Phys 532/822 HW # 6 Solutions ①

1. For a Kerr medium,

$$\Delta n = \frac{1}{2} n^3 s E_x^2 = \frac{1}{2} n^3 s \frac{V^2}{d^2}$$

For light polarized along x:

$$\Delta n = \frac{1}{2} n^3 S_{xxxx} \frac{V^2}{d^2} = \frac{1}{2} n^3 S_{11} \frac{V^2}{d^2}$$

For light polarized along y:

$$\Delta n = \frac{1}{2} n^3 S_{yyxx} \frac{V^2}{d^2} = \frac{1}{2} n^3 S_{12} \frac{V^2}{d^2}$$

So phase shift

$$\Delta\phi = k l \Delta n$$
$$= \frac{2\pi}{\lambda} l \frac{1}{2} n^3 (S_{11} - S_{12}) \frac{V^2}{d^2}$$
$$\equiv \pi \left(\frac{V}{V_\pi}\right)^2$$

So

$$\boxed{V_\pi^2 = \frac{\lambda d^2}{n^3 l (S_{11} - S_{12})}}$$

Just as for Pockel's effect, transmission through polarizers is

$$T = \sin^2 \frac{\phi}{2}$$

So

$$\boxed{I_{out} = I_{in} \sin^2 \frac{\pi}{2} \left(\frac{V}{V_\pi}\right)^2 = I_{in} \sin^2\left[\frac{\pi}{2} \frac{n^3 l (S_{11} - S_{12}) V^2}{\lambda d^2}\right]}$$

* If you ignored S_{12}, that's fine

2. Have

$$\left.\begin{aligned} r_{33} &= r_{zzz} \\ r_{13} &= r_{xxz} = r_{yyz} \\ r_{22} &= r_{yyy} = -r_{xxy} = -r_{xyx} \\ r_{51} &= r_{xzy} = r_{yzx} \end{aligned}\right\}$$ symmetries from Table 18.2-2

So there are many choices:

For integrated optical component, need to apply $\vec{E} \perp \vec{k}$, since no access to faces

A. Apply E_z, light polarized along z.

use r_{33} $\quad V_\pi = \frac{d}{\ell} \frac{\lambda_0}{r_{33} n_e^3} = \frac{\lambda_0 d}{\ell} \times 3.17 \times 10^9 \ V/m$

$= 13.5\ V$

B. Apply E_z, light polarized along y (or x)

use r_{13} $\quad V_\pi = \frac{d}{\ell} \frac{\lambda_0}{r_{13} n_o^3} = \frac{\lambda_0 d}{\ell} \times 9.68 \times 10^9 \ V/m$

$= 41.1\ V$

C Apply E_y. Index ellipsoid →

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} - x^2 r_{22} E_y + y^2 r_{22} E_y - 2xz\, r_{51} E_? = 1$$

xz term has only 2nd order effect since $n_x \neq n_z$

So get $\quad \Delta n_x = +\frac{1}{2} n_o^3 r_{22} E_y$

$\Delta n_y = -\frac{1}{2} n_o^3 r_{22} E_y$

so for light polarized along either x or y. get

$$V_\pi = \frac{d}{l}\,\frac{\lambda_0}{r_{22}\,n_0^3} = \frac{\lambda_0 d}{l} \times 34.7 \times 10^9 \ V/m$$

$$= 147.5\,V$$

D Apply E_x, Index ellipsoid →

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} - 2xy\, r_{22} E_x + 2yz\, r_{51} E_x = 1$$

yz term has small effect, but xy term does have linear effect since $n_x = n_y$

New axes $x' = \frac{x+y}{\sqrt{2}}$, $y' = \frac{x-y}{\sqrt{2}}$.

$$\left.\begin{aligned} n_{x'} &= n_0 + \tfrac{1}{2} n_0^3 r_{22} E_x \\ n_{y'} &= n_0 - \tfrac{1}{2} n_0^3 r_{22} E_x \end{aligned}\right\}$$

(just as in KDP example)

So for light polarized along x' or y' get

$$V_\pi = \frac{d}{l}\,\frac{\lambda_0}{r_{22}\,n_0^3} = \frac{\lambda_0 d}{l} \times 34.7 \times 10^9 \ V/m$$

$$= 147.5\,V.$$

So the best choice is case A:

crystal oriented with z perpendicular to $\vec{k}$, and light polarized along z

Get $V_\pi = \frac{850\,nm \times 5\,\mu m}{1\,mm} \times 3.17 \times 10^9 \frac{V}{m} = \boxed{13.5\,V}$

3. a) After the crystal, have

$$E_{out} = E_{in}\, e^{-ikl}$$

$$k = n k_0$$

$$n = n_e - \tfrac{1}{2} n_e^3 r_{33} E_1 \cos \Omega t$$

So
$$E_{out} = E_0\, e^{i(\omega_0 t - n_e k_0 l)}\, e^{+i \frac{k_0 l}{2} n_e^3 r_{33} E_1 \cos \Omega t}$$

$$= \left(E_0\, e^{-i n_e k_0 l}\right)\left[e^{i(\omega_0 t + \frac{k_0 l}{2} n_e^3 r_{33} E_1 \cos \Omega t)}\right]$$

Has desired form, with

$$\boxed{\delta = \pi \frac{l}{\lambda} n_e^3 r_{33} E_1}$$

b) Then

$$\frac{dE_2}{dt} = i(\omega_0 - \delta \Omega \sin \Omega t)\, e^{i(\omega_0 t + \delta \cos \Omega t)}$$

$$\omega(t) = \omega_0 - \delta \Omega \sin \Omega t$$

ranges from $\boxed{\omega_0 - \delta\Omega \quad \text{to} \quad \omega_0 + \delta \Omega}$

c) For small δ,

$$e^{i\delta \cos \Omega t} = 1 + i\delta \cos \Omega t = 1 + \frac{i\delta}{2}\left(e^{i\Omega t} + e^{-i\Omega t}\right)$$

So
$$\boxed{E_L = E_0\left[e^{i\omega_0 t} + \frac{i\delta}{2} e^{i(\omega_0 + \Omega)t} + \frac{i\delta}{2} e^{i(\omega_0 - \Omega)t}\right]}$$

Three frequencies, as claimed.