MEASURING THE SOLAR SYSTEM

Eratosthenes, Aristarchus

The century after Euclid

Two remarkable astronomers, Eratosthenes and Aristarchus, accomplished what seemed impossible in the ancient world, they measured the size of the solar system. It is significant that this took place during the century after Euclid. Euclid’s geometry empowered creative people like these two to do things they would not otherwise have been able to accomplish.

Solving great problems requires first asking the right question. The questions we are discussing were posed by these two in geometric terms. Once set forth in those terms, the problems became quite straightforward to answer as we will see.

So Euclid’s work was not only the greatest mathematical advance of the ancient world, it also allowed others to use those tools to reach farther than ever before imagined in understanding the world around us.

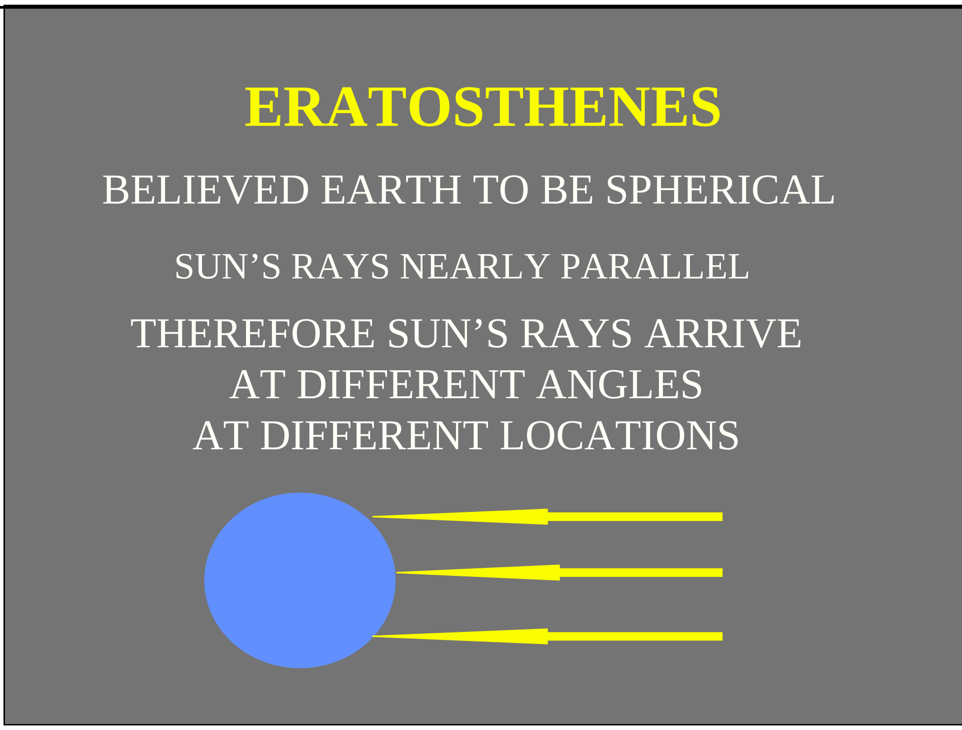
SHADOWS

SIZE OF EARTH

ERATOSTHENES - 225 BC

Eratosthenes was a Renaissance Man of his generation in ancient Greece (almost 2000 years ahead of his time). He was very good, but never the best at whatever he did. He was only the second best mathematician, second best poet, second best musician, etc. In ancient Greece, they used the alphabet, alpha, beta, gamma, etc, to indicate first place, second, third, etc. So his friends gave him the knickname Beta.

Well, Beta left his mark on the world, perhaps more so than his friends. He managed one of the great accomplishments of the ancient world; he measured the size of the earth.



Here is how his thinking went. First he believed the Earth to be spherical. But how so? We all remember the stories of Columbus' crew thinking they would fall off the edge of the earth. Somehow the Greeks had believed the earth to be spherical since way back. Pythagoras described is that way around 500 BCE.

There may have been evidence for this belief: The Greeks were a seagoing nation. When two ships approach each other at sea, the other ship appears to rise up out of the water. From a distance, you only see the top ofthe mast, and finally the whole thing. As they recede from each other, the other ship appears to sink back into the water. This is true whether they approach along a north-south line, or east-west, or any other direction. An Astute Greek might have concluded from this that the earth is spherical.

