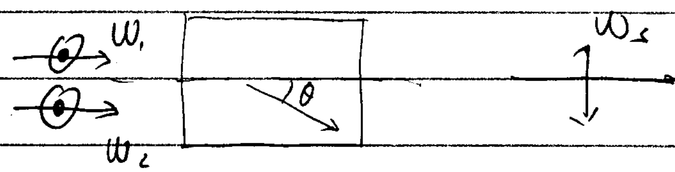
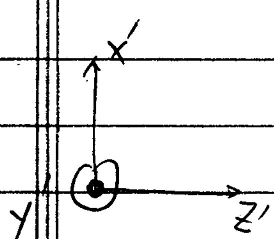
upconversion: $\lambda_2 = 10.6\,\mu m$ + $\lambda_1 = 1064\,nm$
$\rightarrow \lambda_3 = 960\,nm$

in proustite

Saw for phase-matching, two setups are possible

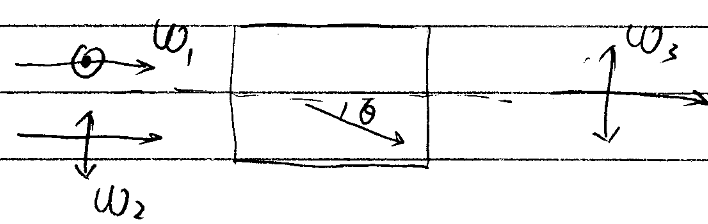
Type I





Get $\theta = 26.3°$

Type II



$\theta = 27.3°$

Almost the same because λ_1 is so big
basically making $n_o(\omega_1) = n_e(\omega_3)$

Still need to get d', work out type I only

Have $E_{y'}(\omega_1)\, E_{y'}(\omega_2)$, need $P_{x'}(\omega_3)$

Recall

$$E_x = \cos\theta\cos\phi\, E_{x'} + \sin\phi\, E_{y'}$$
$$E_y = \cos\phi\, E_{y'} - \cos\theta\sin\phi\, E_{x'}$$
$$E_z = -\sin\theta\, E_{x'}$$

$$P_{x'} = \cos\theta\cos\phi\, P_x - \cos\theta\sin\phi\, P_y - \sin\theta\, P_z$$
$$P_{y'} = \sin\phi\, P_x + \cos\phi\, P_y$$

so here have

$$E_x(\omega_1) = \sin\phi\, E(\omega_1)$$
$$E_y(\omega_1) = \cos\phi\, E(\omega_2)$$
$$E_x(\omega_2) = \sin\phi\, E(\omega_2)$$
$$E_y(\omega_2) = \cos\phi\, E(\omega_2)$$

Pronstite has:

$$\underset{xxz}{d_{15}} = \underset{yyz}{d_{24}} = \underset{zzx}{d_{31}} = \underset{zzy}{d_{32}} = 9.7\times10^{-23}\ C/V^2$$

$$\underset{yyy}{d_{22}} = -\underset{yxx}{d_{21}} = -\underset{xxy}{d_{16}} = 1.6\times10^{-27}\ C/V^2$$

No E_z, so can't use d_{15} or d_{24}.

Have

$$d_{zxx} \rightarrow -\sin\theta\sin^2\phi\, d_{15}$$
$$d_{zyy} \rightarrow -\sin\theta\cos^2\phi\, d_{15}$$
$$d_{yxy} \rightarrow (-\cos\theta\sin\phi)\cos^2\phi\, d_{22}$$
$$d_{yxx} \rightarrow (-\sin\theta\sin\phi)\sin^2\phi\, (-d_{22})$$
$$d_{xxy} \rightarrow 2(\cos\theta\sin\phi)\sin\phi\cos\phi\, (-d_{22})$$

All contribute:

$$d' = -\sin\theta\,[\sin^2\phi + \cos^2\phi]\,d_{15} + \cos\theta\,[-3\sin\phi\cos^2\phi + \sin^3\phi]\,d_{22}$$

$$= -\sin\theta\, d_{15} - \cos\theta\,\sin 3\phi\; d_{22}.$$

Max at $\phi = \pm 30°, \pm 90°$

$$d' = -\sin\theta\, d_{15} - \cos\theta\, d_{22} = -1.9 \times 10^{-22}\ C/V^2.$$

