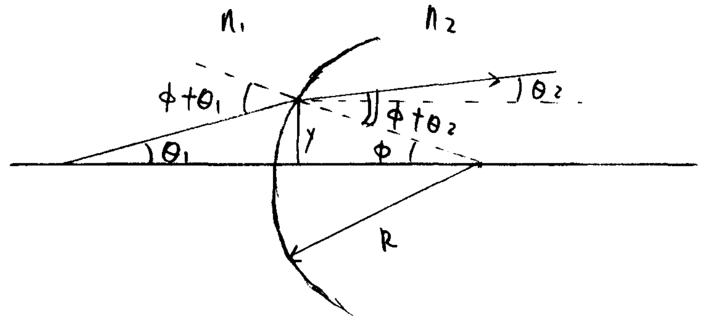
1. a)



Apparently $y_1 = y_2 = y$, so $A = 1$ $B = 0$

Use Snell's law to calculate θ_2

Angle of incident : $\phi + \theta_1$
Angle of refraction : $\phi + \theta_2$

$$n_1 \sin(\phi + \theta_1) = n_2 \sin(\phi + \theta_2)$$

In paraxial approximation: $\phi, \theta_1, \theta_2 \ll 1$

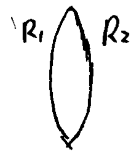
so, $\phi = \frac{y}{R}$, and $n_1\phi + n_1\theta_1 = n_2\phi + n_2\theta_2$

$$\theta_2 = \frac{n_1 - n_2}{n_2}\phi + \frac{n_1}{n_2}\theta_1$$
$$= \frac{n_1 - n_2}{n_2 R} y + \frac{n_1}{n_2}\theta_1$$

so $C = \frac{n_1 - n_2}{n_2 R}$, $D = \frac{n_1}{n_2}$

$$\boxed{M = \begin{bmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}}$$

b)



Take $n=1$ outside lens
$n=n$ inside

total matrix

$$M = M_{R_2} \cdot M_{R_1}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1-n}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{n-1}{nR_1} & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{n-1}{R_2} + \frac{1-n}{R_1} & 1 \end{bmatrix}$$

Compare to matrix for lens $\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

so $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

2. For a stable trajectory, after m round trips, have

$$\begin{cases} y_m = y_{max} \sin(m\phi + \phi_{oy}) \\ x_m = x_{max} \sin(m\phi + \phi_{ox}) \end{cases}$$

where (x_m, y_m) is position on mirror at mth bounce.

and $\phi = \cos^{-1}\left(\frac{A+D}{2}\right)$

for single round trip matrix: $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+\frac{2d}{R} & d \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} 1+\frac{2d}{R} & d \\ \frac{2}{R} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+\frac{2d}{R})^2 + \frac{2d}{R} & (1+\frac{2d}{R})d + d \\ (1+\frac{2d}{R})(\frac{2}{R}) + \frac{2}{R} & \frac{2d}{R} + 1 \end{bmatrix}$$

so $\frac{A+D}{2} = \frac{1}{2}\left[(1+\frac{2d}{R})^2 + \frac{2d}{R} + 1 + \frac{2d}{R}\right]$

$$= b = 1 + \frac{4d}{R} + \frac{2d^2}{R^2}$$

To be periodic, need $m\phi = 2\pi s$

for some integers $\underline{m \;\&\; s}$

Here we want $m=10$, so beam exit on 10^{th} bounce.

Also need s relatively prime to m, otherwise beam will exit early, for example, if $s=2$, then have.

$$5\phi = 2\pi$$

so beam retraces path after 5 bounces, that is, under this configuration, beam will exit after 5 round trips.

This rules out $s = 0, 2, 4, 5, 6, 8, 10$

Also since $\phi = \cos^{-1}\frac{A+D}{2}$, take $0 \leq \phi < \pi$

so, $$2\pi s = m\phi < m\pi$$

$$s < \frac{m}{2} = 5$$

This leaves $s = 1$ and 3.

for $s=1$:

$$\phi = 2\pi \times \frac{1}{10}, \quad b = \cos\phi = 0.809$$

so $$1 + \frac{4d}{R} + \frac{2d^2}{R^2} = b$$

$$\Rightarrow \quad (2d^2)\cdot\frac{1}{R^2} + (4d)\frac{1}{R} + (1-b) = 0$$

$$\frac{1}{R} = \frac{1}{4d^2}\left[-4d \pm \sqrt{16d^2 - 8d^2(1-b)}\right]$$

$$\frac{1}{R} = \frac{1}{d}\left[-1 \pm \sqrt{1-\frac{1}{2}(1-b)}\right]$$

$$= \frac{1}{d}\left[-1 \pm \sqrt{\frac{b+1}{2}}\right]$$

Here $\frac{1}{R} = \frac{1}{d}\begin{cases} -0.0489 \\ -1.9511 \end{cases}$

$\Rightarrow$ $R = \begin{cases} -409\text{ cm} \\ -10.25\text{ cm} \end{cases}$

for $s = 3$

$\phi = 2\pi \times \frac{3}{10}$, $b = \cos\phi = -0.309$

$$\frac{1}{R} = \frac{1}{d}\left[-1 \pm \sqrt{\frac{b+1}{2}}\right]$$

$$= \frac{1}{d}\begin{cases} -1.588 \\ -0.412 \end{cases}$$

$\Rightarrow$ $R = \begin{cases} -12.6\text{ cm} \\ -48.5\text{ cm} \end{cases}$

So possible R's are $\boxed{-10.25\text{cm},\ -12.6\text{ cm},\ -48.5\text{cm},\ -409\text{cm}}$