On the other hand, it might a belief based on culture. The Greeks thought the sphere was the most perfect three-demsional shape. So it might have been natural to think Mother Earth had this shape. So imagining the Earth to be spherical could either have been the result of rational thought based on observations, or an egocentric, aesthetic or religious belief.

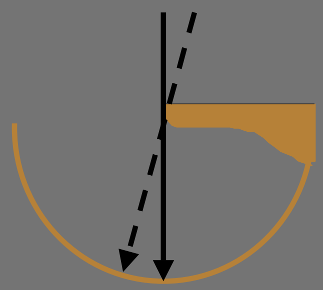
That the sun's rays are nearly parallel at any given location on earth can be seen from the near sharpness of shadows. The angular size of the sun is about 1/2 degree. If it were a point in the sky, the rays would be almost precisely parallel. That all the sun's rays landing at different locations on earth (e.g. the top and bottom arrows above) are nearly parallel to each other is due to the fact that the sun is far from earth compared with the size of the earth.

Thus, as the diagram shows, different locations on earth receive the sun's rays at different angles to the surface, or to the surface normal, which is "the vertical", a line outward from the center of the earth. What Eratosthenes realized is that by measuring how this angle changes with location, he could determine the size of the earth.

Eratosthenes knew that at noon on June 21(the summer solstice) the sun was directly overhead at Syene, Egypt. (Present-day Aswan, where there is now a large dam in the Nile) This means that Syene is on the tropic of cancer. This undoubtedly was common knowledge, having been noticed by the fact that a deep well will reflect the sun's rays directly back in your face at noon on that date. Or that a vertical stick in the ground will cast no shadow.

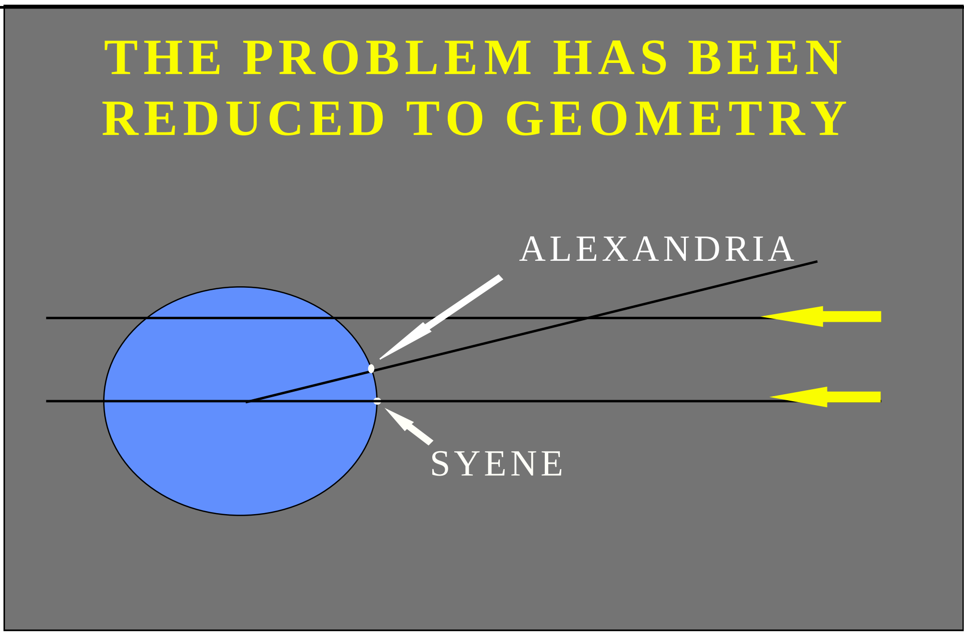
SIZE OF EARTH

- ERATOSTHENES MEASURED THE ANGLE OF THE SUN AT ALEXANDRIA, 5000 STADES NORTH, ON THE SAME DATE, FINDING IT WAS 1/50 OF A FULL CIRCLE FROM OVERHEAD.



