04/25/05 Lecture 35 ①

Examples, $LiNbO_3$ can be used to double Nd:YAG laser 1064 nm → 532 nm

has

λ	n_o	n_e
1064 nm	2.2338	2.1539
532 nm	2.3310	2.2378

$$n_o(\omega) \simeq n_e(2\omega)$$

To get just right, can adjust temperature
find that n_o & n_e are soft functions of T

For $LiNbO_3$, get indices equal at $T = 60°C$

Of course, need correct d element:

Output is $E_z(2\omega)$, input is $E_x(\omega)$ and/or $E_y(\omega)$

So need d_{zxx}, d_{zyy}, or d_{zxy}
d_{31} d_{32} d_{36}

Have $d_{31} = d_{32} = 4 \times 10^{-23} \frac{C}{V^2}$ so it works

Get $\boxed{E_z(\omega_3) = -i \frac{\omega_3}{n} \eta_0 L \frac{d_{31}}{2} E_x(\omega_1)^2}$

Methods of phase-matching:

A. Find crystal with $\Delta k \approx 0$ for appropriate polarizations of beams

Fine adjust with temperature
"non-critical phase matching"

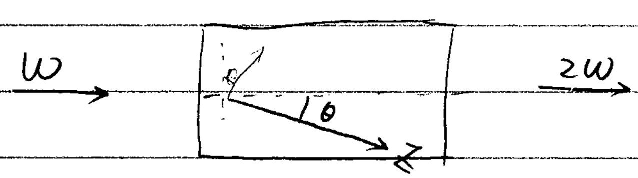
Not always possible / convenient

B. Use L small enough that Δn doesn't matter.
Typically $L < 1mm$

make up for small L with large $E(\omega_1)$, $E(\omega_2)$
$\Rightarrow$ great for pulsed lasers

C. use birefringence, but adjust angle of crystal.
"critical PM"

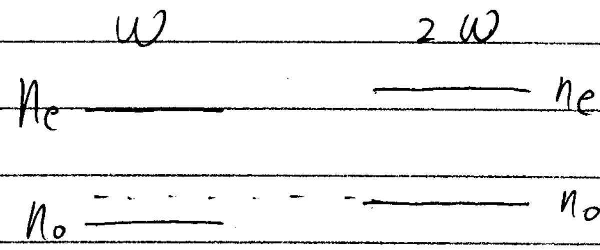
Consider SHG $\omega \to 2\omega$ in uniaxial crystal.



Have $n_a = n_o$ $\qquad \dfrac{1}{n_b^2} = \dfrac{\cos^2\theta}{n_o^2} + \dfrac{\sin^2\theta}{n_e^2}$

n_b ranges from n_o to n_e

Picture
($n_e > n_o$)



Want to adjust θ to make $n_b = n_a = n_o$

Here, means pick ω light to be e- polarized ↕
2ω light to be o- polarized ⊙

Then solve for θ:

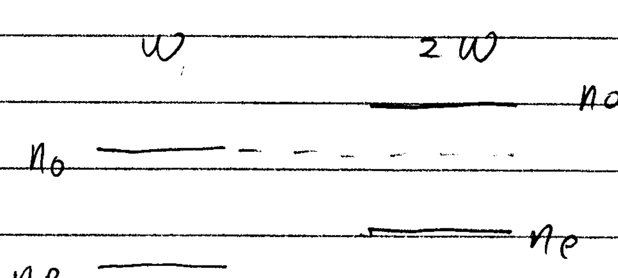
$$\frac{1}{n_o(2\omega)^2} = \frac{\cos^2\theta}{n_o(\omega)^2} + \frac{\sin^2\theta}{n_e(\omega)^2}$$

$$= \frac{1}{n_o(\omega)^2} - \frac{\sin^2\theta}{n_o(\omega)^2} + \frac{\sin^2\theta}{n_e(\omega)^2}$$

$$\sin^2\theta = \frac{\frac{1}{n_o(\omega)^2} - \frac{1}{n_o(2\omega)^2}}{\frac{1}{n_o(\omega)^2} - \frac{1}{n_e(\omega)^2}}.$$

If $n_o > n_e$, reserved

use 2ω light
e-polarized.

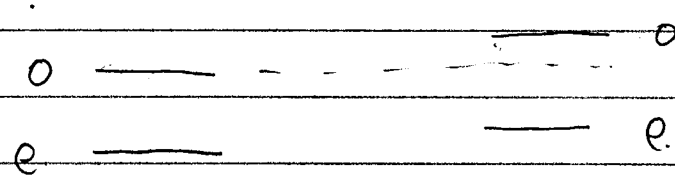


get $$\sin^2\theta = \frac{\frac{1}{n_o(\omega)^2} - \frac{1}{n_o(2\omega)^2}}{\frac{1}{n_e(2\omega)^2} - \frac{1}{n_o(2\omega)^2}}.$$

Example.

