

## PTOLEMY

Best planetary model in the ancient world

Maps

Trigonometry

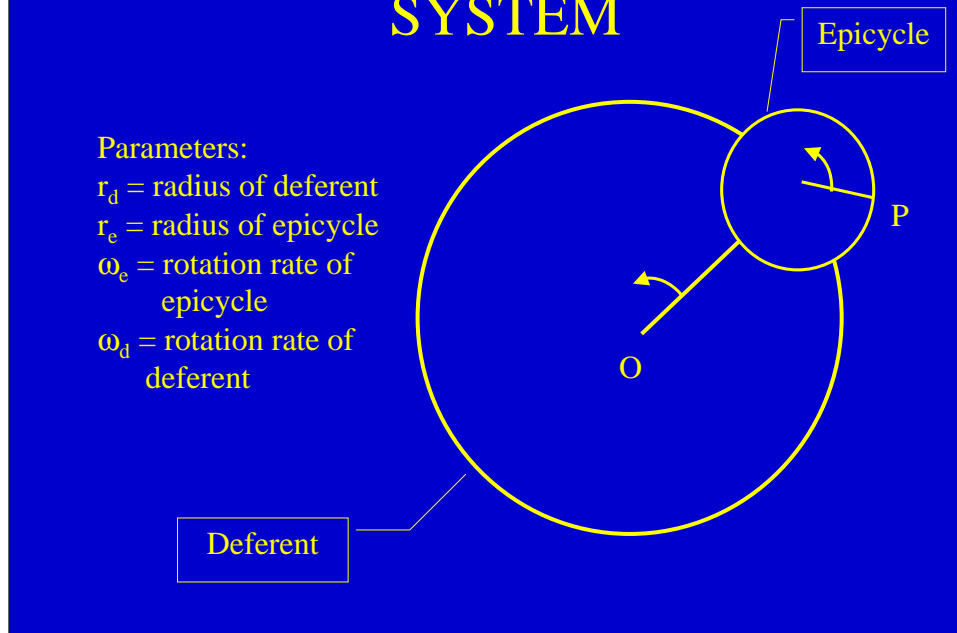
The most highly developed description of the planets was produced by Ptolemy working around 150 AD in Alexandria. He was undoubtedly associated with the great library and museum located there. He was also an accomplished map maker. He made the first map of the world with latitude and longitude indicated. It extended from England to China, and when the Age of Exploration began in Europe 1400 years later, his map was still the best available.

Ptolemy also helped develop trigonometry, producing the first table of trig functions from ancient times to survive. He evaluated what we now call the sine of an angle, producing a table of values from 0 to 90 degrees in steps of 0.25 degree. His value for what we call the sine of 36 degrees is  $1/120(70+32/60+3/3600) = 0.5877847$ . Today a calculator gives 0.5877853. The difference is about one part per million.

Back around 380 BCE, Plato had asked a question of his students. Knowing that the planets behave irregularly, e.g. they exhibit retrograde motion, and yet believing that celestial objects must be eternal and move in perfect circles, he asked: “What combination of uniform circular motions about the earth describes the apparent motions of the planets?” His students and later astronomers constructed various descriptions in their attempts to answer this question. Plato’s question remained the one astronomers tried to answer until Kepler recast it in the early 1600’s.

Ptolemy worked out a very accurate description of the motions of the sun, moon, and planets as seen from earth, and in doing so used combinations of circles as Plato had decreed. His system was accurate and durable enough to last 1400 years.

## DEFERENT-EPICYCLE SYSTEM

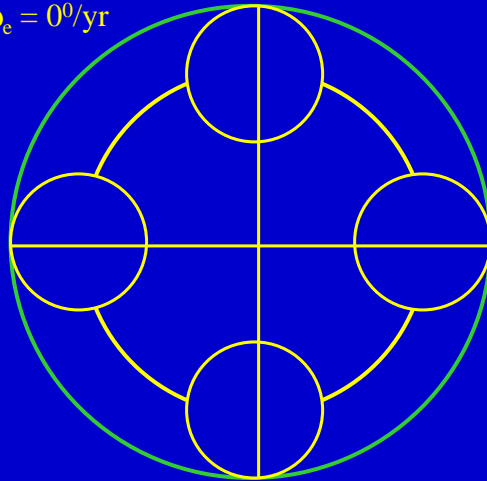


At the core of Ptolemy's approach is the Deferent-Epicycle system. At the center of the construction is the earth, labeled O here for observer. Centered on O is a large circle called the deferent. The sun, moon, or planet moves around this deferent in a specified time that is different for each celestial body. But the body does not move on the deferent itself, but moves on another circle, the epicycle, whose center moves on the deferent. P here represents the planet. The rotation on the deferent is traditionally taken to be counter clockwise as shown. The motion on the epicycle could be in either direction.