Eratosthenes lived in Alexandria, 5000 stades North of Syene near the mouth of the Nile on the Mediterranean. At noon on June 21 (the summer solstice) he measured the angle of the sun’s rays there. He did so using a Greek sundial, shown above. As constructed then, sundials were hemispheres with a gnomon (pointer) over the center. A plumbob suspended from the end of the gnomon indicates the vertical direction and angular positions are marked from that origin.

At noon on the summer solstice, Eratosthenes found the sun’s shadow was $1/50^{th}$ of a circle from the vertical. Using toda’ys angular measure, this would be $360/50 = 7.2^{0}$.



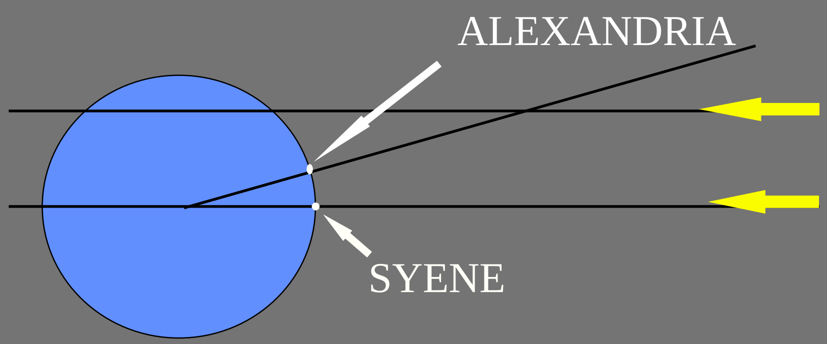
The problem has now been reduced to one of geometry. In the above diagram the sun's rays incident on Syene are headed straight for the center of the Earth. The sun's rays at Alexandria are parallel to those at Syene: here Eratosthenes is assuming that the distance to the sun is large compared with the distance between Syene and Alexandria. The shadow angle Eratosthenes measured at Alexandria is the angle between the sun's rays and the vertical black line shown. But this is equal to the angle at the center of the earth between radial lines drawn from Syene and Alexandria. Geometry once again allows us to measure an angle at a location that is impossible to get to. We still cannot go anywhere near the center of the earth. If the earth were the size of an apple, the deepest we have gone (by drilling) is not far enough to have penetrated the skin of the apple.

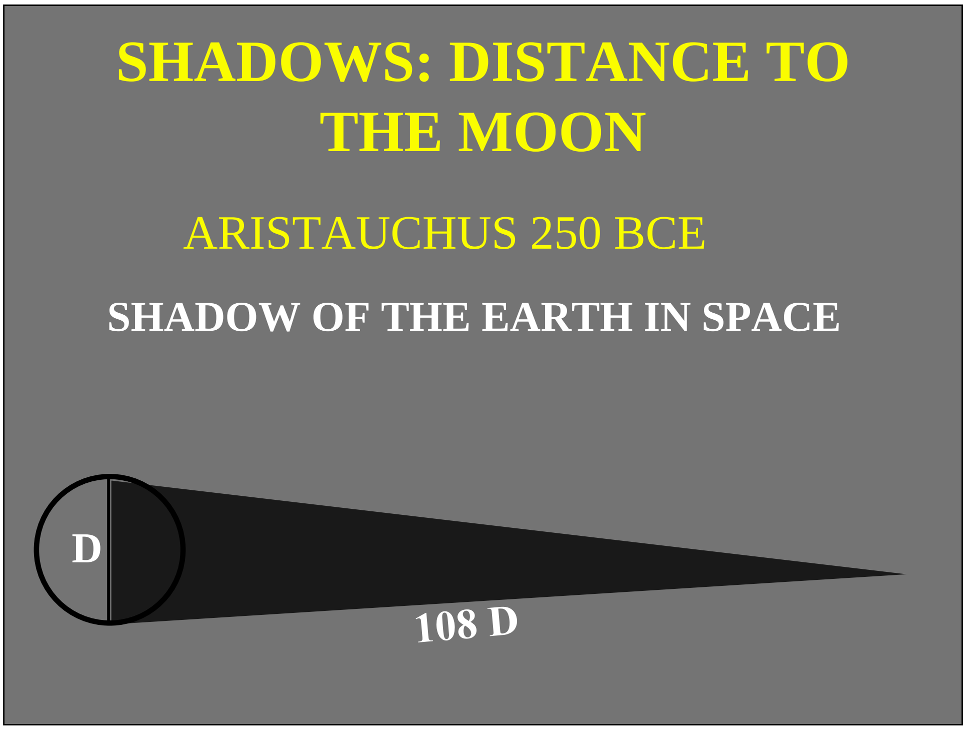
Since the angle at the center of the earth is $1/50^{th}$ of a circle, the circumference of the earth must be 50 times the distance from Syene to Alexandria. This means the earth's circumference must be 50*5000 = 250,000 stades.

