02/07/05 Lecture 9 ①

Inside a laser cavity, frequencies are restricted to

$$\nu_n = \frac{c}{L}\left(n + \frac{\Delta\varphi}{2\pi}\right)$$

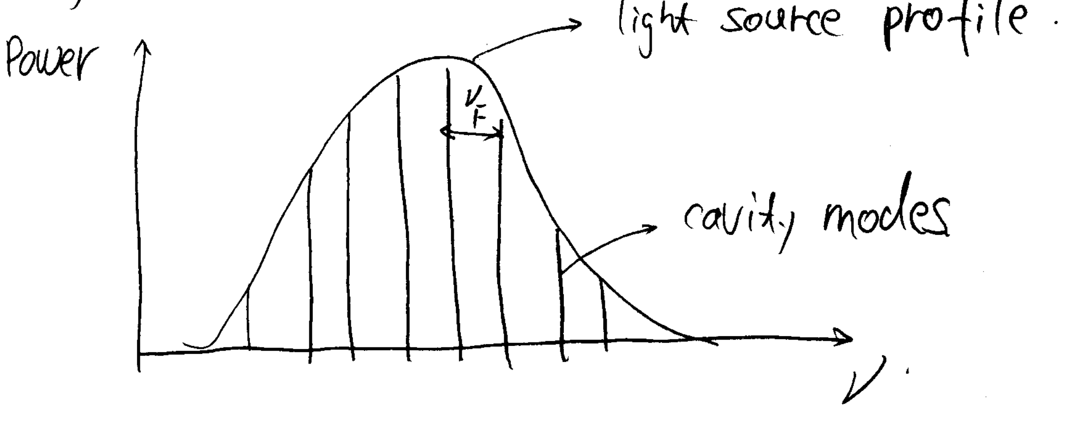
$$\equiv \nu_F\left(n + \frac{\Delta\varphi}{2\pi}\right)$$

where $\nu_F = \frac{c}{L}$: free spectral range.

longitudinal mode spacing.

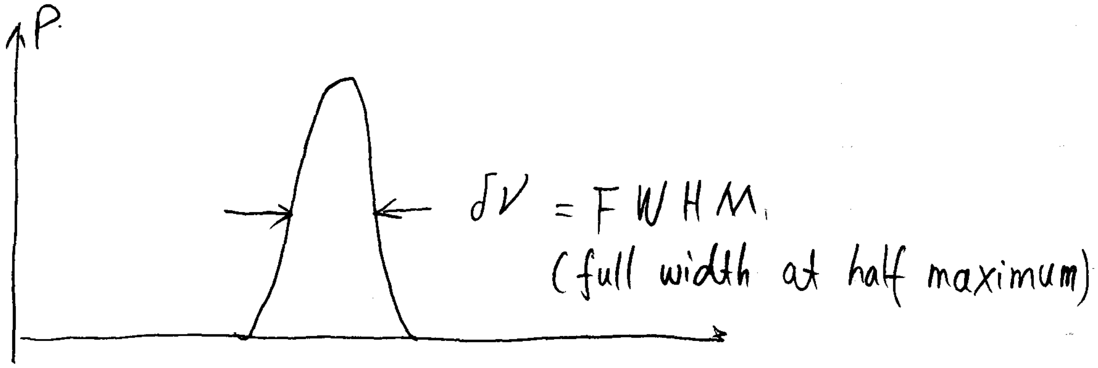
n : labels "longitudinal modes."

So in a laser cavity, even if the input light source has very broad spectral profile, the only output from the cavity would be mode frequencies ν_n ;



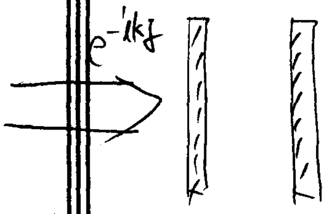
Next question: what is the width of these modes?

Zoom in



The modes are not infinitely narrow, but have finite width, this is due to the finite loss of the cavity.

Recall from optics, for Fabry-Perot resonator:



mirrors have reflectance coefficient, r. (for amplitude)

So $U_{refl} = r U_{inc}$

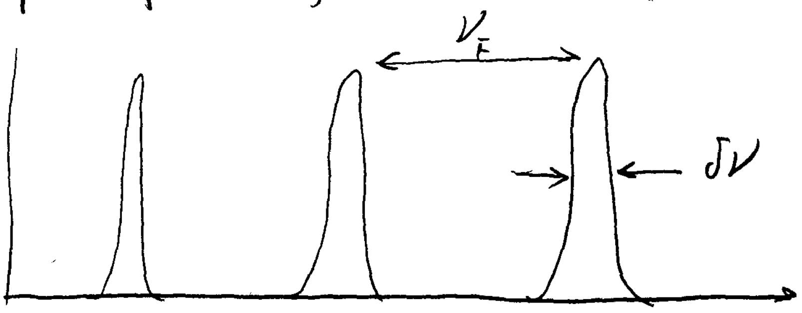
So Reflectance (intensity) $R = |r|^2$.