The parameters needed to specify a single deferent and epicycle are shown. The rotation rates could be specified in degrees per unit time (e.g. year), or in rotations per unit time.

## EXAMPLE

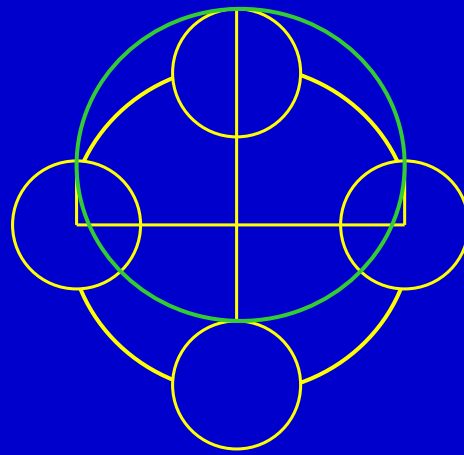
$$\omega_d = 360^\circ/\text{yr}, \quad \omega_e = 0^\circ/\text{yr}$$



As a simple example, and to illustrate one of the conventions used, consider the parameters shown above. The deferent rotation rate is 360 degrees/year. The epicycle rotation rate is zero. What direction then does the epicycle radius point as the epicycle moves about the deferent circle? The convention is that it remains parallel to the deferent radius as it rotates. In other words the epicycle angle is measured from the deferent radius at any given point in the motion.

As the example above shows, the motion described is that of a circle with radius  $r_d + r_e$ . This is simply a larger circle, which could have more easily been described by simply increasing the radius of the deferent in the first place.

## EXAMPLE



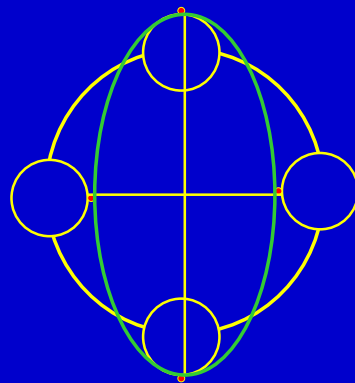
$$\omega_d = 360^\circ/\text{yr ccw}, \omega_e = 360^\circ/\text{yr cw}$$

Now consider another simple example: Both the deferent and epicycle rotate at the same rate, but in opposite directions. What shape results in this case? After the deferent has rotated 90 degrees ccw, the epicycle will have rotated 90 degrees cw. So the epicycle radius remains pointing straight up. And as we continue, the epicycle radius continues to point straight up since it always rotates back as much as the deferent has rotated forward.

The result is a circle with radius equal to the deferent radius, but with center shifted up by an amount equal to the epicycle radius. A shifted circle.

It is clear that the epicycle gives us some flexibility, but so far nothing striking has resulted.

## EXAMPLE



$$\omega_d = 1 \text{ rev/yr ccw} \quad \omega_e = 2 \text{ revs/yr ccw}$$

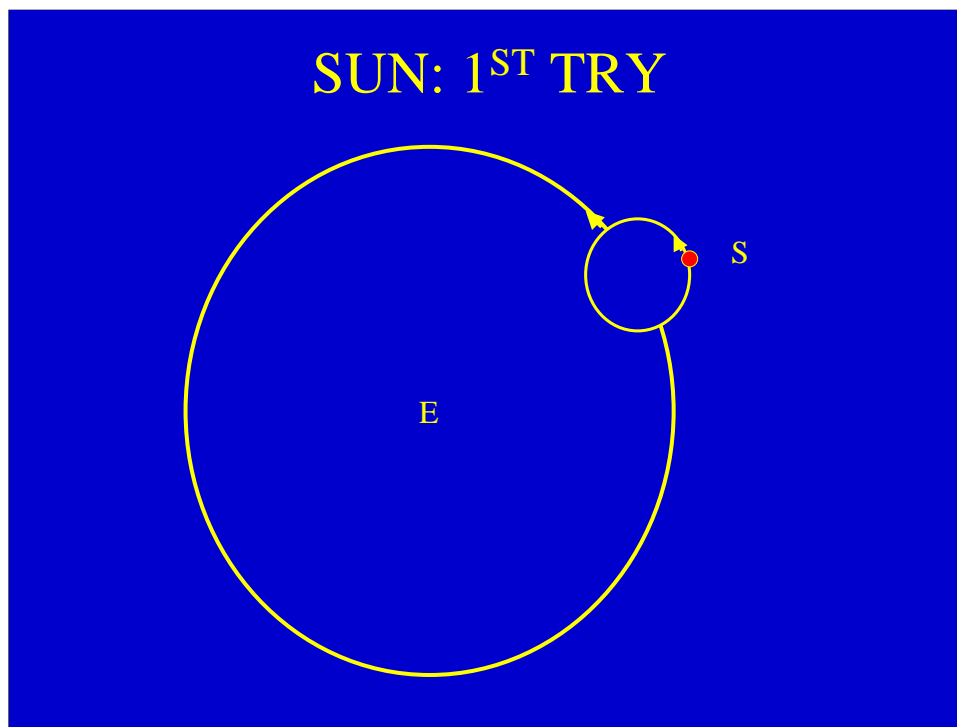
Now let's look at an example in which the shape of the orbit is changed by the epicycle motion. Above we see a case in which the epicycle rotates twice as fast as the deferent and in the same direction.