The Greeks did not have standardized units of measure at this time. The units of length were directly associated with body parts. The foot was a basic unit of length. Archaeologists have found that depending on the city, the foot could be 0.30, 0.32, or 0.33 meters in size. For longer distances, the Olympic measure, the stadium, was used, which was supposed to be 600 feet. If we use today's definition of a foot, 250,000 stades equals 28,400 miles. The accepted value of the earth's circumference today is about 25,000 miles so Eratosthenes' value is about right.

The method he used is completely sound and is a good example of Greek analytic methods in astronomy. Geometry was the only mathematics they knew (recall that they did not even have a useful way of doing arithmetic), and continuing the Pythagorean idea that mathematics underlies natural phenomena, they used it wherever they could.

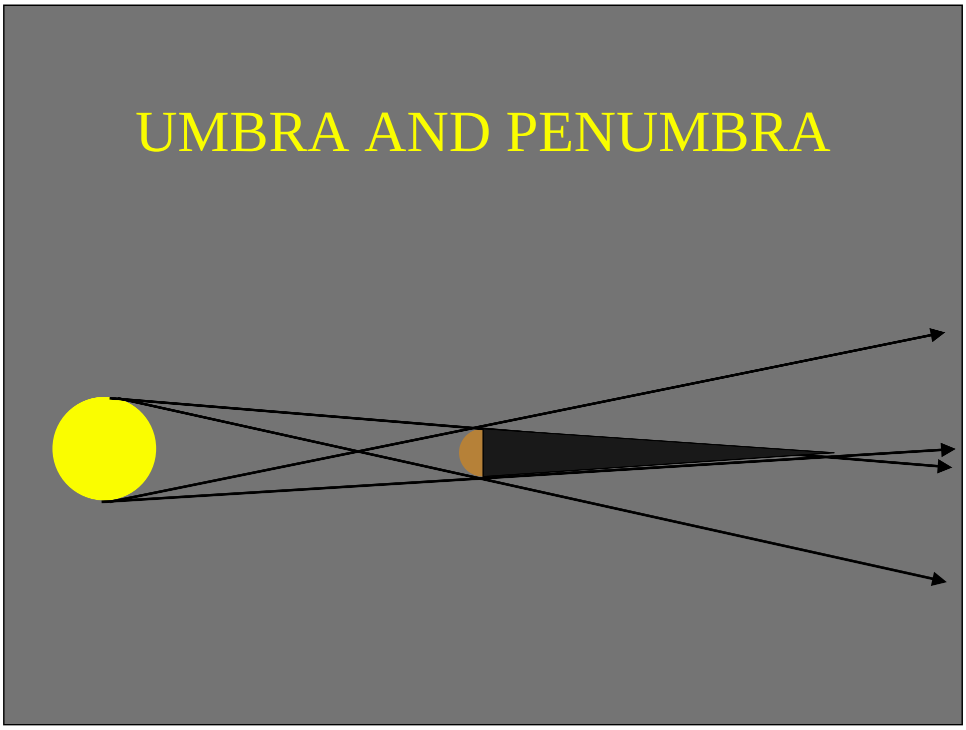
ALEXANDRIA MUST BE 1/50 OF THE EARTH'S CIRCUMFERENCE NORTH OF SYENE, SO THE CIRCUMFERENCE MUST BE 250,000 STADES, OR ABOUT 28,000 MILES





Aristarchus of Samos was one of the most revolutionary thinkers of ancient Greeck science. Not only did he devise methods of finding the distances to the Moon and Sun, but he also proposed that the sun was at the center of our solar system, with the earth rotating and orbiting around it. This **heliocentric** model did not gain acceptance among other astronomers, and for centuries to come, the **geocentric** picture remained the standard model of the solar system.

