01/28/05

# Lecture 5



Wave approach to laser beam:

Approximate wave equation by paraxial Helmholtz equation:

$$\vec{E} = \hat{\varepsilon} A(\vec{r}) e^{-ikz} e^{i\omega t}$$

$$\nabla_T^2 A - 2ik \frac{\partial A}{\partial z} = 0$$

Found a solution

$$A(\vec{r}) = \frac{A_1}{q} e^{-ik\frac{\rho^2}{2q}}.$$

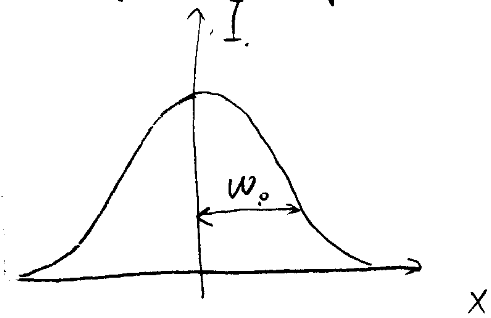
Here. $q = z + iz_0$ · complex

$$A(\vec{r}) = \frac{A_1}{z+iz_0} e^{-\frac{kz_0\rho^2}{2(z^2+z_0^2)}} e^{-ik\frac{\rho^2 z}{2(z^2+z_0^2)}}$$

Then $I(\vec{r}) \propto |A|^2 = \frac{A_1^2}{z^2+z_0^2} e^{-\frac{kz_0\rho^2}{z^2+z_0^2}}$

At $z = 0$ $= \frac{|A_1|^2}{z_0^2} e^{-\frac{k\rho^2}{z_0^2}}$

Gaussian function of $\rho$




Define $1/e^2$ radius = beam waist = $W_0$

Then $I \sim e^{-\frac{2\rho^2}{W_0^2}}$

$$W_0^2 = 2\frac{z_0}{k} = 2\frac{z_0}{2\pi/\lambda} = \frac{\lambda z_0}{\pi}$$

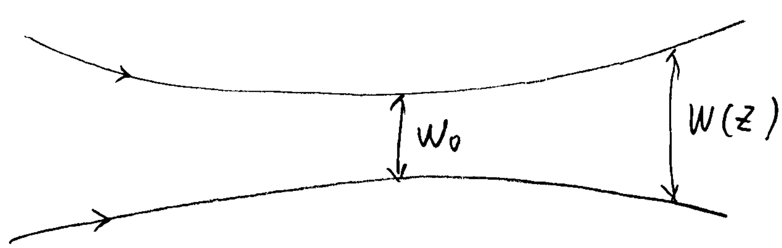
or $\boxed{z_0 = \frac{\pi W_0^2}{\lambda}}$ Rayleigh length.

At any other $z$, write

$$I(z) = I_0 \frac{W_0^2}{w(z)^2} e^{-\frac{2\rho^2}{W(z)^2}}$$

where $w(z)^2 = W_0^2 \left(1 + \frac{z^2}{z_0^2}\right)$

So intensity looks like



Describes a laser beam focassed at $z=0$

Called a "Gaussian beam"

Good model for a laser beam.

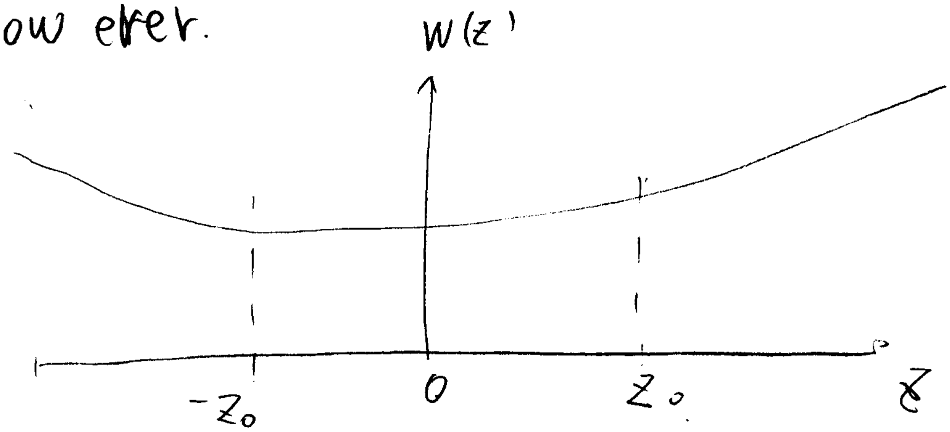
Look at properties of this beam.

a) Beam has focus at $z=0$.

$z<0$, beam is converging to focus
$z>0$, beam is diverging to focus.

Laser seems like a collimated beam w/o diverging or converging, but due to their wave nature, lasers have to have diffraction

However.




looks collimated.

At $z = -Z_0$ to $Z_0$

$W(z)$ ranges from $W_0$ to $\sqrt{2} W_0$

Over Rayleigh range $-z_0 < Z < Z_0$

beam is considered collimated.