So when the deferent has rotated 90 degrees, the epicycle has rotated 180 degrees, and so the planet, indicated by the small red circle, is on the right side of the epicycle circle, closer to the origin (observer). After the deferent has rotated 180 degrees, the epicycle has rotated 360 degrees, and now points along the deferent radius like it did at the starting point. And after 270 degrees rotation of the deferent, the epicycle radius again points opposite to that of the deferent.

The resulting orbital shape is that of an oval, with greater diameter in the vertical direction and smaller in the horizontal direction than for the original deferent circle. As we will see, Copernicus used the deferent-epicycle in his model of the solar system, and this oval shape was important for him.

Further changes in orbital shape can readily be achieved. As an example a pattern obtained from Crowe's book is shown on the overhead projector. The pattern shows a deferent with epicycle circles drawn on it every 15 degrees. Each epicycle has dots on it every 15 degrees. So we can choose a ratio of rotation rates, such as three, and trace out the resulting orbit. In this case we see that we can describe retrograde motion.

The Deferent-Epicycle system is clearly flexible and powerful. It is also straightforward to use.



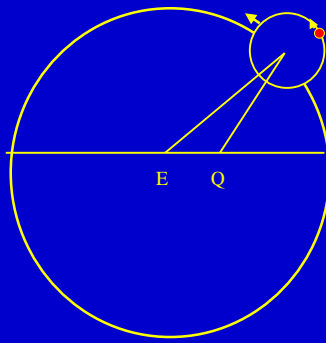
Now let's see how we can use the deferent-epicycle system to approach a real problem: The motion of the sun.

An obvious starting point is to use a deferent with a period of rotation of one year. This will describe the progression of the sun through the zodiac. But we also need to account for the variation of the seasons on earth – i.e the motion of the sun above and below the equator. The plane of the deferent above is parallel to the plane of the celestial equator (also the earth's equator). To describe the fact that the ecliptic is tilted at an angle of 23.5 degrees to the equator, an epicycle can be used.

In this case the problem becomes three-dimensional. We need an epicycle whose plane is tilted. Then if the rotation rate of the epicycle is also one year, we will have achieved the desired result.

So now we have described the sun moving uniformly about earth on a tilted plane. This however is not the whole story.

## SUN AGAIN: EQUANT OR ECCENTRIC CIRCLE



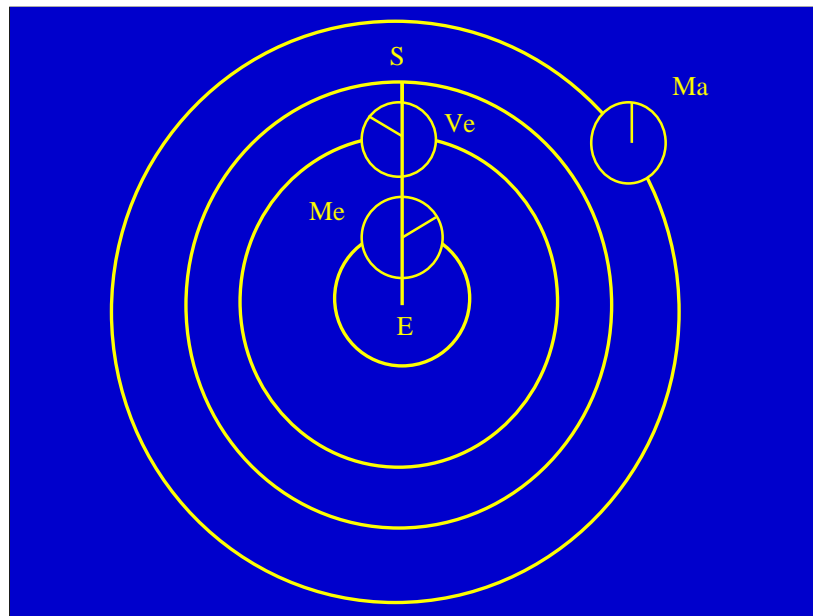
It was recognized early on by astronomers that the lengths of the seasons are not equal. Thales wrote about it, and he probably learned this from the Egyptians. In any case the time between the vernal equinox and autumnal equinox is about a week longer than the time from autumnal to vernal. In other words the summer season is a week longer than the winter season for those of us who live in the northern hemisphere. But the sun moves 180 degrees around in its orbit between the equinoxes, so it must be going slower in the summer than in the winter.