Double ruby laser in KDP ($\lambda = 694\,nm \rightarrow 347\,nm$)

Have	694 nm	347 nm
n_o	1.5055	1.5357
n_e	1.4658	1.4897



From formula $\sin^2\theta = 0.6462$

$\theta = 53.5^\circ$

Again, need correct nonlinear coefficients

Have $d_{14} = d_{25} = 3.9 \times 10^{-24} \dfrac{C}{V^2}$.

$(d_{xyz} = d_{yxz})$

$d_{36} = 5 \times 10^{-24} \dfrac{C}{V^2}$

(d_{zxy})

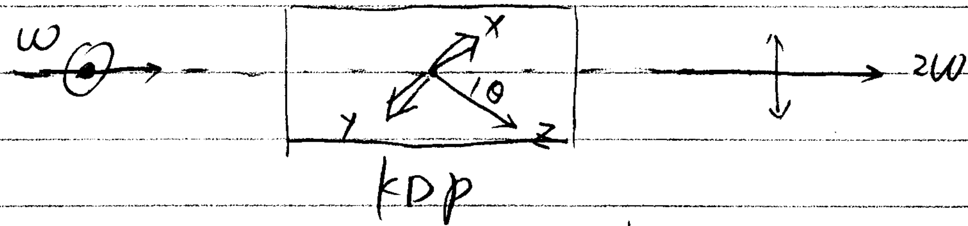
How we use these?

Need ω beam ordinary, so $E_z(\omega) = 0$

rules out d_{14} and d_{25}

For d_{36} need both $E_x(\omega)$ and $E_y(\omega)$

Rotate axes by $45°$



Then $E_x(\omega) = E_y(\omega) = \frac{1}{\sqrt{2}} E(\omega)$

However, produce $P_z(2\omega)$

only vertical component of P couples to output wave : coupling $= \sin\theta$

Finally write $\quad E(2\omega) = -2i \frac{2\omega}{n} \eta_0 L d' E(\omega)^2$

d' now contains all coupling factors

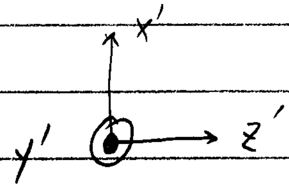
$$d' = \underbrace{d_{36} \times \tfrac{1}{2}}_{\text{degenerate frequencies}} \times \underbrace{2}_{E_x E_y + E_y E_x} \times \underbrace{\tfrac{1}{2}}_{E_x = \frac{1}{\sqrt{2}} E(\omega),\ E_y = \frac{1}{\sqrt{2}} E(\omega)} \times \underbrace{\sin\theta}_{\vec{P} \cdot \vec{E}(2\omega) = \sin\theta}$$

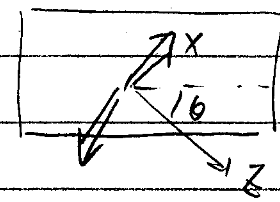
$$= \frac{1}{2} \sin\theta \, d_{36}$$

<u>3D vectors</u>

3D vectors

For beam coordinates (x', y', z')





with $x' = e$ direction
$y' = o$ direction

Generally, x axis of crystal rotated out of $x'z'$ plane by ϕ

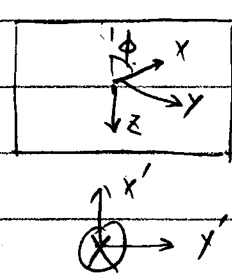
($\phi = 45°$ in KDP example)

Then have

$$E_x = \cos\theta \cos\phi \, E_{x'} + \sin\phi \, E_{y'}$$
$$E_y = \cos\phi \, E_{y'} - \cos\theta \sin\phi \, E_{x'}$$
$$E_z = -\sin\theta \, E_{x'}$$

use these factors to figure out d'

Looking into crystal



Notes

* sometimes have multiple d elements contributing
  Then $d' =$ sum for each process

* usually range of ϕ's works, pick ϕ that
  gives biggest d'

* more complicated for $\omega_1 \neq \omega_2$, more choices
  for polarizations

Actually, even for SHG, could have input light
with one component at o & one at e

Terminology:

Type I : both input beams have same
polarization (o or e)

Type II : two input beams are different
(o and e)

If type II can phase match, so can type I
(but not vice versa)

But sometimes need Type II to get good d_{ijk}.