Power output & conversion efficiency

We derived:

$$E(\omega_3) = -2i\omega_3 \frac{L}{n_3} \eta_3\, d' E(\omega_1)\, E(\omega_2)$$

crystal length L
n_3 = index for ω_3 light
$\eta_0 = 377\ \Omega$

convert to intensity:

$$I \leq \frac{n}{2\eta_0} |E|^2.$$

$$\text{So } I(\omega_3) = \frac{n_3}{2\eta_0} 4\omega_3^2 \frac{L^2}{n_3^2} \eta_0^2 d'^2 |E(\omega_1)|^2 |E(\omega_2)|^2$$

use $|Z(\omega_1)|^2 = \frac{2\eta_0 I(\omega_1)}{n_1}$

$$I(\omega_3) = 2\omega_3^2 \frac{L^2}{n_3} \eta_0 d'^2 \frac{2\eta_0}{n_1} I(\omega_1) \frac{2\eta_0}{n_2} I(\omega_2)$$

$$\boxed{I(\omega_3) = 8\eta_0^3 \frac{L^2\omega_3^2}{n_1 n_2 n_3} d'^2 I(\omega_1) I(\omega_2)}$$

Convert to powers: $P(\omega_1) = \pi W_0^2 I(\omega_1)$

W_0 : beam waist.

$$P(\omega_3) = \frac{8}{\pi}\eta_0^3 \frac{\omega_3^2 d'^2}{n_1 n_2 n_3} \frac{L^2}{W_0^2} P(\omega_1) P(\omega_2)$$

Linear function of either $P(\omega_1)$ or $P(\omega_2)$

for SHG

$P(2\omega)$ & $P^2(\omega)$ quadratic function

So efficiency $\zeta \equiv \frac{P(2\omega)}{P(\omega)} \propto P(\omega)$

Difficult for low powers

Easy for high power.

Well suited for pulsed lasers.

To improve efficiency, use large L and small W_0

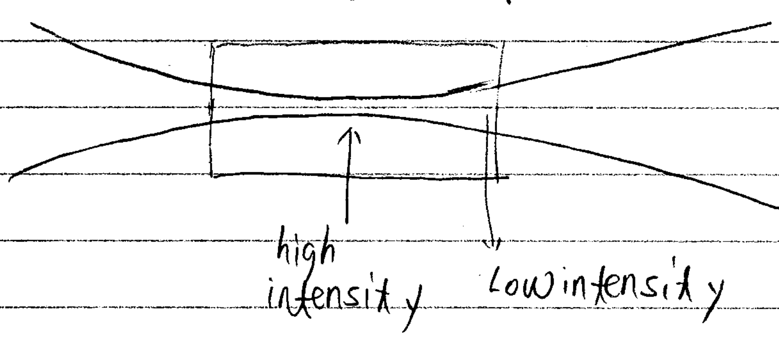
L limited by two effects:

* phase - matching accuracy becomes difficult
* Beam walk off: in birefrigent crystal light at ω & 2ω diverges

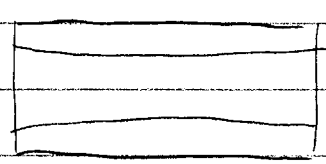
weird effect, consider in HW.

Practical limit, $L \lesssim 2.5\,cm$

Also a limit to W_0: if focus beam too tightly it diverge rapidly



Optimum W_0 has $z_0 = \frac{\pi W_0^2}{\lambda} = \frac{L}{2}$.



Then $\pi W_0^2 = \frac{\lambda L}{2}$ which λ? largest one.

corresponding ω_{min} is the smallest frequency.

Then $$\Lambda = \frac{2\pi\, c}{n_{min}\, \omega_{min}}$$

$$w_0^2 = \frac{c_0 L}{n_{min}\, \omega_{min}}$$

and optimum power

$$P(\omega_3) \approx \frac{8}{\pi} \eta_0^3 \frac{n_{min}}{n_1 n_2 n_3} \omega_3^2\, \omega_{min} \frac{L}{c_0} d'^2 P(\omega_1) P(\omega_2)$$

Simplifies for SHG

$$\omega_1 = \omega_2 = \omega$$
$$\omega_3 = 2\omega$$
$$n_1 = n_2 = n_3$$

$$P(2\omega) = \frac{32}{\pi} \eta_0^3 \frac{\omega^2 L}{n^2 c_0} d'^2 P(\omega)^2$$