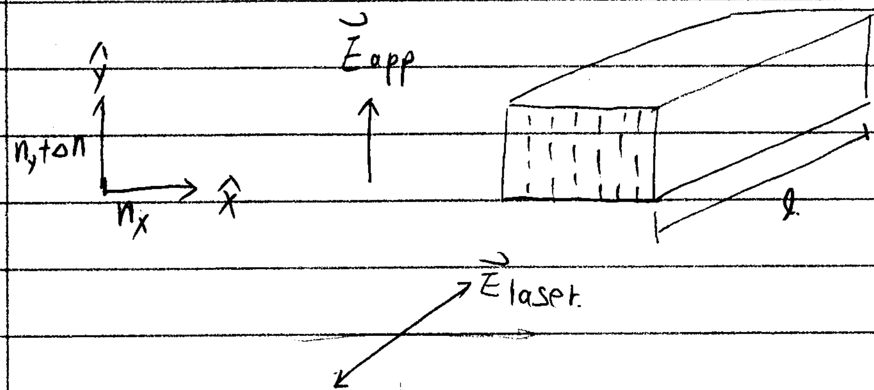
04/01/05 Lecture 25

Modulation with EO effect:

Suppose $\vec{E}$ modifies index for y-polarized light.



Send laser through crystal with polarization at 45°

$$\vec{E}_{laser} \approx \hat{\varepsilon} E_0 e^{i(\omega t - kz)}$$

$$\hat{\varepsilon} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$$

In crystal, $k = n k_0 = n \frac{\omega}{c_0}$

So output field is $\vec{E}'_{laser} = \frac{E_0}{\sqrt{2}} e^{i\omega t}\left[\hat{x} e^{-ink_0 l} + \hat{y} e^{-i(n+\Delta n)k_0 l}\right]$

$$= \frac{E_0}{\sqrt{2}} e^{i(\omega t - kl)}\left[\hat{x} + e^{-i\phi}\hat{y}\right]$$

$\phi = \Delta n k_0 l$ depends on E_{app}

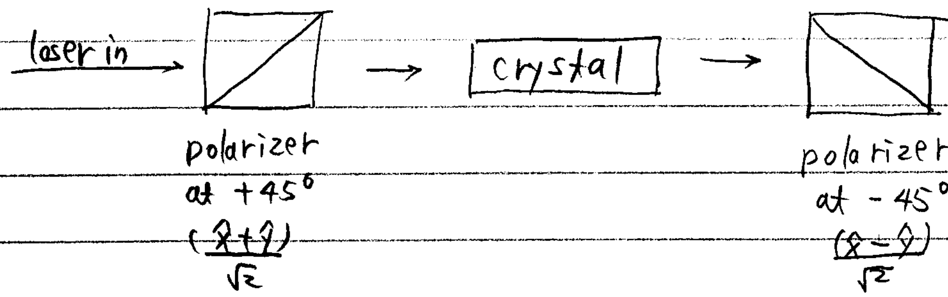
$$= -\tfrac{1}{2} r n^3 k_0 l E$$

or $-\frac{1}{2} s n^3 k_0 l E^2$

So can alter polarization of the light

Amplitude modulation (AM),

put between polarizers:



Output intensity:

$$I_{out} = I_{in} \left| \left(\frac{\hat{x}-\hat{y}}{\sqrt{2}}\right) \cdot \left(\frac{\hat{x}+e^{-i\phi}\hat{y}}{\sqrt{2}}\right) \right|^2$$

$$= I_{in} \frac{1}{4} \left| 1 - e^{-i\phi} \right|^2$$

$$= I_{in} \frac{1}{4} \left[(1-\cos\phi)^2 + \sin^2\phi \right]$$

$$= I_{in} \frac{1}{4} \left[2 - 2\cos\phi \right]$$

$$= I_{in} \sin^2\frac{\phi}{2}$$

controlling ϕ controls I
$\rightarrow$ AM

Frequency modulation: (FM)

