Phys 532/822 Assignment 4 Due February 25

1. **Stimulated vs. Spontaneous Emission:** (a) An atom contains two energy levels connected by a transition with wavelength $\lambda_0 = 0.7\mu$m, spontaneous lifetime $t_{sp} = 3$ ms, and a Lorentzian lineshape with width $\Delta\nu = 50$GHz. The atom is prepared in the upper state, and is placed in a resonator with volume $V = 100$ cm^3 that has a cavity mode at the center frequency ν_0. How many photons must be present in this cavity mode so that the rates for stimulated and spontaneous emission are equal.

   (b) If an atom in free space has a transition with frequency ν_0, spontaneous lifetime t_{sp}, and light scattering cross section $\sigma(\nu)$, find a simple expression for the intensity required to make the stimulated transition rate W equal the spontaneous emission rate.

   822 students only

2. **Emission into a Laser Cavity:** Equation (12.2-1),(12.2-3), and (12.2-5) can only be applied to closed cavities with well-defined volumes V. In a laser cavity, confined modes can be characterized by their intensity distribution $I(\mathbf{r})$, and the photon interaction rates can be more generally expressed as, for example,

$$p = \frac{I(\mathbf{r}) c \sigma(\nu)}{\int I(\mathbf{r}) dV} \qquad (1)$$

   for spontaneous emission. Consider an atom positioned at the center of a symmetric cavity with mirror spacing d and radii of curvature such that the TEM$_{00}$ (Gaussian mode) beam waist is W_0. The atom is in the upper state of a transition with oscillator strength S. Calculated the probability that a particular photon emitted by the atom enters a TEM$_{00}$ mode. You can assume that the free spectral range of the cavity is small compared to the atomic transition linewidth and that the total spontaneous emission rate P_{sp} is not altered by the cavity.