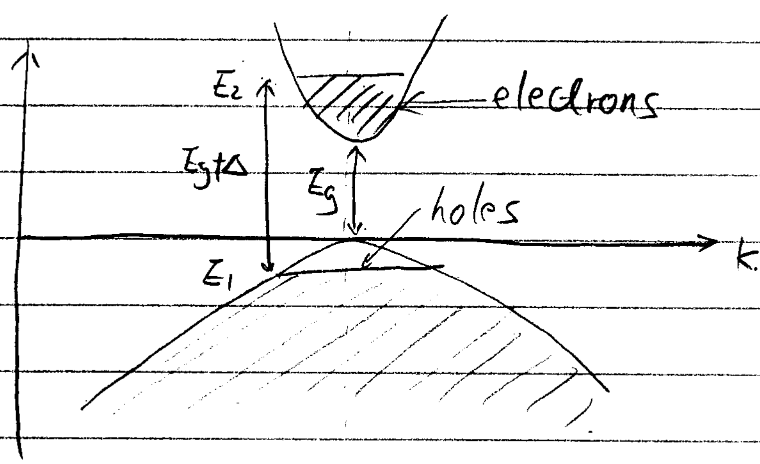
So conduction electrons rapidly settle to <u>bottom</u> of band ($t \sim 10^{-12}$ s)
Holes rise to <u>top</u> of valence band.

So in $\underline{\underline{Si}}$, excited electrons are __not__ likely to decay radiatively. "indirect gap" semi conductor

GaAs: e^-'s can decay: "direct gap"

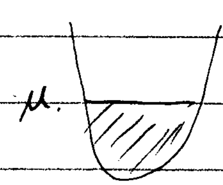
Need direct gap for laser.

System with inversion looks like:

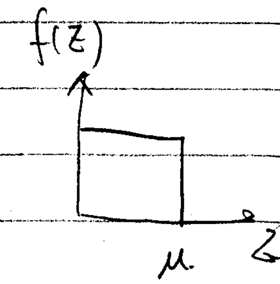


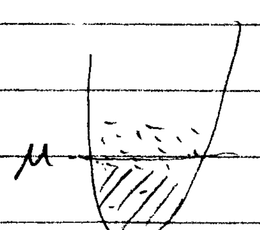
looks like gain possible between E_g & $E_g+\Delta$

Fermi function $\quad f(E) = \dfrac{1}{e^{(E-\mu)/kT}+1}$

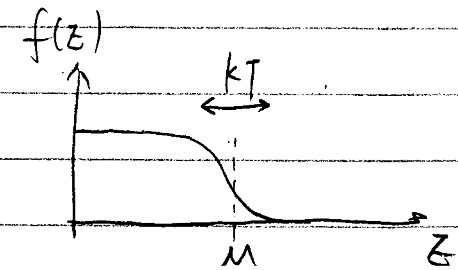


At $T=0$ $\quad f(E) = \begin{cases} 1 & E < \mu \\ 0 & E > 0 \end{cases}$





At finite T.
population pop up



Near band gap, can approximate $E(k)$ by parabola.

$$E_1 = -\frac{\hbar^2 k^2}{2m_v}$$

$$E_2 = E_g + \frac{\hbar^2 k^2}{2m_c}$$

But m_v & m_c are just coefficients with units of mass

GaAs: $m_c = 0.07\, m_e$
$m_v = 0.5\, m_e$

Want to calculate gain coefficient.

For any two states, $W = \frac{\lambda^2}{8\pi t_s} \frac{I}{h\nu} g(\nu)$

But need to integrate over band:

$$W_{c\to v} = \int \frac{\lambda^2}{8\pi t_s} g'(\nu) \frac{I(\nu)}{h\nu} \rho(\nu_0') f_e(\nu_0')\, d\nu_0'$$

$$g'(\nu) = \frac{\Delta\nu/2\pi}{(\nu-\nu_0')^2 + (\Delta\nu/2)^2}$$ $\Delta\nu$ from collisions

$\rho(\nu_0')$ = joint density of states
$\rho(\nu_0')\,d\nu_0'$ = # of states within transition freq. ν_0'

$f_e(\nu_0')$ = prob. that states have right occupations
(upper state occupied, lower state empty.)

