Phys 532/822 HW # 3 Solutions

1. (a) starting just before mirror R_1, the matrix of the cavity is

$$M_1 = \begin{bmatrix} 1 & 570 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 660 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{2}{150} & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 220 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{2}{100} & 1 \end{bmatrix}\begin{bmatrix} 1 & 130 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{2}{100} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.256 & 1450.8 \text{ mm} \\ -0.0019 \text{ mm}^{-1} & 1.36 \end{bmatrix}$$

Cavity is stable, if $\left|\frac{A+D}{2}\right| < 1$

Here, $\frac{|A+D|}{2} = 0.052$, so it is stable

(b) For a variable R_1, the matrix is

$$M_1 = \begin{bmatrix} 1 & 570 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & 130 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{2}{R_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 27.76 & 1450.8 \text{ mm} \\ 0.0253 \text{ mm}^{-1} & 1.36 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{2}{R_1} & 1 \end{bmatrix}$$

Let $M_1 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{2}{R_1} & 1 \end{bmatrix} = \begin{bmatrix} \alpha + \frac{2\beta}{R_1} & \beta \\ \gamma + \frac{2\delta}{R_1} & \delta \end{bmatrix}$

so stability requires $\left|\alpha + \frac{2\beta}{R_1} + \delta\right| < 2$

then
$$-2 < \alpha + \delta + \frac{2\beta}{R_1} < 2$$
$$-2 - \alpha - \delta < \frac{2\beta}{R_1} < 2 - \alpha - \delta$$

must be $R_1 < 0$, (since $\beta > 0$, but $(-2-\alpha-\delta)$ & $(2-\alpha-\delta) < 0$)

therefore
$$\frac{2\beta}{2-\alpha-\delta} < R_1 < \frac{2\beta}{-2-\alpha-\delta}$$

i.e. need
$$-107\text{ mm} < R_1 < -93.2\text{ mm}$$

2. The round trip matrix just before mirror R_1 is

$$M_1 = \begin{bmatrix} -1.256 & 1450.8\text{ mm} \\ -0.0018667\text{ mm}^{-1} & 1.36 \end{bmatrix}$$

Then at that point,

$$q = \frac{1}{C}\left[\frac{A-D}{2} \pm i\sqrt{1-\left(\frac{A+D}{2}\right)^2}\right]$$
$$\doteq 700\text{ mm} + i\,535\text{ mm}$$

so for leg 3-4-1, have focus $\boxed{700\text{ mm}}$ behind mirror R_1 or $\boxed{130\text{ mm}}$ behind mirror R_4

[Note mirror R_4 has no effect on beam shape, but merely fold it, since $R_4 = \infty$]

For this focus, beam waist

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}} = \boxed{370\ \mu m}$$

For leg 1-2, just propagate q through mirror R_1

$$\text{Matrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R_1} & 1 \end{bmatrix} \qquad R_1 = -100\,mm$$

so $$q' = \frac{q}{\frac{2}{R_1} q + 1} = -52.3\ mm + i\ 1.887\,mm$$

therefore for this leg, focus is $\boxed{52.3\ mm}$ in front of mirror R_1

beam waist $$W_0 = \sqrt{\frac{\lambda z_0}{\pi}} = \boxed{21.9\ \mu m}$$

Finally, leg 2-3

$$\text{Matrix} \begin{bmatrix} 1 & 0 \\ -0.02\,mm^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & 130\,mm \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 130\,mm \\ -0.02\,mm^{-1} & -1.6 \end{bmatrix}$$

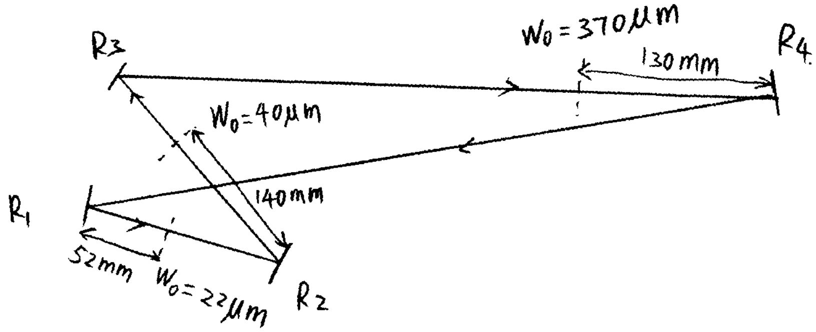
$$q'' = \frac{q' + 130}{-0.02 q' - 1.6}$$

$$= -139.8\ mm + i\ 6.12\ mm$$

Focus is $\boxed{139.8\ mm}$ in front of mirror R_2,

and beam waist $W_0 = \sqrt{\frac{\lambda z_0}{\pi}} = \boxed{39.5\ \mu m}$

Picture :



(b) When beam travels from R_3 to R_4, part of beam will transmit R_4 as output beam. therefore beam is diverging from focus with beam waist

$$W_0 = 370\ \mu m$$

So the divergence angle :

$$\theta = \frac{\lambda}{\pi W_0} = \boxed{0.7\ m\,rad}$$

3. Free spectral range :

$$\nu_F = \frac{c}{L}$$

$$L = (130 + 220 + 660 + 570)\,mm = 1.58\ m$$

$$\nu_F = 190\ MHz.$$

Guoy phase :

leg 1-2: $z_1 = -52.3\,mm$, $z_2 = +77.7\,mm$

$z_0 = 1.887\,mm$

$$\Delta\zeta_{12} = \tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)$$

$$= 1.546 + 1.535 = 3.081 \text{ rad.}$$

leg 2-3: $z_1 = -139.\,mm$ $z_2 = 80.2\,mm$

$z_0 = 6.12\,mm$

$$\Delta\zeta_{23} = \tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)$$

$$= 3.022 \text{ rad.}$$

leg 3-4-1: $z_1 = -530\,mm$, $z_2 = 700mm$

$z_0 = 535\,mm$

$$\Delta\zeta_{31} = \tan^{-1}\left(\frac{z_2}{z_1}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)$$

$$= 1.699 \text{ rad}$$

so $\Delta\zeta = \Delta\zeta_{12} + \Delta\zeta_{23} + \Delta\zeta_{31} = \boxed{7.802 \text{ rad}}$

4. After one round trip through cavity, intensity is reduced to

$$(0.995)^3 \times 0.95 = 0.9358$$

so loss per pass $\Gamma = 1 - 0.9358$

$$\boxed{\Gamma = 0.0642}$$

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then $\boxed{\delta\nu = \nu_F \frac{\Gamma}{2\pi} = 1.94\,MHz}$

Finesse : $\boxed{\mathcal{F} = \frac{\nu_F}{\delta\nu} = \frac{2\pi}{\Gamma} = 97.8}$

$$\boxed{Q = \frac{\nu}{\delta\nu} = \frac{c}{\lambda\,\delta\nu} = 1.93 \times 10^8}$$

$$\boxed{\tau_p \doteq \frac{1}{2\pi\,\delta\nu} = 82\ ns}$$

$$\boxed{\alpha = \frac{1}{L}\ln\frac{1}{1-\Gamma} = 0.042\ m^{-1}}$$