Aristarchus' method of finding the distance to the moon begins with careful observations of a lunar eclipse, in which the Moon passes through the shadow of the earth. If the sun were a point source of light, then the earth's shadow would be a cylinder extending to infinity through space. Instead it is a sphere of finite size, having an angular diameter of about ½ degree as seen from earth. We can imagine building up the shadow of earth by assembling many point sources until we have a sphere or disk the size of the sun. Each of these sources, i.e. each part of the sun, creates a cylindrical shadow with a slightly different direction. These shadows overlap each other for only a finite distance in space. The shadow of a round object has the shape of a cone, with length equal to 108 times the diameter of its base.

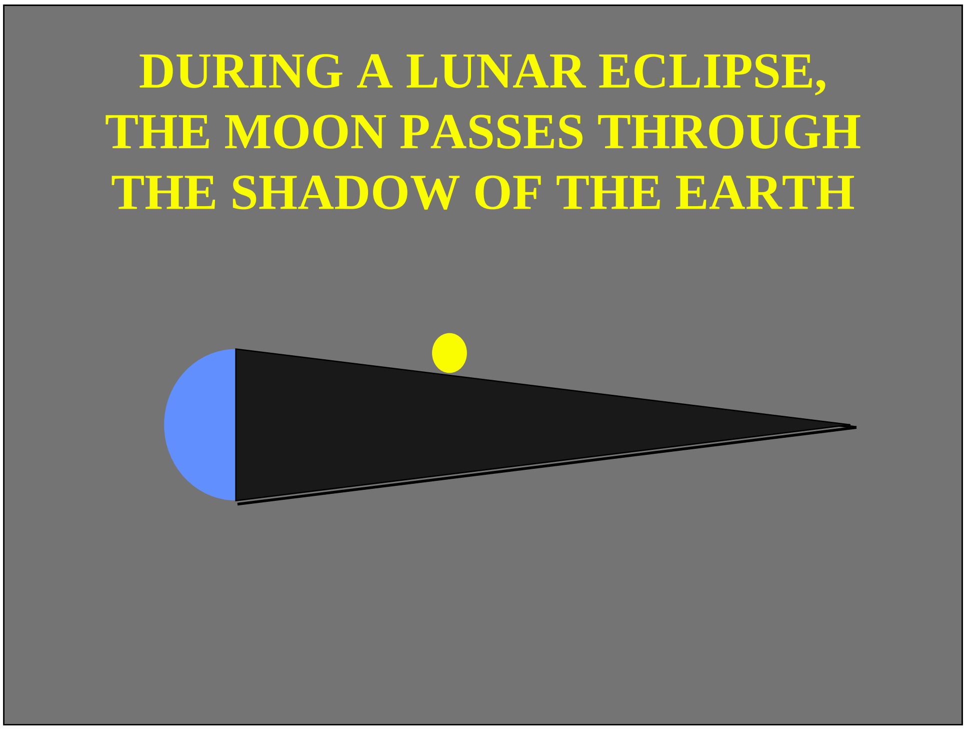


The dark part of the shadow that we have been discussing, is called the **umbra**. Surrounding it is a region of partial shadow called the **penumbra**. This is what gives shadows of trees and houses a fuzzy edge. The farther an object is from its shadow, the fuzzier the shadow becomes, eventually disappearing altogether.

The diagram above shows a sketch of the sun and earth with the umbra of earth extending behind it while becoming smaller and smaller and eventually disappearing. This conical shape of the umbra of a round object is easy to observe with a quarter or other round object.

During an eclipse of the moon, we hardly notice the penumbra. When the moon nearly disappears, it is in the umbra of the earth.

Notice that the vertex angle of the cone of the umbra equals the angular diameter of the sun as seen from earth. This will be important shortly.



Aristarchus began timing lunar eclipses. He measured the time required for the moon to enter the umbra, and the time for the leading edge of the moon to cross the umbra. Doing this accurately required making many eclipse observations because the moon rarely goes directly across the umbra passing through its center. It is more likely to be off center, or just graze the umbra for a partial eclipse. Other astronomers repeated these measurements once Aristarchus had shown why it was interesting to do so. They eventually determined that the moon requires 2.5 times as long to cross the umbra as it takes to enter it.