The transmission of the cavity is

$$I_{out} = \frac{I_{max}}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\pi\nu}{\nu_F}\right)}$$

See discussion pgs 70 - 72, 316 - 317

The output of Fabry-Perot cavity looks like



finesse $\mathcal{F}$ of the cavity is defined as:

ratio of peak spacing to width:

$$\mathcal{F} = \frac{\nu_F}{\delta\nu}$$

Particularly, for Fabry-Perot cavity.

$$\mathcal{F} = \frac{\pi r^{1/2}}{1-r}.$$

when $r \approx 1$, $\mathcal{F} = \frac{\pi}{1-r}$ is large.

To generalize, finesse $\mathcal{F}$ of a cavity is always the ratio of mode spacing to mode width. However, for more complicated cavity, the finesse $\mathcal{F}$ cannot be calculated directly like Fabry-Perot cavity.

Several other parameters can be used to characterize the loss of a cavity.

Q : quality factor of resonator.

define : $Q = \frac{\nu}{\delta\nu}$ ← frequency of the mode

↑ width of the mode

Can show that

$$Q = \frac{2\pi\nu\mathcal{E}}{(-d\mathcal{E}/dt)}$$

where $\mathcal{E}$ = energy stored in cavity

$-\frac{d\mathcal{E}}{dt}$: rate energy lost from cavity.

Define cavity decay time τ_p, also τ_p = photon lifetime

since $\frac{d\mathcal{E}}{dt} = -\frac{\mathcal{E}}{\tau_p}$

Then $\boxed{Q = 2\pi\nu\tau_p}$

and $\boxed{\delta\nu = \frac{1}{2\pi\tau_p}}$

That is the width of cavity mode is the inverse of photon lifetime in a cavity. Make sense.

Moreover:

finesse $\mathcal{F} = \dfrac{\nu_F}{\delta\nu}$, $Q = \dfrac{\nu}{\delta\nu}$

so $Q = \dfrac{\nu}{\nu_F}\mathcal{F}$

Also. Γ = fractional energy loss per round trip

$$\Delta\mathcal{E}\Big|_{\substack{\text{round}\\\text{trip}}} = -\Gamma\mathcal{E}$$

but round trip time is $\dfrac{L}{c} = \dfrac{1}{\nu_F}$

so $\Delta\mathcal{E} = \dfrac{d\varepsilon}{dt}\dfrac{1}{\nu_F}$

then $\Gamma = \dfrac{1}{\tau_p \cdot \nu_F}$

Finally, define "distributed loss coefficient α

by $\Gamma = e^{-\alpha L}$

$$\alpha = \frac{1}{L}\ln\frac{1}{\Gamma}$$

Characterize cavity losses by:

	Typical values in a laser
$\delta\nu$ = FWHM of mode (linewidth)	1 MHz
$\mathcal{F}$ = finesse = $\nu_F/\delta\nu$	100
Q = Qulity factor = $\nu/\delta\nu$	10^9
τ_p = photon lifetime = $\frac{1}{2\pi\,\delta\nu}$	1 μs
Γ = loss per pass = $2\pi\,\delta\nu/\nu_F$	5%
α = distributed loss = $\frac{1}{L}\ln\frac{1}{\Gamma}$	10^{-3} cm^{-1}

Γ is usually easiest to calculate

common sources of loss:

i) Imperfect mirrors

Reflectances $R_1, R_2, \cdots R_N$ (intensity, not amplitude)

then loss: $\Gamma = 1 - R_1 R_2 \cdots R_N$

Need at least one mirror partially transmitting to let light out: "output coupler", ~95%

other mirrors typically $\simeq 99.5\%$

2) Absorption by laser medium, other intracavity elements
usually characterized by α.

3) Loss of light around edges of mirror:
G. beams really extend to ∞ $I \propto e^{-2\rho^2/W^2}$

so for finite mirror size, some light escapes: diffraction losses.

For mirror diameter $\simeq \pi W^2$, get about 1% loss

High order modes:

The Gaussian beam being discussed is the lowest order mode, that is, u repeats itself after one round trip. This mode is called TEM00

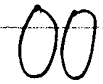
High order modes are still confined in the cavity but where u repeats itself after two or more round trips.

The higher order modes are usually labeled by l, m. called TEM_{lm}.

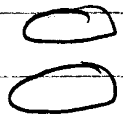
The higher modes usually have multiple spots

TEM_{00}

TEM_{10}



TEM_{01}



TEM_{11}