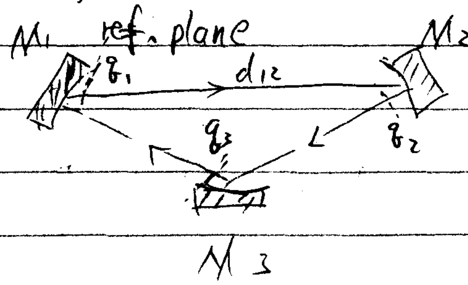
Lecture 8

Multiple mirror cavities

With several mirrors beam in each "leg" of cavity will be different



Say pick up ref plane as shown

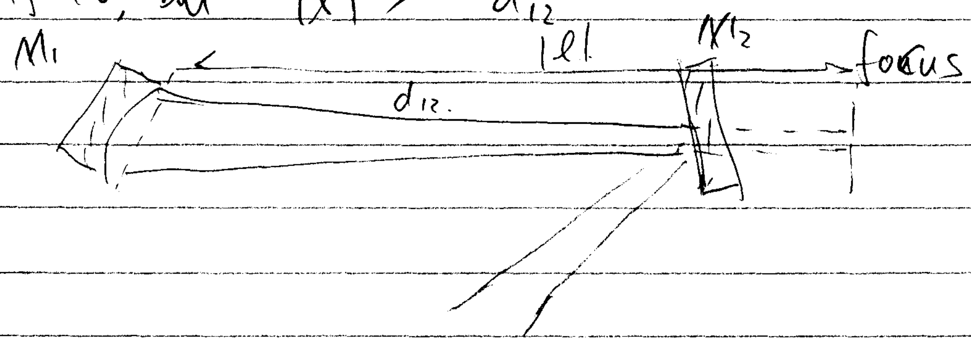
Get $M_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ = round trip

then $q_1 = \frac{1}{C}\left[\frac{A-D}{2} \pm i\sqrt{1-\left(\frac{A+D}{2}\right)^2} \right.$

$= z + i z_0$

see focus point $\frac{A-D}{2C} = l$ (w/o mirror)

if $l<0$, but $|l| > d_{12}$



if $R_2 = \infty$, plane mirror.

the focus point will still be $-l$ ahead but folded.

But if curved mirror, need to account for mirror to find focus

Now to get q_2: use.

$$M' = \begin{bmatrix} 1 & 0 \\ \frac{2}{R_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_{12} \\ \frac{2}{R_2} & 1 + \frac{2d_{12}}{R} \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

and $q_2 = \dfrac{A' q_1 + B'}{C' q_1 + D'}$

Similarly $q_3 = \dfrac{A'' q_2 + B''}{C'' q_2 + D''}$

for $M'' = \begin{bmatrix} 1 & 0 \\ \frac{2}{R_3} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_{23} \\ 0 & 1 \end{bmatrix}$.

Calculation of multiple mirror cavity.

① Choose reference plane, calculate round trip matrix. M_1

② use M_1 to find the beam parameter q_1 of the "leg" which includes the reference plane.

③ use partial matrix to propagate q_1 into other legs one by one, ...

To find a mode of cavity, not only does q have to be self-consistent, really need wave $u(\vec{r})$ to be self consistent.

$$u = \frac{A_1}{q} e^{-ik\frac{\rho^2}{2q}} e^{-ikz}$$

$$= A_0 \frac{W_0}{W(z)} e^{-\frac{\rho^2}{W(z)^2}} e^{-ik\left(z + \frac{\rho^2}{2R(z)} + i\zeta(z)\right)}$$

We insisted that q repeat itself after round trip but this doesn't account for accumulated phase.

two contributions e^{-ikz} and $e^{i\zeta(z)}$

So in one round trip, acquire phase

$$\phi_1 = kL - \sum_{\text{legs } i} \Delta\zeta_i \equiv kL - \Delta\zeta$$

where L = round trip length of optic axis

$\Delta\zeta_i$ = Guoy phase acquired on leg i.
$= \zeta(z_{i+1}) - \zeta(z_i)$

for mirrors at z_i, z_{i+1}

Example: symmetric cavity

first leg: $\Delta\zeta_1 = \tan\frac{z_2}{z_0} - \tan\frac{z_1}{z_0}$

z_1, z_2: mirror locations, $z_1 = -\frac{d}{2}$, $z_2 = +\frac{d}{2}$

$$\Delta\zeta_1 = 2\tan\frac{d}{2z_0}$$

2^{nd} leg : $\Delta\xi_2 = \tan\frac{z_1}{z_0} - \tan\frac{z_2}{z_0}$

but, relative to reflected beam

$z_1 = +\frac{d}{2}$ (mirror after focus)

$z_2 = -\frac{d}{2}$ (mirror before focus)

$$\Delta\xi_2 = 2\tan\frac{d}{2z_0} = \Delta\xi_1$$

$$\text{so } \Delta\xi = 4\tan\frac{d}{2z_0}$$

For more complex cavities, each leg has different $\Delta\xi_i$, add up all to get $\Delta\xi$

To ensure u is self consistent, need

$e^{-i\phi_1} = 1$ so $kL - \Delta\xi = 2\pi n$

for integer n

This restricts k

$$k_n = \frac{2\pi n + \Delta\xi}{L}$$

So frequencies restricted

$$\nu_n = \frac{ck}{2\pi} = \frac{c}{L}\left(n + \frac{\Delta\xi}{2\pi}\right) \equiv \nu_F\left(n + \frac{\Delta\xi}{2\pi}\right)$$

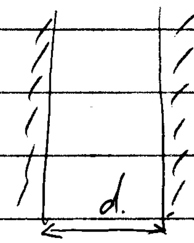
where $\nu_F = \frac{c}{L}$ = Free spectral range.

Recall simple example: two planar mirrors

Then modes are plane waves.

$$\Delta\zeta = 0$$

So $\nu_n = \frac{c}{2d}\, n$



or $\frac{c}{\lambda_n} = \frac{c}{2d}\, n \Rightarrow d = n \cdot \frac{\lambda}{2}$

integer # of half waves

$\Delta\zeta$ is correction for Gaussian beam.

$\nu_n = \nu_F \left(n + \frac{\Delta\zeta}{2\pi}\right)$ is generalization to arbitrary cavity

often don't really care about absolute frequency ν_n, so much as mode spacing:

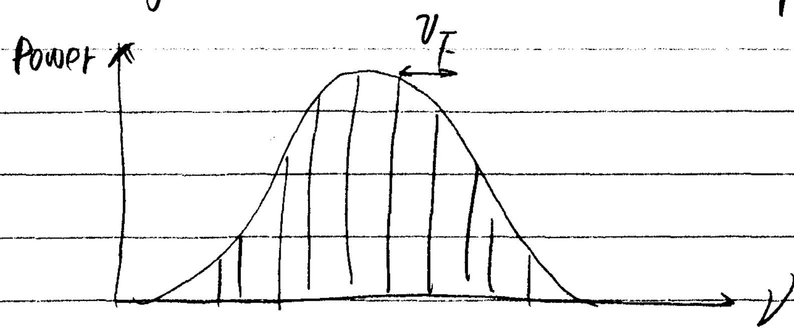
$$\Delta\nu = \nu_F = \frac{c}{L}$$ very simple.

n labels "longitudinal modes"

ν_F: longitudinal mode spacing.

So if we build a laser.

take light source with some spectral width.



enclose in cavity, only get output at mode freqs. ν_n.