To get $\rho(\nu_0')$ & $f_e(\nu_0')$, need to know

how E_2 = energy of electron state
E_1 = energy of hole state
are determined by ν_0' (transition frequency)

Have $E_2 - E_1 = h\nu_0'$

$$E_2 = E_g + \frac{\hbar^2 k^2}{2m_c}, \quad E_1 = -\frac{\hbar^2 k^2}{2m_v}$$

So $E_g + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_c} + \frac{1}{m_v}\right) = h\nu_0'$

Define $\frac{1}{m_r} = \frac{1}{m_c} + \frac{1}{m_v}$, m_r = reduced mass

then $k^2 = \frac{2m_r}{\hbar^2}(h\nu_0' - E_g)$

which means:

$$E_2 = E_g + \frac{\hbar^2 k^2}{2m_c} = E_g + \frac{m_r}{m_c}(h\nu_0' - E_g)$$

$$E_1 = \frac{m_r}{m_v}(E_g - h\nu_0')$$

So to get $\rho(\nu_0')$, use

$$\# \text{ states} = \rho(\nu_0')\,d\nu_0' = \rho_c(E_2)\,dE_2$$

$$\rho_c(E_2) = \frac{(2m_c)^{3/2}}{2\pi^2\hbar^3}(E_2 - E_g)^{1/2}$$

$$= \frac{(2m_c)^{3/2}}{2\pi^2\hbar^3}\left(\frac{m_r}{m_c}\right)^{1/2}(h\nu_0' - E_g)^{1/2}$$

and $\frac{dE_2}{d\nu_0'} = \frac{m_r}{m_c} h$

gives $$\boxed{\rho(\nu_0') = \frac{(2m_r)^{3/2}}{\pi \hbar^2} (h\nu_0' - E_g)^{1/2}}$$

And to get $f_e(\nu_0')$

$$f_e(\nu_0') = f_c(E_2) \times [1 - f_v(E_1)]$$

$$f_c(E_2) = \frac{1}{e^{(E_2 - \mu_c)/kT} + 1}$$

$$f_v(E_1) = \frac{1}{e^{(E_1 - \mu_v)/kT} + 1}$$

So, back to equation for W:

normally width of $g(\nu) \approx 10^{12}$ Hz, is small compared to anything else, peaked at $\nu_0' = \nu$

So take

$$W_{c\to v}(\nu) \approx \frac{\lambda^2}{8\pi t_s} \frac{I(\nu)}{h\nu} \rho(\nu) f_e(\nu) \underbrace{\int g'(\nu_0')\, d\nu_0'}_{=1}$$

But: can have absorption too:

some probability f_a that c state is empty & v state occupied

like before, now $f_a = f_v(E_1)[1 - f_c(E_2)]$

So really have <u>net</u> transition rate.

