Phys 532/822 HW #7 solutions CD

1. a) Bragg angle $\theta = \frac{\lambda}{2\Lambda}$

full angle between beams is $2\theta = \frac{\lambda}{\Lambda}$,

where Λ = sound wavelength $= \frac{V_s}{\nu_s} = \frac{617\ m/s}{80\ MHz} = 7.7\ \mu m$

λ = light wavelength

$= \frac{\lambda}{n} = 332\ nm$ here.

So $2\theta = \frac{332\ nm}{7.7\ \mu m} = 0.043\ rad = 2.47°$

But this is the angle in crystal. In air, larger by n

since $\theta_{air} = n\,\theta_{glass}$

$$\boxed{2\theta_{air} = 5.8°}$$

b) Beam waist of 200μm, estimate wave requires time

$$T = \frac{200\mu m}{617\ m/s} = \boxed{0.32\ \mu s}$$ to cross beam

For one full modulation cycle, need to turn light on and off, so max rate is

$$f = \frac{1}{2T} = \boxed{1.5\ MHz}$$

2. Peak power: $\frac{1J}{5ns} = \boxed{2 \times 10^8 W}$

Peak intensity: $I = \frac{2P}{\pi w_0^2} = \boxed{5 \times 10^{16} \frac{W}{m^2}}$

Peak E-field: $I = \frac{|E_0|^2}{2\eta_0}$ $\quad \eta_0 = 377\,\Omega$

So $E_0 = \sqrt{2\eta_0 I} = \boxed{6.2 \times 10^9 \frac{V}{m}}$

Over long times, $P_{avg} = \frac{1J}{1s} = \boxed{1W}$

3 a) Flash energy $E_f = 20J$, $0.2J$ absorbed.

ion in excited state has:

$$h\nu = \frac{hc}{\lambda} = 1.9 \times 10^{-19} \text{ J energy}$$

So, # of excited ions is:

$$\frac{0.2\,J}{1.9 \times 10^{-19} J} = 1.1 \times 10^{18}$$

Volume of laser medium is $10\ cm^3$, so

$$N_2 = \frac{1.1 \times 10^{18}}{10\ cm^3} = 1.1 \times 10^{17}\ cm^{-3} = \Delta N$$

So $\gamma_0 = \dfrac{\left(\frac{1.064\,\mu m}{1.5}\right)^2 (1.1 \times 10^{17}\,cm^{-3})}{8\pi\ (1\,ms)\ (2 \times 10^{11}\,Hz)} = 10.7\ m^{-1}$

and $g_0 = e^{2\gamma_0 l} = e^{2.1} = \boxed{8.5 \gg \Gamma = 0.3}$

b) Peak power:

$$P = h\nu \cdot \bar{V} \cdot \frac{1}{\tau_p} \frac{\Delta N_i}{2}$$

$\bar{V}$: mode volume $\simeq l \times \pi w_0^2$

Take $\pi w_0^2 = A$, optimum case

Then $\bar{V}\,\Delta N_i = 1.1 \times 10^{18}$

$$\tau_p = \frac{1}{\Gamma v_F} = \frac{1}{\Gamma} \cdot \frac{2d}{c} = \frac{1}{0.3} \frac{2 \times 0.3\,m}{3 \times 10^5\,\frac{m}{s}} = 6.7\,ns$$

$$P_{max} \simeq (1.9 \times 10^{-19}\,J)(1.1 \times 10^{18}) \left(\frac{1}{2 \times 6.7\,ns}\right)$$

$$= \boxed{1.6 \times 10^7\ W}$$

Pulse duration $= \boxed{\tau_p = 6.7\,ns}$