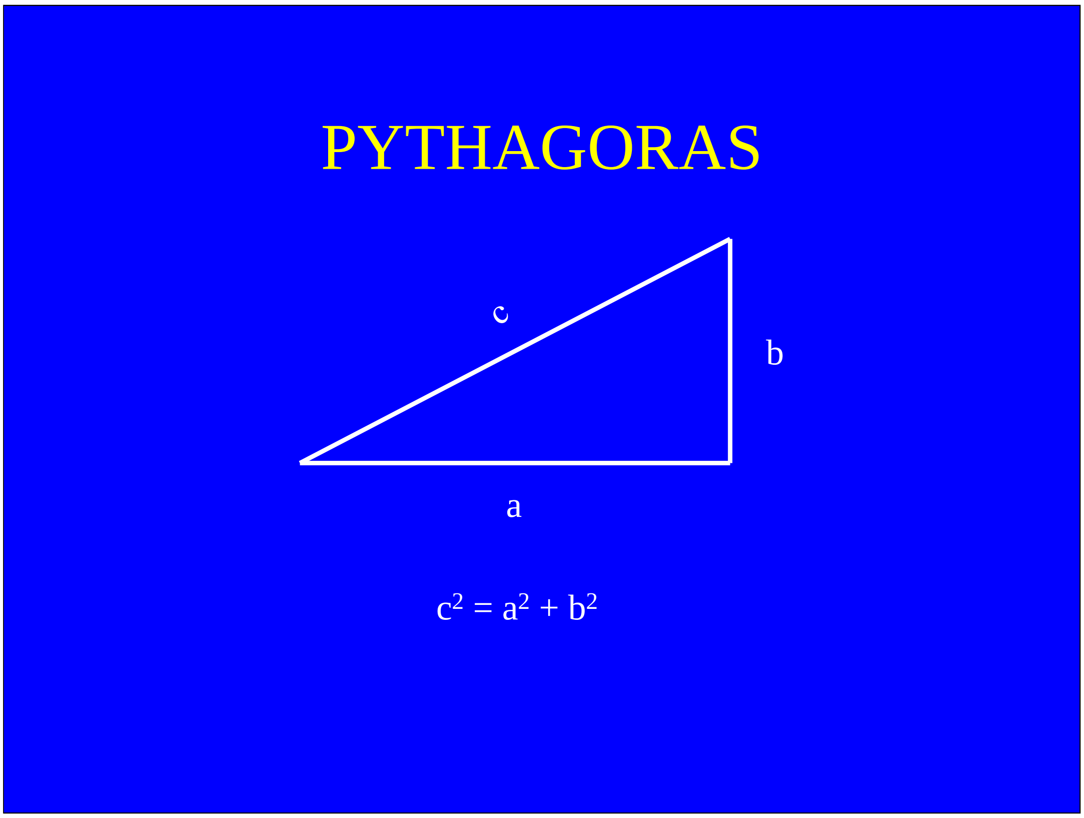
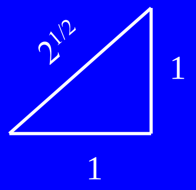


This relationship had been known since very ancient times. For example a number of Pythagorean triplets are found on Babylonian clay tablets: 3, 4, 5; 5, 12, 13; and as high as 56, 90, 106 (try it on your calculator). The clay tablet with 15 of these triplets has been dated to 1700 BCE, or 1200 years before Pythagoras. The ancient Chinese also recorded Pythagorean triplets about the same time. These triplets were probably used in construction to help produce a right angle. There is no evidence that anyone proved this theorem before the Pythagoreans. We are sure they did prove it because they used the theorem to discover irrational numbers.

A rational number is one that can be expressed as a fraction.

IRRATIONAL NUMBERS

Any rational number can be written as n/m where n and m are integers with all common factors removed.



Is $2^{1/2}$ such a number? Let's assume it is:

$2^{1/2} = n/m$. squaring, $2 = n^2/m^2$ and $n^2 = 2m^2$

So n^2 is an even integer. That means n is an even integer.
<u>Therefore m is an odd integer.</u>

Since n is even, we can write $n = 2n'$
So $2m^2 = 4n'^2$ and $m^2 = 2n'^2$ <u>Therefore, m is an even integer.</u>

Contradiction: So our assumption must be wrong. $2^{1/2}$ cannot be written as a fraction.

Consider the simplest most symmetric right triangle: one with equal base and height, each equal to one unit of length. Then the hypotenuse must, according to the Pythagorean theorem, equal the square root of two. This is a number which, when squared, equals two.