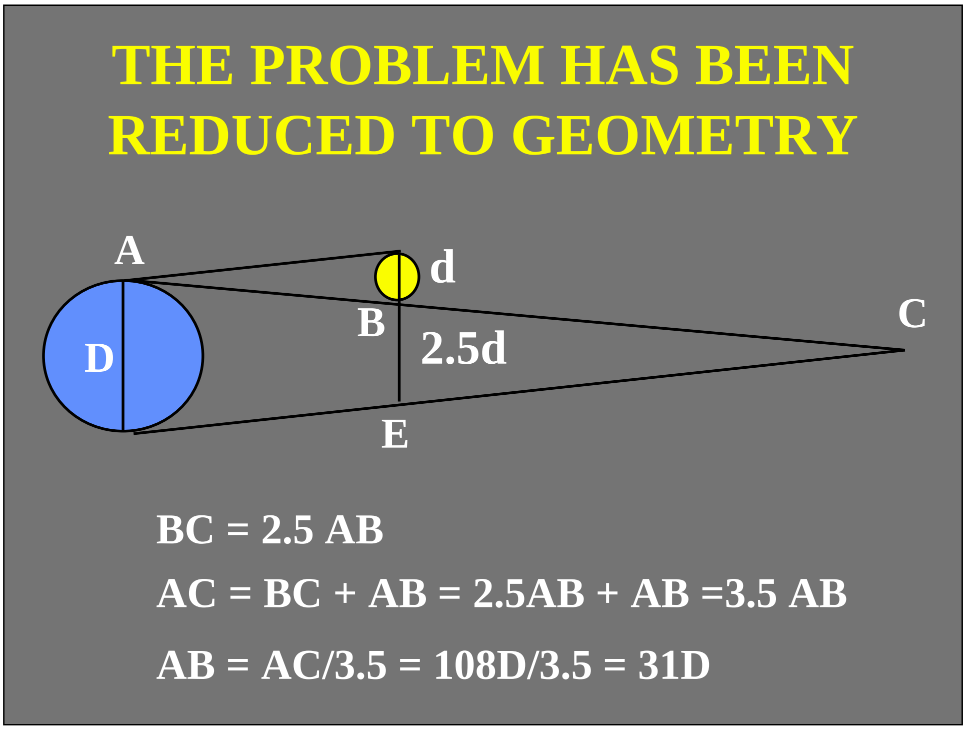
OBSERVATIONS

THE SHADOW IS 2 1/2 TIMES THE MOON'S DIAMETER

ANGULAR SIZE OF MOON = ANGULAR SIZE OF SUN

A second observation was also important, namely that the moon's angular diameter is the same as that of the sun. Modern values for the angular diameters of the moon and sun are 0.51 and 0.53 degrees, respectively, so the Greek conclusions were quite accurate.

These two observations are all Aristarchus needed to evaluate the distance to the moon. To do so he relied once again on geometry.