let laser polarized along $\hat{y}$

so $\hat{\varepsilon} = \hat{y}$

Then $E_{out} = E_0 \hat{y} e^{i(\omega t - kl)} e^{i\phi}$

Apply oscillating voltage, $\phi = \phi_0 \sin \Omega t$

Output field has oscillating phase
$\Rightarrow$ oscillating frequency

$$E_{out} = E_0 \hat{y} e^{i(\omega t - kl)} e^{i\phi_0 \sin \Omega t}$$
$$\approx E_0 \hat{y} e^{i(\omega t - kl)} [J_0(\phi_0) + 2i J_1(\phi_0) \sin \Omega t]$$
$$= E_0 \hat{y} [J_0(\phi_0) e^{i(\omega t - kl)} + J_1(\phi_0) e^{i[(\omega+\Omega)t - kl]} - J_1(\phi_0) e^{i[(\omega-\Omega)t - kl]}]$$

Several other uses for EO effect described in book,
§ 18.1

Now look at details of EO effect

Complicated because EO effect generally occurs in birefringent crystals:
n depends on direction of polarization.

S&T 18.2, 6.3

Examples : KH_2PO_4 (KDP)

In birefringent cystals, generally the electric flux $\vec{D}$ and electric field $\vec{E}$ can be written as

$$\vec{D} = \boldsymbol{\epsilon} \vec{E}, \qquad D_i = \sum_j \epsilon_{ij} E_j$$

$\boldsymbol{\epsilon}$ is a tensor of second rank known as the electric permittivity tensor.

In the coordinate system defined as principal axes.

$$D_1 = \epsilon_1 E_1, \quad D_2 = \epsilon_2 E_2, \quad D_3 = \epsilon_3 E_3$$

Note: the coordinate system x, y, z (or 1, 2, 3) will always be assumed to lie along the crystal's principal axes.

The permittivities ϵ_1, ϵ_2 and ϵ_3 correspond to refractive indices

$$n_1 = \left(\frac{\epsilon_1}{\epsilon_0}\right)^{1/2}, \quad n_2 = \left(\frac{\epsilon_2}{\epsilon_0}\right)^{1/2}, \quad n_3 = \left(\frac{\epsilon_3}{\epsilon_0}\right)^{1/2}.$$

known as the principal refractive indices (ϵ_0 is the permittivity of free space)

* if $n_1 = n_2 = n_3$, the medium is optically isotropic.

* if $n_1 = n_2 = n_o$ (ordinary), $n_3 = n_e$ extraordinary
uniaxial crystals
positive uniaxial : $n_e > n_o$
negative uniaxial : $n_e < n_o$

∴ KDP : negative uniaxial
$n_1 = n_2 = n_o = 1.51$, $n_3 = n_e = 1.47$

The relation between $\vec{D}$ and $\vec{E}$ can be inverted and written in the form

$$\vec{E} = \boldsymbol{\epsilon}^{-1} \vec{D}$$

or

$$\epsilon_0 \vec{E} = \boldsymbol{\eta} \vec{D}$$

where $\boldsymbol{\eta} = \epsilon_0 \boldsymbol{\epsilon}^{-1}$, impermeability tensor.

Both tensors $\boldsymbol{\epsilon}$ and $\boldsymbol{\eta}$ share the same principal axes (directions for which $\vec{E}$ and $\vec{D}$ are parallel.)

In pricipal coordinate system, $\boldsymbol{\eta}$ is diagonal with principal values :

$$\frac{\epsilon_0}{\epsilon_1} = \frac{1}{n_1^2}, \quad \frac{\epsilon_0}{\epsilon_2} = \frac{1}{n_2^2}, \quad \frac{\epsilon_0}{\epsilon_3} = \frac{1}{n_3^2}$$

Either $\boldsymbol{\epsilon}$ and $\boldsymbol{\eta}$ describes the optical properties of the crystal completely