04/27/05 Lecture 36

phase matching make $\Delta k = 0$

critical PM: achieve by adjusting angles of crystal

Two tasks:

1) Find required θ

2) Find effective d'

For (2), often need to consider 3D vectors

In KDP:

crystal coordinates (x, y, z) - axis $= z$

Beam coordinates (x', y', z')

— propagate along z'

$x' =$ e direction, $y' =$ o direction ($\perp z$)

Generally, x axis rotated out of $x'z'$ plane by angle ϕ

($\phi = 45°$ in KDP example)

For these ~~coordinates~~, have

$$E_x = \cos\theta \cos\phi \, E_{x'} + \sin\phi \, E_{y'}$$
$$E_y = \cos\phi \, E_{y'} - \cos\theta \sin\phi \, E_{x'}$$
$$E_z = -\sin\theta \, E_{x'}$$

use these to figure out d'

$$P_{x'} = -\sin\theta\, P_z + \cos\theta\cos\phi\, P_x - \sin\phi\cos\theta\, P_y$$
$$P_{y'} = \cos\phi\, P_y + \sin\phi\, P_x$$

Define $\quad E_j(\omega_1) = \beta_j\, |\vec{E}(\omega_1)|$
$\qquad\qquad E_k(\omega_2) = \gamma_k\, |\vec{E}(\omega_2)|$

β_j, γ_k = coefficients from E_x vs. $E_{x'}$ relations

Also, define P' = component of P parallel to $E(\omega_3)$
$= (P_{x'}$ or $P_{y'}$

$= \sum_i \alpha_i P_i \qquad i = x, y, z$

α_i : again from relations

Then $d' = \sum_{ijk} \alpha_i \beta_j \gamma_k d'_{ijk}$

In KDP example,

using $d_{36} = d_{zxy}$ coefficient,
so light at ω is ordinary, $\Rightarrow$ has component $E_{y'}(\omega)$

So $\quad E_x(\omega) = \sin\phi\, E_{y'}(\omega)$
$\qquad E_y(\omega) = \cos\phi\, E_{y'}(\omega)$

Light at 2ω is polarized along x'
but produce P_z

$$P_{x'} = -\sin\theta\, P_z$$

Get further factor of $\frac{1}{2}$ from from frequency degeneracy, but $\times 2$ for sum over polarizations

So $$d' = -\sin\phi \cos\phi \sin\theta \, d_{36}$$
$$= -\tfrac{1}{2} \sin 2\phi \sin\theta \, d_{36}$$

Best for $\phi = 45°$, as said last time

Note:

* sometimes have multiple d elements contributing then $d' =$ sum for each

* usually range of ϕ's works, pick optimum

* more complicated for $\omega_1 \neq \omega_2$: more choices for polarizations

Same idea for phase matching, pick option that gives largest d'

Example: Sum frequency generation $\omega_3 = \omega_1 + \omega_2$.
for $\lambda_1 = 1.064\ \mu m$ (Nd:YAG)
$\lambda_2 = 10.6\ \mu m$ (CO_2 laser)

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \qquad \lambda_3 = 0.96\ \mu m$$

"up conversion" Do experiment with CO_2 laser
scattered light hard to detect
Up convert to near IR: better detectors

Consider using pronstite Ag_3AsS_3
* Negative uniaxial
* Symmetry class 3m (like $LiNbO_3$)
* Transparent 0.6 μm to 13 μm

Indices:

λ	n_o	n_e	$\frac{n_o}{\lambda}$	$\frac{n_e}{\lambda}$
10.6 μm	2.697	2.503	0.254 μm^{-1}	0.236 μm^{-1}
1.06 μm	2.816	2.582	2.657	2.436
0.96 μm	2.836	2.598	2.955	2.706

Need to make $\frac{n_3}{\lambda_3} = \frac{n_1}{\lambda_1} + \frac{n_2}{\lambda_2}$