Since $z_0 = \frac{\pi W_0^2}{\lambda}$, can be pretty big

$W_0 = 1 \text{ mm}$ $\lambda = 632 \text{ nm}$

$\underline{Z_0 = 5 \text{ m}}$

makes it easy to not notice divergence

b) Gaussian beam defined by two parameters

Rewrite solution.

$$u(\vec{r}) = \frac{A_1}{q} e^{-ik\frac{\rho^2}{2q}} e^{-ikz}.$$

Then $$\frac{1}{q} = \frac{1}{z+iz_0} = \frac{z-iz_0}{z^2+z_0^2}.$$

* Im part $$-\frac{z_0}{z^2+z_0^2} = \frac{1}{z_0}\frac{1}{1+\frac{z^2}{z_0^2}}$$

$$= \frac{1}{z_0^2}\frac{W_0^2}{W(z)^2}.$$

But $W_0^2 = z_0\lambda/\pi$.

so $$\text{Im}\,\frac{1}{q} = -\frac{\lambda}{\pi W(z)^2}$$

* Also define $$\text{Re}\,\frac{1}{q} = \frac{z}{z^2+z_0^2} = \frac{1}{R(z)}$$

Then $$e^{-ik\frac{\rho^2}{2q}} = e^{-k\frac{\rho^2\lambda}{2\pi W(z)^2}} e^{-ik\frac{\rho^2}{2R(z)}}$$

$$= e^{-\frac{\rho^2}{W(z)^2}} e^{-ik\frac{\rho^2}{2R(z)}}$$

⑥

$e^{-\frac{\rho^2}{W(z)^2}}$ gives amplitude, Gaussian spot.

$e^{-ik\frac{\rho^2}{2R(z)}} e^{-ikz}$ give phase

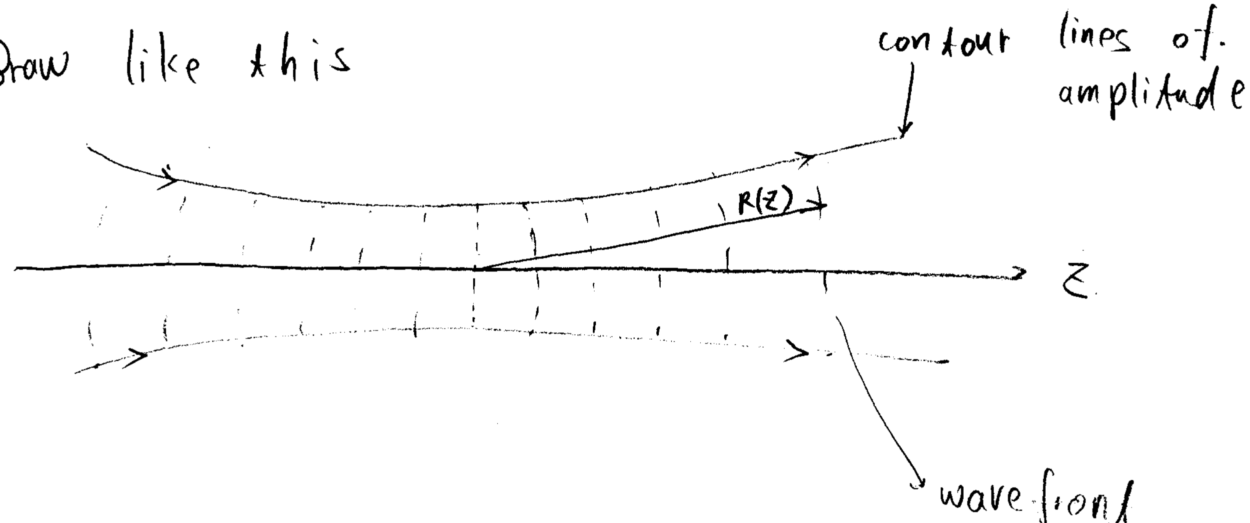
↳ Look like paraxial part of a spherical wave.

spherical wave $\sim e^{-ikr}$ $r = \sqrt{\rho^2 + z^2}$

$= z + \frac{\rho^2}{2z}$

$R(z)$ : distance to center of curvature.

Draw like this




At focous $R = \frac{z^2 + z_0^2}{z} = \infty$

wave front is flat

Far from focus, $z \gg Z_0$   $R \to z$.

Wave front centered at focus

So near focus, laser beam looks a plane wave. far from focus. ~ spherical wave centered at focus

Note Laser beam is completely specified by two parameters. $W(z)$ and $R(z)$.

or by complex curvature $q$

Give $W$. $R$, we have

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi W^2},$$ so $q$ is determined

Interesting properties:

$q(z) = z + iZ_0$   describe beam in terms of focus location $(z)$ & beam waist ($Z_0$ or $W_0$)

while $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi W^2}$

describes beam in terms of local properties

local width: $W$

local radius of curvature $R$.

One more contribution to phase

$$u(\vec{r}) = \frac{A_1}{q} e^{-\frac{\rho^2}{W(z)^2}} e^{-ik(z + \frac{\rho^2}{2R})}$$

$\frac{1}{q}$ in front has phase too:

$$\frac{1}{q} = \frac{1}{z + iz_0} = \frac{z - iz_0}{z^2 + z_0^2}.$$

by convention

$$= -i\frac{z_0 + iz}{z^2 + z_0^2}$$

$$= \frac{1}{\sqrt{z^2 + z_0^2}} e^{-i\frac{\pi}{2}} e^{i\zeta}$$

where $\tan\zeta = \frac{z_0}{z_0}$

or $$\frac{1}{q} = \frac{e^{-i\frac{\pi}{2}}}{W_0} \frac{W_0}{w(z)} e^{i\zeta(z)}$$

Let us write.

$$u(\vec{r}) = A_0 \frac{W_0}{W(z)} e^{-\frac{\rho^2}{W(z)^2}} e^{-ik\left(z + \frac{\rho^2}{R(z)}\right)} e^{i\zeta(z)}$$

$$A_0 = A_1 \frac{e^{-i\pi/2}}{W_0}$$

$\zeta$ called Guoy phase.

varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.