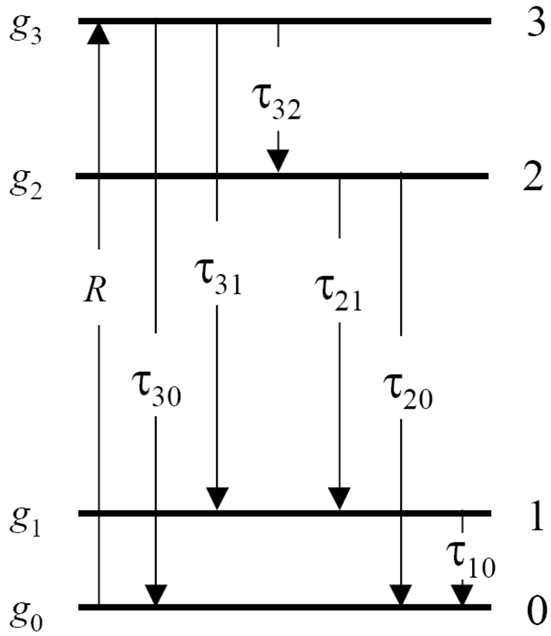
1. **Absorption Coefficient:** Sodium atoms have a doublet of excited states labeled $3p_{1/2}$ and $3p_{3/2}$ that are connected to the $3s_{1/2}$ ground state by transitions at 589.6nm and 589.0nm respectively. The excited states have a radiative lifetime of 16ns and degeneracies of 2 ($3p_{1/2}$) and 4 ($3p_{3/2}$), while the ground state has degeneracy 2. At a temperature of 400K, Na vapor has a number density $N = 3 \times 10^{11}$ cm^{-3} and the collision rate between atoms is of order 1000 s^{-1}. Identify the dominant source of line broadening in this case, and then find the peak absorption coefficient for each of the two lines.

2. **Rate Equations:** Use the rate equations to calculate the population inversion $\Delta N_0 = N_2 - (g_2/g_1)N_1$ for the four-level system shown. The only transition being driven is the $0 \leftrightarrow 3$ pump transition, and you can assume that the pump rate is low enough to neglect depletion of state 0. Pumping can therefore be characterized by the total rate R (units of (atoms/s) /m^3), as shown. Feel free to simplify expressions by using the total lifetimes τ_i for each state. For instance,

$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}. \tag{1}$$

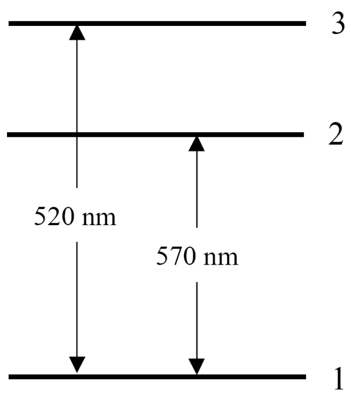


3. **Gain Coefficient:** Suppose that the system of the above problem describes a solid state laser medium with the following data:

$\lambda_{03} = 500$ nm	$\lambda_{12} = 800$ nm	$\Delta\nu_{03} = 5 \times 10^{13}$ Hz
$\Delta\nu_{12} = 10^{11}$ Hz	$\tau_{30} = \tau_{32} = 100$ ns	$\tau_{31} = 2$ ms
$\tau_{21} = 1$ ms	$\tau_{20} = 1$ ms	$\tau_{10} = 25$ ns
$g_0 = g_1 = 1$	$g_2 = 2$	$g_3 = 4$

Take the $0 \leftrightarrow 3$ and $1 \leftrightarrow 2$ transitions to be radiative, so that τ_{ij} gives the appropriate spontaneous emission lifetime. The medium consists of ions in a 1-cm long crystal, where the total density of ions in $N_a = 10^{19}$ cm^{-3} and the index of refraction of the host crystal is $n = 1.7$. What pumping intensity on the $0 \leftrightarrow 3$ transition is required to achieve a total gain through the crystal of 2.5? Neglect any saturation effects.

4. **Gain Characterization via Absorption Coefficients:** Suppose a three-level gain medium has the structure shown (assume equal degeneracies). Transitions from level 3 to level 2 proceed via collisions at a rate of 10^{12} s^{-1}. This gives an elastic collision rate of 10^{12} s^{-1} for atoms in level 2 as well, which is the dominant source of line broadening. The density of the medium is $N_a = 10^{17}$ cm^{-3}, and the host index is 1.3. When the medium is in the ground state, the peak absorption coefficients for $1 \rightarrow 3$ and $1 \rightarrow 2$ transitions are 10 cm^{-1} and 0.1 cm^{-1} respectively.
(a) What is the spontaneous lifetime for the $2 \rightarrow 1$ transition?
(b) A 10-cm long section of this medium is placed in a two-mirror cavity with $T = L = 0.05$, and the pump transition is driven with monochromatic light of intensity I_p. (Assume the pump light enters from the side of the medium, and that the medium is thin enough that reduction in pump intensity due to absorption is negligible.) What is the threshold pump intensity for laser oscillation?
(c) At this pump intensity, what is the saturation intensity for the $2 \rightarrow 1$ transition?
(d) If the pump intensity is twice the threshold intensity, and the beam width in the medium is 100 μm, estimate the output power produced.



5. **Number of Longitudinal Modes:** Suppose a gas laser uses Ar^+ ions at a temperature of 2000K, leading to a Doppler broadened gain profile. The laser wavelength is 514 nm, and it uses a two-mirror resonator of length 0.5 m. The gas density is low, so the index of refraction is 1. Estimate the number of longitudinal modes that will oscillate if the small signal gain is 1.5 times the total cavity loss coefficient.