Let's see if this number is rational. We begin by assuming that it is, ie that we can express it as a ratio of two integers, with all common factors removed (the fraction is reduced to lowest terms). squaring this equation and rearranging, we see that n^2 is an even integer. But that means n is an even integer (Try it: the square root of an even integer is always an even integer). Therefore m must be an odd integer, otherwise a factor of two could be removed from the numerator and denominator of our fraction n/m.

Since n is even, we can divide it by two producing another integer. Call it n'. Using our basic equation, we see that it requires that m be an even integer. So we have proved that m is both odd and even. This is a contradiction, so something must be wrong. Our initial assumption that the square root of two can be written as a fraction is the only suspect element in this argument. Everything else just follows the rules of arithmetic.

This kind of proof is called "reduction to absurdity". We assumed something and then showed it led to something absurd. Therefore the thing we assumed must be wrong.

This is an excellent example of the use of deductive logic, which was developed by the Greeks, and used extensively by them. The proof that irrational numbers exist was a major discovery in mathematics.

The Pythagoreans were not happy with this result. They had believed that integers were at the root of everything in the universe. A new kind of number that cannot be expressed with integers was such unhappy news that they tried to keep it secret.

CHARACTERISTICS OF GREEK CULTURE

1. The Assembly, with its rational debate.
2. Maritime economy helping prevent isolation.
3. In contact with widespread Greek-speaking world.
4. Independent merchant class.
5. The Iliad and the Odyssey.
6. Polytheistic literary religion without entrenched priesthood.
7. The persistence of these factors for many centuries.

THALES

“Of all things that are, the most ancient is God, for he is uncreated; the most beautiful is the universe, for it is God’s workmanship; the greatest is Space, for it contains everything; the swiftest is the Mind, for it speeds everywhere; the strongest is Necessity, for it masters all; and the wisest Time, for it brings everything to light.”

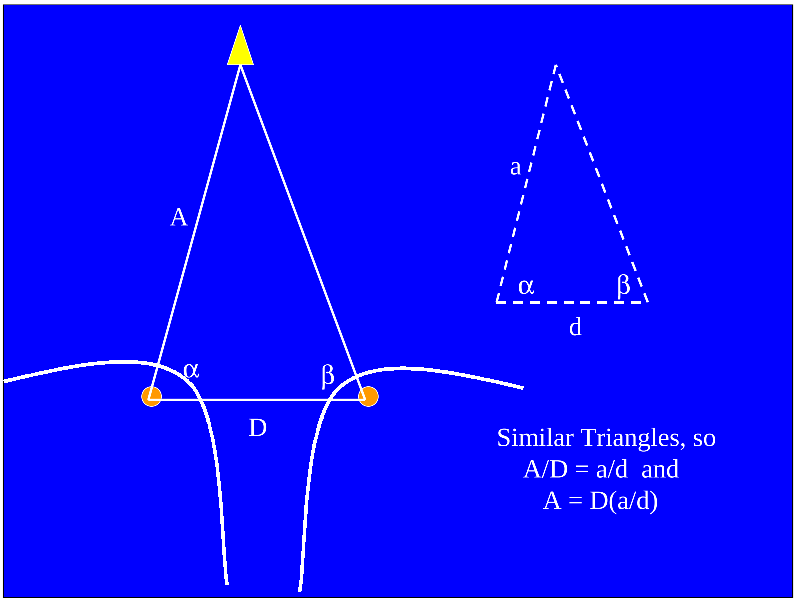
First principle of the universe: Water. Similar to ideas Of Egyptians, Israelis, and Babylonians. Earth is Condensed water, air is expanded water.

BASIC PROPERTY OF LIGHT

- **TRAVELS IN STRAIGHT LINES ALLOWING TRIANGULATION AND SURVEYING**

When I look at an object across the room I can see it because light from some source scattered off of it, perhaps in all directions, but some of it came towards me. That beam of light traveled in a straight line from the object to me. I can imagine that my line of sight draws a straight line through the air.

If I look at two objects there are two straight lines in the air, and some angle between them. This is the basic idea underlying triangulation and surveying. We can create imaginary triangles in the air and use what we know about geometry to measure distances.



Here we see a sketch of the mouth of the Meander river as it flows into the Mediterranean Sea. Thales had two observation posts built, one on each side of the river's mouth (green circles). When a ship appears at sea, it is desirable to know how far away it is to know how much time there is to prepare the dock, if it carries a cargo, or prepare defenses, if it is an enemy ship.

The distance D between the two observation posts can be measured. And observers at the two posts can measure the angles alpha and beta between the ship and the other observation posts. So know we have created an imaginary triangle out over the sea.