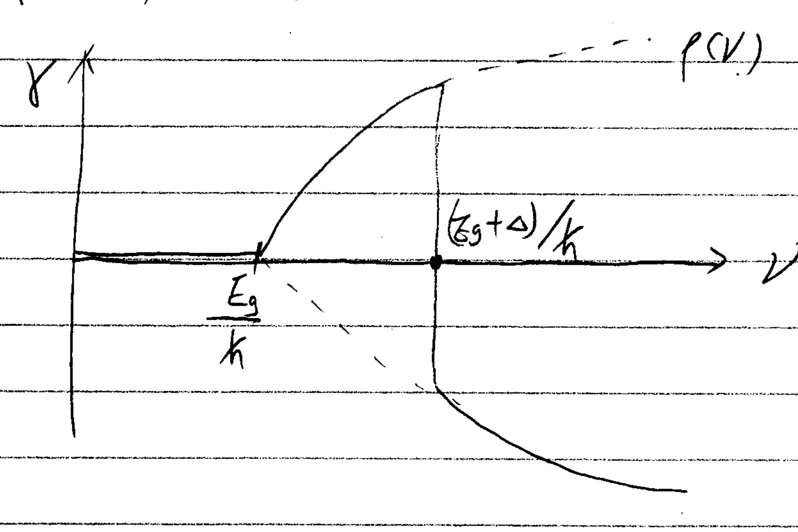
$$W = W_{c\to v} - W_{v\to c}$$
$$= \frac{\lambda^2}{8\pi t_s}\frac{I(\nu)}{h\nu}\rho(\nu)\underbrace{\left[f_e(\nu) - f_a(\nu)\right]}$$

$$f_c(E_2) - f_c(E_2)f_v(E_1)$$
$$- \left[f_v(E_1) - f_v(E_1)f_c(E_2)\right]$$
$$= f_c(E_2) - f_v(E_1)$$
$$= f_g(\nu)$$

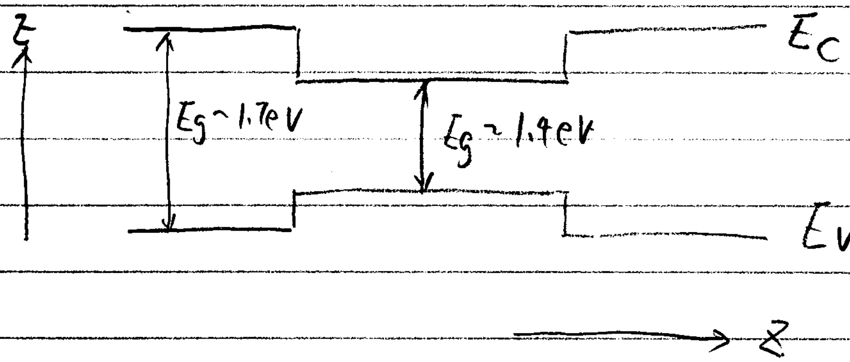
From W calculate gain just as before.

$$\boxed{\begin{aligned}\gamma_0(\nu) &= W(\nu)\cdot\frac{h\nu}{I(\nu)}\\ &= \frac{\lambda^2}{8\pi t_s}\rho(\nu)f_g(\nu)\end{aligned}}$$

