Output power of 2^{nd} order process

$$P(\omega_3) = \frac{8}{\pi} \eta_0^3 \frac{n_{min}}{n_1 n_2 n_3} \omega_3^2 \omega_{min} \frac{L}{C_0} d'^2 P(\omega_1) P(\omega_2)$$

ω_{min} = lowest of $\omega_1, \omega_2, \omega_3$

$n_{min} = n(\omega_{min})$

(optimum focus)

Recall, this fomula is only valid for $P_3 \ll P_1, P_2$ but what if we want to relax this constraint.

Have DE's for electric fields

say $\omega_3 = \omega_1 + \omega_2$

$$\frac{dE(\omega_3)}{dz} = -i \frac{\omega_3}{n_3} \eta_0 d' E(\omega_1) E(\omega_2) \qquad \omega_3 = \omega_1 + \omega_2$$

$$\frac{dE(\omega_2)}{dz} = -i \frac{\omega_2}{n_2} \eta_0 d' E^*(\omega_1) E(\omega_3) \qquad \omega_2 = \omega_3 - \omega_1$$

$$\frac{dE(\omega_1)}{dz} = -i \frac{\omega_1}{n_1} \eta_0 d' E^*(\omega_2) E(\omega_3) \qquad \omega_1 = \omega_3 - \omega_2$$

Normolize. define

$$a_q = \left(\frac{n_q}{2\hbar \omega_q \eta_0}\right)^{1/2} E(\omega_q)$$

Then $|a_q|^2 = \frac{n_q}{2\hbar\omega_q \eta_0} |E(\omega_q)|^2$

but $I_q = \frac{n|E|^2}{2\eta_0}$

So $|a_q|^2 = \frac{I(\omega_q)}{\hbar\omega_q}$ = photon flux density

Normalize, get

$$\frac{da_3}{dz} = -i\left(2\hbar \frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}\eta_0^3\right)^{1/2} d' a_1 a_2$$

Define $g = \left(2\hbar \frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}\eta_0^3\right)^{1/2} d'$

Then $\frac{da_3}{dz} = -ig\, a_1 a_2$

also $\frac{da_2}{dz} = -ig\, a_1^* a_3$

$$\frac{da_1}{dz} = -ig\, a_2^* a_3$$

Can't solve in general, but can solve in limit:

(1) ω_1, ω_2 are input

and $a_1 \sim a_2 \gg a_3$

so take a_1 & a_2 = const $\frac{da_1}{dt} \approx \frac{da_2}{dt} \approx 0$

$$\frac{da_3}{dz} = -i g a_1 a_2.$$

up - conversion

(2) if a_1 & a_2 input,

but $a_2 \gg a_1, a_3$

Take $a_2 \approx$ constant

define $\gamma = g a_2$

$$\frac{da_3}{dz} = -i\gamma a_1$$

$$\frac{da_1}{dz} = -i\gamma^* a_3$$

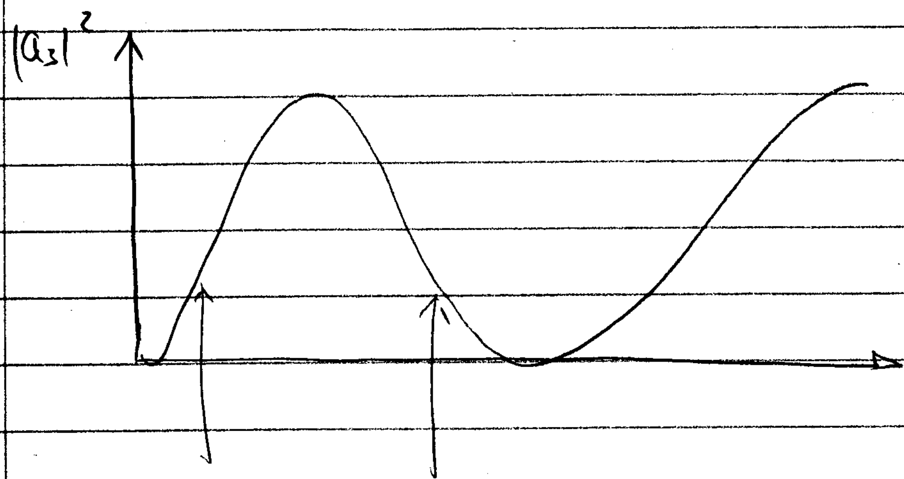
so $$\frac{d^2 a_3}{dz^2} = -i\gamma(-i\gamma a_3) = -\gamma^2 a_3$$