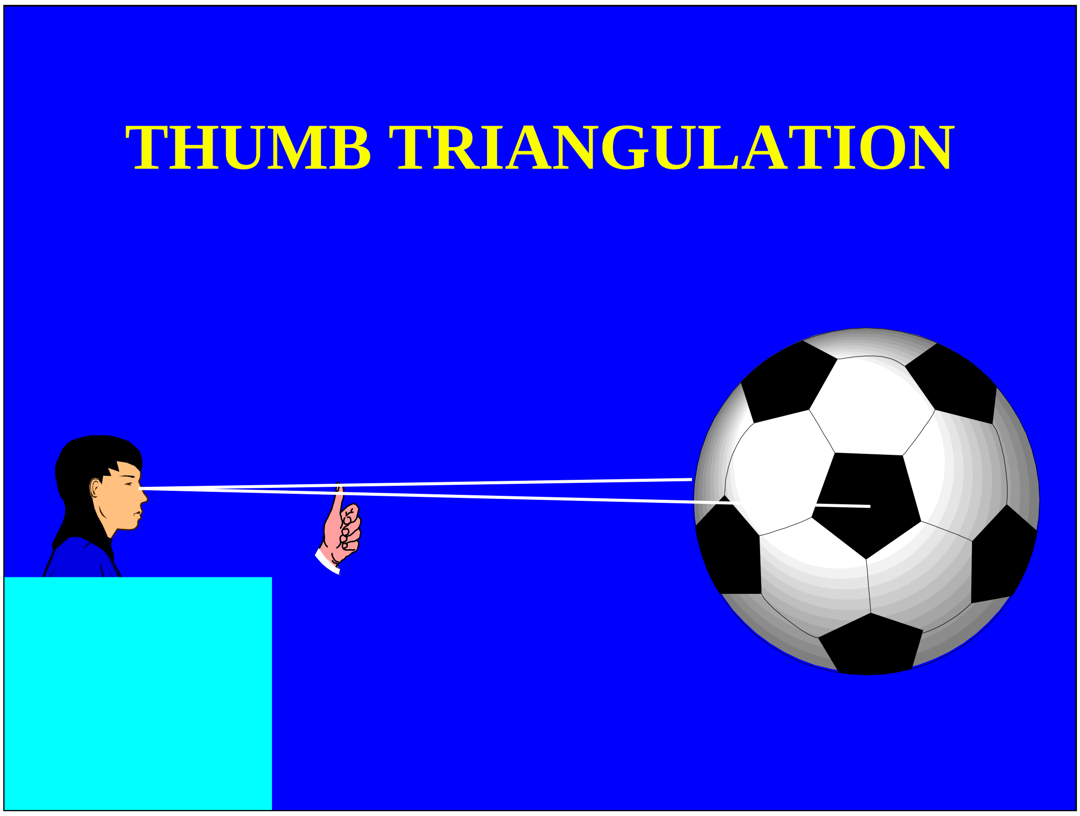
We can then construct a similar triangle on a flat surface, shown with dashed lines. It is constructed to have the same angles alpha and beta, and hence it has the same vertex angle as the larger triangle at the ship's position. Since the two triangles are similar, the ratios of corresponding sides must be equal. Therefore $A/D = a/d$, and we can solve for what we want to know, A, the distance to the ship: $A = D(a/d)$.

The Egyptians needed these ideas about similar triangles in order to survey the land after the spring flood of the Nile. Somehow they managed to generalize their surveying techniques and to learn some of the properties of similar triangles. This was an inspired abstraction from the practical work at hand, and doubtless simplified the surveying work.

The Greeks were not aware of the properties of similar triangles, but when Thales brought them to Miletus, they took hold. Over the next 300 years, the Greeks combined ideas of plane geometry with their own development of deductive logic to produce the greatest mathematical tract of the ancient world, Euclid's Elements. This series of books completely laid out the subject of plane geometry. It has not changed one bit since. The geometry taught in schools today is the same as that described by Euclid.

We can go beyond Euclidean geometry by using a curved surface instead of a plane, such as spherical geometry, appropriate for navigating on the surface of the earth, or by using more dimensions than two (three, four, five,…). These extensions are sometimes referred to as non-Euclidean geometry. They are not a correction to what Euclid wrote, but an addition to it.