At $T = 0$, looks like



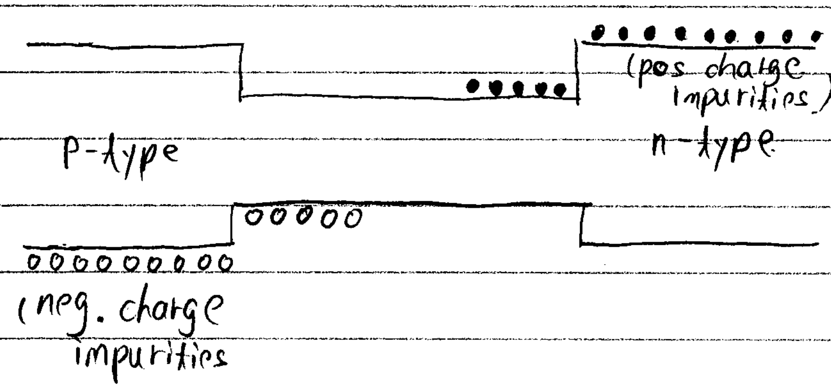
AlGaAs & GaAs have different band gaps



Dope AlGaAs with impurities

p-type: trap electrons, generate holes in valence

n-type: donate electrons to conduction band

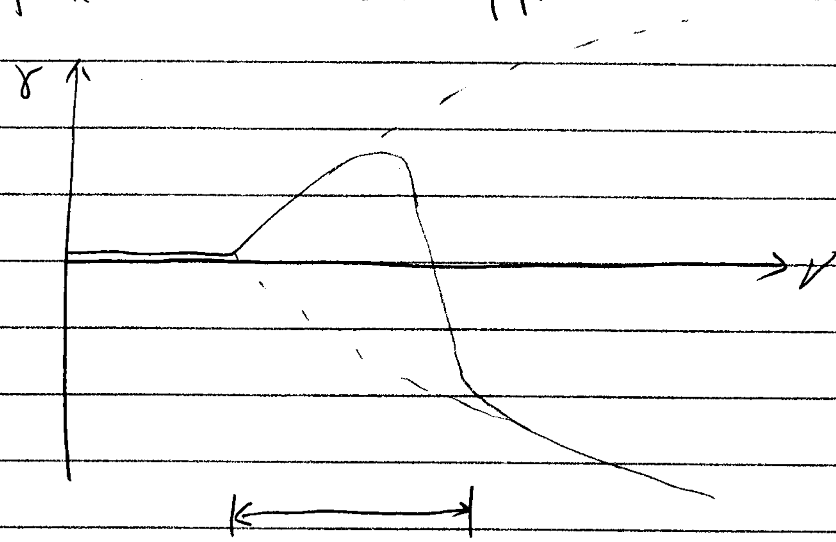


Holes & electrons diffuse into GaAs to lower energy

But restrained by charge build up in AlGaAs

Apply electric field & current, overcome restraint.

At finite T, rounded off.



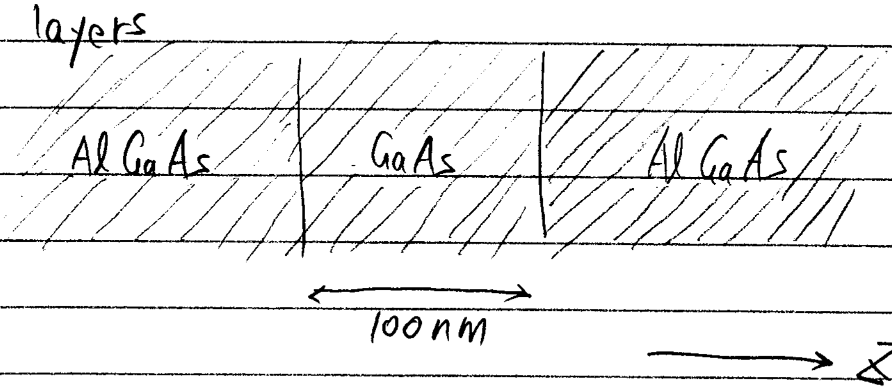
range of gain ~ $\Delta\nu$

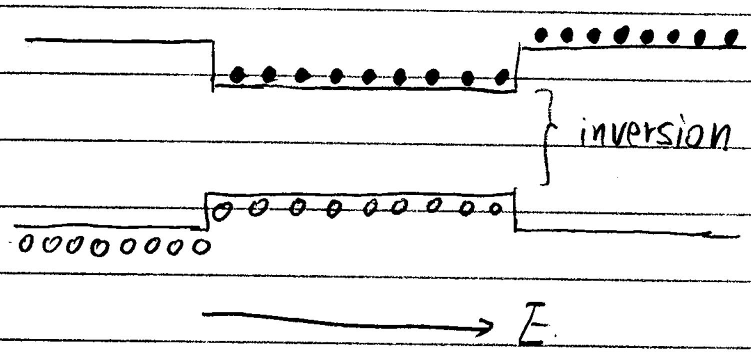
Typically gain over ~10^{13} Hz.

How to achieve inversion

use "heero structure"

layers





To calculate inversion:

current density $J =$ A/m^2

gives electron density $N_c = \frac{J}{e} \times \frac{\tau}{d}$

τ = electron lifetime

d = layer thickness