The simple deferent-epicycle model just described cannot account for this inequality because in that model the sun moves uniformly about earth.

Here is a solution Ptolemy proposed: In addition to the deferent and epicycle, a stationary axis bisecting the deferent is drawn with a point Q, called the equant, located on it, not too far from earth. The center of the epicycle moves uniformly about the point Q, not about earth. That is, the angle between Q and the axis changes at a constant rate, not the angle between earth and the axis. This will result in the sun moving more slowly on the right half of the deferent than on the left half. Note: The epicycle rotation must vary in the same way.

So the sun's motion is circular (about E), and uniform (about Q). This is bending Plato's "uniform circular" requirement, and not very esthetically pleasing, but it fits the data well. The equant is unesthetic but improves accuracy. Which is more important in a scientific theory?

It is significant that Ptolemy found another way to equally well describe the sun's motion. It is called an **eccentric circle**. The idea is to move the earth away from the center of the sun's deferent. The sun then moves uniformly about the center of the deferent, but not uniformly about earth. This simply interchanges the roles played by E and Q in the above diagram. The sun moves uniformly about Q (now the center of its deferent circle) and moves in a non-uniform manner about E. In this case the sun's motion is circular and uniform about Q, but neither circular nor uniform about E.



Here is a summary picture showing the planets out to Mars according to Ptolemy's description. All five of the planets have motions tied to the Sun-Earth motion in two distinct ways. Mercury and Venus move on epicycles whose centers lie on the radius from the earth to the sun. They move with the sun and are only seen near it, in accord with observation.

The outer three planets, Mars, Jupiter, and Saturn, each have an epicycle whose center is independent of the earth-sun radius. But their epicycle radii are parallel to the earth-sun radius. This is shown in the diagram above for Mars. As these planets move on their deferents, their epicycle motion is tied to the earth-sun motion in this way.

So Mercury and Venus have equants with a period of one year, and Mars and the other outer planets have epicycles with a period of one year.

Why are the motions of all these planets tied to the earth-sun motion? They are each independent objects in their own right; why do their motions depend on the earth-sun motion?

In ancient Greece, such "why" questions were not asked, at least not by Ptolemy. He was concerned with describing the motions of the planets, not explaining them. Asking such a question means going to a deeper level, and requires having a theory of what causes motion in the first place. Isaac Newton was the first to provide such a theory.



## ALMAGEST

“The Greatest”

Largest science compilation to date.

Best planetary model for next 1400 years

Describes order of presentation

Gives arguments why earth is spherical.

Argues earth is stationary

Ptolemy’s great tome survives because it was translated by Arabic scholars and reintroduced into Europe in medieval times. The title is a combination of the Arabic definite article and the Greek root meaning greatest. The implication is that it was called something like this by the Greeks before the Arabic translation. Ptolemy simply called it “mathematical compilation”. It deserves the title it was given by others however. Ptolemy compiled 400 years of astronomical observations, and used them to build the best planetary model of the ancient world. He set a high standard by writing such a thorough book. It was by far the largest compilation of mathematical and astronomical results ever assembled. This influenced Copernicus significantly.

Ptolemy begins, in a preface in book 1, explaining why the study of astronomy appealed to him in contrast to other branches of philosophy: “From all this we concluded: that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them, and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously.” This sounds familiar. What we choose to study is often decided as much by negative comparisons as by positive attributes.