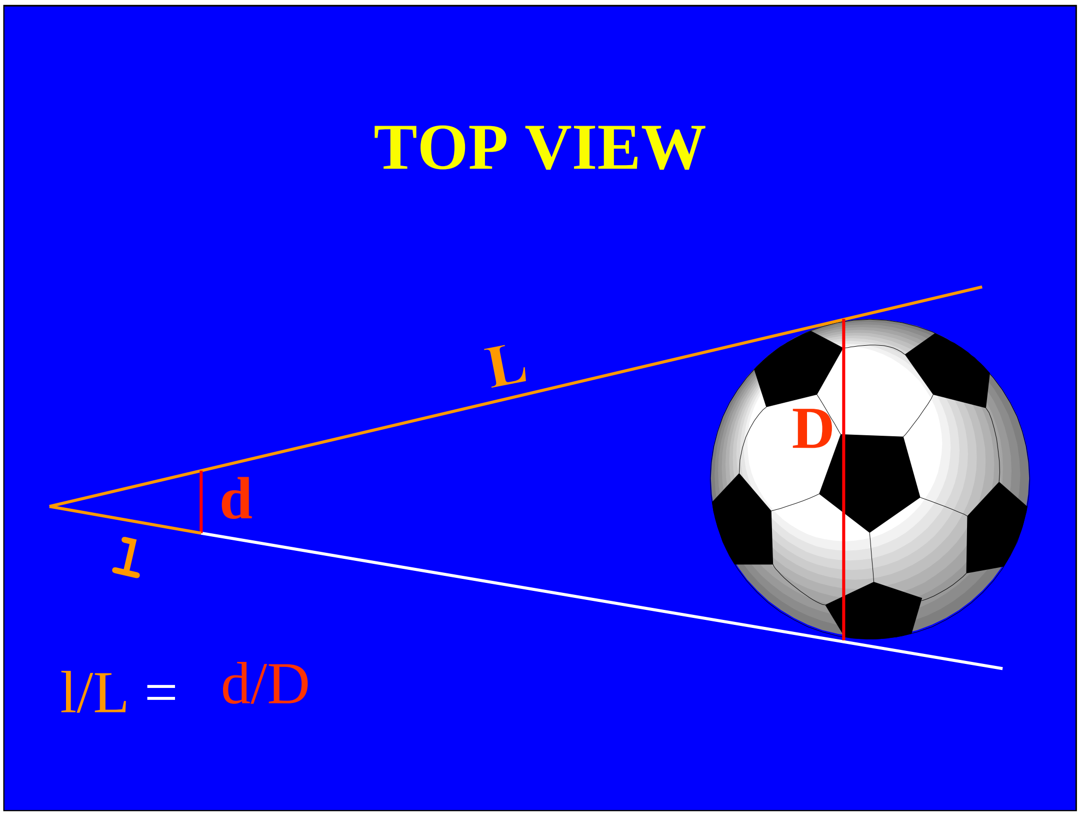
We do not know whether Euclid was himself an accomplished mathematician, or an accomplished encyclopedist, summarizing the work of others before him. In either case he produced a masterpiece.



We can make measurements similar to those made by Thales somewhat more casually using our hands, fingers, and arms. If you hold your thumb up at arm's length, and sight along its left and right sides you are creating two straight lines is space separated by a definite angle. The angle depends on the length of your arm and the width of your thumb. What counts is the ratio between these two dimensions.

Other useful ratios are the width of your spread hand or the distance between two knuckles to the length of your arm. Knowing these ratios will enable you to deal with any giant soccer ball you may encounter without having to go too near.

Now let's review the geometry needed to understand triangulation.



Imagine that the two lines of sight past your thumb form a triangle with the thumb as its base. If you are the right distance from the soccer ball so that your thumb just fills the ball, then those lines of sight form another triangle with the ball as its base.

These two triangles are similar triangles. Any two triangles are similar if their corresponding angles are equal. That means they have the same shape; They differ only in size. One may be measured in inches, the other in miles.

In this case both triangles are isosceles (have equal sides) so the base angles are equal to each other.(The triangles are symmetric). The two triangles have the same apex angle at your eye. Now using the fact that the sum of the angles in any triangle must be 180 degrees, we can see that the base angles of the two triangles must equal each other, and the two triangles are similar.

Another way to see it is this: The two triangles' bases are parallel. Each of the sides of the triangles is a straight line crossing the two parallel lines. When two parallel lines are crossed by a straight line (transversal) then the included angles are equal.

Finally, if two triangles are similar, the ratios of their corresponding sides are equal. So the ratio of the width of the ball to your distance from it equals the ratio of the width of your thumb to the length of your arm. That is why you should know this ratio about yourself.

HAND AND FINGER TRIANGULATION

Angular width at arms length of:

Thumb: 2 degrees: l/d = 30

Little finger: 1 degree: l/d = 60

Spread fingers: 20 degrees: l/d = 3

Triangulation is a useful way to estimate the size of something or the distance to something, and as we have seen, you don’t have to build observation posts as Thales did to do this. In addition to using your thumb, your little finger, and your spread fingers are useful.

For most people, the thumb held at arms length subtends an angle of about 2 degrees measured from the eye. The little finger is about half this wide, and so subtends an angle of about 1 degree.

When you spread your fingers wide, the angle between the tip of your thumb and the tip of your little finger is about 20 degrees.

Remembering these numbers will make it possible for you to make angle estimates of objects around you.