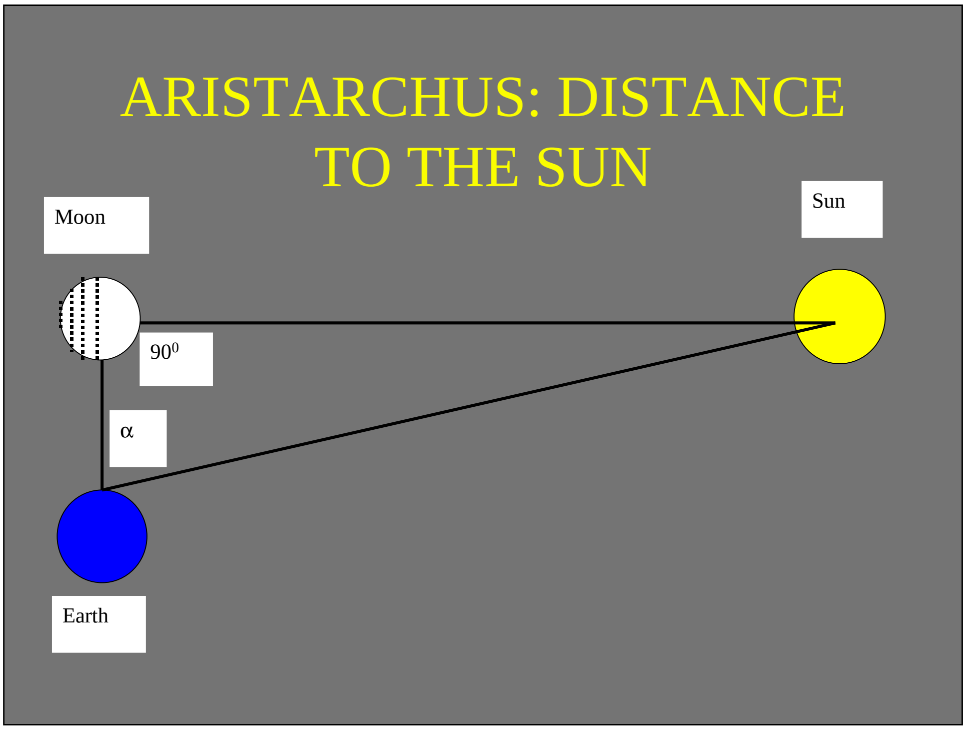
Here is a sketch of the earth is space with its umbra behind it. The moon is about to enter the umbra from above, going into eclipse. We label the earth's diameter D and the moon's diameter d. We know from the Greek timing observations that the line BE equals 2.5d. We also know that the vertex angle at C is equal to the angular diameter of the sun, and that the vertex angle at A has the same value. Therefore triangle CEB is similar to triangle AdB.

This means that the ratios of corresponding sides of these triangles are equal. Therefore CB/BA = BE/d = 2.5. Now AC is the length of the umbral cone, which we know to be 108 times D, the diameter of earth. But AC is also equal to the sum of AB and BC, and we know that BC is 2.5 times AB. So AC, the length of the cone, is 3.5 times AB, the distance to the moon.

Solving for AB we find it is equal to 31 earth diameters. This is remarkably close to the modern value of 30.3 earth diameters.

What Aristarchus has done here is use the earth's diameter as a yardstick to measure the distance to the moon. The idea itself is an important step in Greek astronomy. When he wanted to measure an astronomical distance, likely to be very large, he did not try to use feet or stades, which are appropriate for earth-bound distances. He used the earth itself.

Note also that Aristarchus did this before Eratosthenes had measured the size of the earth. So his yardstick had not been calibrated at the time he proposed using it.



Emboldened by his success with the moon, Aristarchus pushed on. Now that he knows the distance to the moon, could he use this as a yardstick to measure another, larger astronomical distance? The obvious challenge was the distance to the sun. He did devise a method. The geometry is easier, but the measurements themselves much more demanding.

Here is the idea: When the moon is exactly half full (so the shadow line on the moon is straight), the angle between the earth and the sun as seen from the moon, must be 90^0. The diagram above depicts this situation. Now the trick is to measure the angle alpha, the angle between the sun and moon as seen from earth. This requires, of course, being able to see both the sun and moon at the same time. When the sky is clear, this is usually possible for a couple of hours when the moon is half full.

The measurement is difficult because alpha is so close to 90^0. Aristarchus thought alpha was 87 degrees; it is actually 89 degrees 52 minutes. In any case, if we can measure alpha, we can find the distance to the sun in terms of the distance to the moon using similar triangles. We would draw a triangle on a flat surface with the measured value of alpha, and measure the ratio of the two sides corresponding to the earth-sun distance, and the earth-moon distance.

SUMMARY: GREEK DISTANCES IN EARTH DIAMETERS

Date	Astronomer	Moon Distance	Sun Distance
-260	Aristarchus	9.5	180
-130	Hipparchus	33.7	1245
-70	Posidonius	26.2	6545
+150	Ptolemy	29.5	605
Modern Values		30.3	11,745

Above is a table summarizing numerical results for various Greek astronomers using Aristarchus' methods for the moon and sun.

Triangulation was used for the moon until recent times. Now we measure distances to the planets by bouncing radar pulses off them. The distance to the moon can be measured even more accurately because of a set of corner reflectors left there by US astronauts. Laser pulses returned by the reflectors give a very precise value. A project is underway to measure this distance to within 1mm as a test of general relativity.