Draw picture for both sides of equation

2.955 ———— $\frac{n_o}{\lambda_3}$

$\frac{n_o}{\lambda_1} + \frac{n_o}{\lambda_2}$ ———— 2.911

$\frac{n_o}{\lambda_1} + \frac{n_e}{\lambda_2}$ ———— 2.893

2.706 ———— $\frac{n_e}{\lambda_3}$

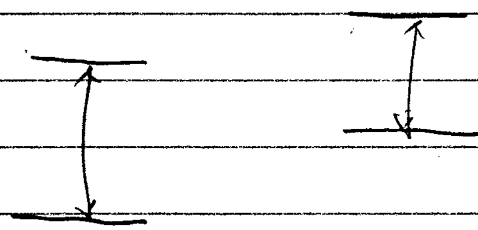
$\frac{n_e}{\lambda_1} + \frac{n_o}{\lambda_2}$ ———— 2.690

$\frac{n_e}{\lambda_1} + \frac{n_e}{\lambda_2}$ ———— 2.672

Immediately see that ω_3 light must be e.

Can't achieve $\frac{n_1}{\lambda_1} + \frac{n_2}{\lambda_2} = \frac{n_o}{\lambda_3}$

Less obvious, need ω_1 light to be o
for picture.



can't both be e

But ω_2 could be o or e.

I: ω_2 : o solve

$$\frac{n_{3e}(\theta)}{\lambda_3} = \frac{n_{1o}}{\lambda_1} + \frac{n_{2o}}{\lambda_2}$$

$$\frac{1}{\lambda_3}\left(\frac{\cos^2\theta}{n_{3o}^2} + \frac{\sin^2\theta}{n_{3e}^2}\right)^{-1/2} = \frac{n_{1o}}{\lambda_1} + \frac{n_{2o}}{\lambda_2}.$$

Easiest to solve on computer.
plot both sides & see where they cross

Get $\theta_I = 26.3^\circ$

II $\omega_2 = e$, have

$$\frac{n_{3e}(\theta)}{\lambda_3} = \frac{n_{1o}}{\lambda_1} + \frac{n_{2e}(\theta)}{\lambda_2}.$$

Again, solve numerically, get $\theta_{II} = 27.3^\circ$.

Choose based on d's

have $d_{15} = d_{24} = d_{31} = d_{32} = 9.7 \times 10^{-23} \frac{C}{V^2}$

$d_{22} = -d_{21} = -d_{16} = 1.6 \times 10^{-22} \frac{C}{V^2}$.

Consider I

I: have $E_{y'}(\omega_1)$, $E_{y'}(\omega_2)$, need $P_{x'}(\omega_3)$

So d_{xxz}, $d_{yyz} \to 0$ no $Z_{z'}$ at ω_1 or ω_2

$d_{zxx} \to d' = (-\sin\theta) \sin^2\phi \; d_{15}$
$d_{zyy} \to d' = (-\sin\theta) \cos^2\phi \; d_{15}$
$d_{yyy} \to d' = (-\sin\theta \sin\phi) \cos^2\phi \; d_{22}$
$d_{yxx} \to d' = (-\sin\phi \cos\theta) \sin^2\phi \; (-d_{22})$
$d_{xxy} \to d' = (\cos\theta \cos\phi) \sin\phi \cos\phi \; (-d_{22})$

All these contribute:

$$d' = -\sin\theta\,[\sin^2\phi + \cos^2\phi]\, d_{15}$$
$$+ \cos\theta\,[-\sin\phi\cos^2\phi + \sin^3\phi - \sin\phi\cos^2\phi]\, d_{22}$$

$$= -\sin\theta\, d_{15} + \cos\theta\sin\phi\,[\sin^2\phi - 2\cos^2\phi]\, d_{22}$$

optimum at $\phi = 28^\circ$

$$d' = -87 \times 10^{-24}\ \frac{C}{V^2}$$