$$a_3(z) = a_3(0)\cos\gamma z + \frac{1}{\gamma}\left.\frac{da_3}{dz}\right|_0 \sin\gamma z$$

$$= a_3(0)\cos\gamma z - i\, a_1(0)\sin\gamma z$$

So if $a_3(0) = 0$, $\boxed{a_3(z) = -i\, a_1(0)\sin\gamma z}$

oscillates



$\omega_1 + \omega_2 \to \omega_3$
sum frequency

$\omega_3 - \omega_2 \to \omega_1$
Differen freq gen

power goes back to ω_1

Because if $\omega_3 = \omega_1 + \omega_2$ is phase matched so is $\omega_1 = \omega_3 - \omega_2$.

Max conversion at $\Gamma Z = \frac{\pi}{2}$

$$|a_3|^2 = |a_1(0)|^2$$

$$\frac{I_{out}(\omega_3)}{\hbar\omega_3} = \frac{I_{in}(\omega_1)}{\hbar\omega_1}$$

Each photon at ω_1 is converted to a photon at ω_3. sensible quantum result.

So nonlinear response can be effective in converting

(3) ω_1 & ω_3 input. $\omega_3 = \omega_1 + \omega_2$

$a_3 \gg a_1 \; a_2$

Define $\gamma = g a_3$

$$\frac{da_1}{dz} = -i\gamma a_2^*$$

$$\frac{da_2}{dz} = -i\gamma a_1^*$$

$$\frac{da_1}{dz} = -i\gamma a_2^*$$

$$\frac{da_2}{dz} = -i\gamma a_1^*$$

$$\frac{d^2 a_1}{dz^2} = -i\gamma(i\gamma^* a_1) = +|\gamma|^2 a_1$$

solution (for $a_2(0) = 0$.)

$$a_1(z) = a_1(0) \cosh \gamma z$$
$$a_2(z) = -i a_1(0) \sinh \gamma z$$

Both grow exponentially with z, $\rightarrow$ until they get comparable to a_3, then more complicated.

This process, using ω_3 to amplify ω_1, is regarded as OPA (optical parametric amplifier.)

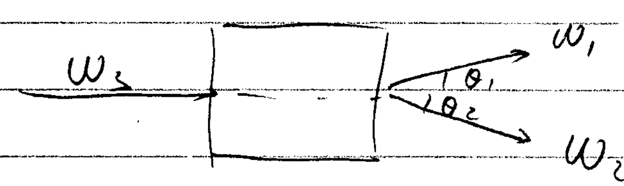
(5.) What if ω_3 is the only input?

that is $\quad a_1(0) = a_2(0) = 0$

QM say that can have $\omega_3 \to \omega_1 + \omega_2$
even for no input of ω_1
spontaneous parametric fluorescence

Explanation: zero point energy acts like ≈ 1 photon in every mode of field
So always an effective input of about one photon

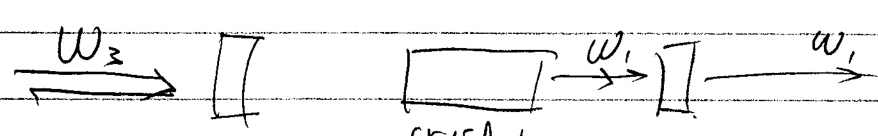
General picture:



Typically occurs with non-collinear phase-matching.

For any $\omega_1 + \omega_2 = \omega_3$ can find angles θ_1 & θ_2 so that $\vec{k}_1 + \vec{k}_2 = \vec{k}_3$

Put into a cavity



crystal

The eigenmode of the cavity, whose frequency has the right phase-match, will start oscillate (OPO).
(optical parametric oscillator)