The above table raises an interesting question: In studying contributions made in the past by scientists working on a quantitative subject, how important is accuracy? Here we see that Aristarchus concluded that the distance to the sun is less than modern values by a factor of 65. It may seem that when someone making a quantitative measurement is off by this much, their result is of no value whatsoever. This is not so, however, and here we see a good example of why.

Before Aristarchus, there was only guesswork about the distance to the sun. Being wrong by a factor of 65 is better than having no idea at all. And, more important, Aristarchus provided a method. Later astronomers could and did improve on it. A new method can be more important than an accurate measurement. In addition, the astronomers making this measurement knew their values were very uncertain because alpha is so close to 90 degrees.

Sometimes, however, accuracy can be very important. We will encounter a very good example of this when we discuss Tycho Brahe's measurements of planetary positions and how Kepler interpreted them.

There is an unstated assumption in all of the above work: It is assumed that light travels the same way in space near the moon or sun, as it does here on earth. For example, the knowledge that the length of the umbral cone of a circular object is 108 times its diameter came from measurements made of disks and spheres here on earth. How do we know light travels the same way in the space behind earth?

This is the first use of the idea of Universality. That the laws of physics are the same elsewhere in the Universe as here on earth. Isaac Newton is usually given credit for this idea. Unfortunately none of Aristarchus' own writings have survived, so we do not know whether he was aware that he was making this assumption.

ARISTARCHUS' HELIOCENTRIC MODEL

SUN IS AT CENTER

EARTH ROTATES DAILY PRODUCING THE APPARENT DAILY MOTIONS OF THE CELESTIAL OBJECTS

EARTH ORBITS SUN ONCE PER YEAR PRODUCING THE PROGRESSION THROUGH THE ZODIAC

Aristarchus was not only an accomplished astronomer, he was also a daring thinker. Ancient astronomers had believed earth to be stationary and at the center of the universe since the first recorded speculations on this topic. This was established conventional wisdom. This location for earth had a kind of religious appeal as well, since putting earth at the center of things put humans there too, which was a kind of proof of our importance in the scheme of things.

Aristarchus however proposed that the sun was at the center, with the earth rotating and orbiting around it. How did he come upon this idea? One possibility is the following. He had measured the distance to the moon in terms of earth's diameter. Then he measured the distance to the sun using the distance to the moon as a yardstick. Could he use the distance to the sun as a measuring stick for something else?

The distance to the starry vault is the obvious next step. He realized this distance could only be measured if the earth moved. If it did, then the angular positions of the stars would change as earth moves about the sun. This is called **stellar parallax**. This change in angular positions of objects because the observer moved is something we experience every day. As you walk along a sidewalk, the angular positions of trees, houses, buildings, etc change, not because they moved, but because you did.

So Aristarchus realized that an observation of stellar parallax would both confirm the heliocentric model and determine the distance to a star. Attempts to observe the parallax failed, however.

SUMMARY

1. Lack of stellar parallax.
2. Required overthrowing a long-established conventional wisdom.
3. The earth <u>feels</u> like it is stationary. If we evaluate the earth's orbital speed required by the heliocentric model, even using Aristarchus' small value for the distance to the sun, we find $v = 2\pi R/365 = 25{,}000$ miles/day. This was an unheard of speed, and we don't notice it. Galileo was the first person to correctly address this question and to explain it.
4. Religious and cultural reasons. Accepting this radical change in world view would require changing the jobs of many of the gods (e.g. Zeus, who carried the Sun across the sky). It also displaces humans from the center of the universe.

Aristarchus' heliocentric model had little impact on other Greek astronomers, or later people as well. Why was such a good idea ignored? What were their reasons for rejecting it?

We have just seen that they failed to observe a stellar parallax. Archimedes concluded that either the heliocentric model is wrong, or the universe is much larger than had been thought. This was the correct conclusion. There was no compelling argument either way, and under those circumstances, the old, accepted, comfortable idea usually wins out.

The Greeks could not understand how, if the earth moves around the sun at enormous speeds, we do not notice this motion. They imagined that falling objects would be deflected to the side, and that clouds would be blown away.

Aristarchus did not have a smoking gun, and without it, his idea died on the vine.