Then he describes the order in which he will discuss the various celestial objects – sun and moon, fixed stars, then planets. He then speculates how the ancients may have come up with the idea of the starry vault, and argues against the primitive idea that the stars are extinguished each day and rekindled each night. He presents arguments as to why the earth must be spherical, is in the middle of the starry vault, and is very small compared with the distance to it. Then he argues that the earth does not move or rotate.

We will compare some of these arguments with those made by Copernicus in his great work presenting a quite different model of the universe.

## COPERNICUS

Canon lawyer dedicated to the Church.

But spent evenings working out a model of the solar system that disagreed with Church dogma.

Ptolemy was the last of the great Greek astronomers. Worse yet, the knowledge of the Greeks was nearly lost to the world. The great library and museum at Alexandria was sacked and the books burned. People simply ceased becoming educated about these questions. This is the beginning of what is called the *Dark Ages* in Europe. We must jump ahead by nearly 1.5 millenia until further progress occurred. This kind of instability of knowledge, evidenced by dark ages, is not unique to Europe. Egypt, in its long history, suffered several periods when nothing important seemed to happen.

By the year 1500 the only thing that had changed regarding science was that Thomas Aquinas had joined Aristotelian ideas including those about the solar system with Christian theology. So the geocentric model had acquired a new meaning: It had become Church dogma, and to question it was to doubt both the Church and Aristotle.

But times were changing. The Renaissance movements spread from Italy throughout Europe producing a new ideal person – a confident individual full of curiosity. The invention of printing assured that the works of a writer could be rapidly distributed to a growing audience. Even the works of the ancient Greeks, preserved and returned to Europe by Arab scholars, were more widely available than they had been when they were first written.

Nicolaus Copernicus (1473-1543) was born in Torun, Poland. He attended Krakow university (beginning at age 9) after which he became a canon of the cathedral at Frauenburg in Poland. He continued his education at three Italian universities, studying law and medicine, and earning a doctorate in canon law. He was clearly very dedicated to the Catholic church, intending from an early age to make it his career. He spent the rest of his life performing various administrative duties for the chapter of canons at Frauenburg, and, remarkably, in his spare time, remodeling the universe according to a model that flatly disagreed with Church dogma. His interest in astronomy was well known to people around him. He established a room for astronomical observation at the Cathedral, and told people about his ideas of the universe through a preliminary draft, written 30 years before the final publication of his completed work in 1543.

## COPERNICUS

Realist (Copernicus): Model is meaningless unless it is or could be physically real

Instrumentalist (Ptolemy): A model is useful if it describes observations. It need have no reality.

Copernicus was concerned with the old problem of Plato – the construction of a model of the planets using the fewest possible uniform circular motions. He was not a radical with a new idea who wanted to replace the old model, instead he just wanted to clean up the old model. He was particularly unhappy with the equant, introduced by Ptolemy, which he thought inelegant. He pointed out that his proposal of a **heliocentric model**, with the earth and other planets revolving around a fixed sun, was not a new idea. As we have seen, Aristarchus had proposed such a model nearly two millenia earlier.

Copernicus was a realist. He wanted to believe in the reality of a model of the solar system. It was not enough for him to simply fit the data as Ptolemy had done. The fact that Ptolemy could describe the motion of the sun equally well with an equant or an eccentric circle, and seemed happy with both, bothered him greatly. He could believe in neither.

This disagreement highlights a distinction between two points of view:

**REALIST:** Understands a model as being physically real; if not, it has no meaning.

**INSTRUMENTALIST:** A model is useful as a means of fitting data. It need have no reality.

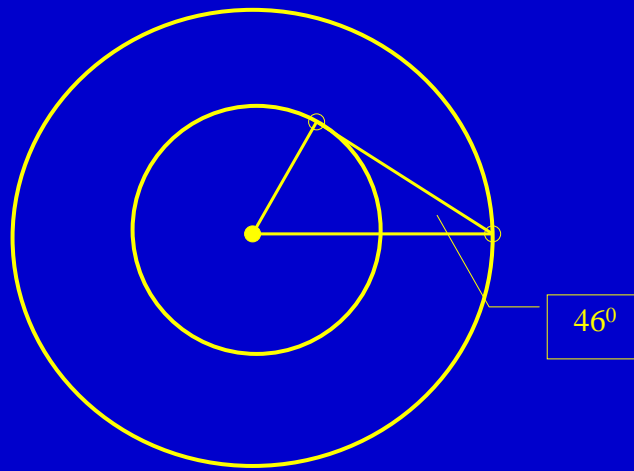
According to Copernicus, the earth rotates daily on its axis, thereby making the sun, moon, stars, and planets appear to revolve about the Earth. Not only does this simplify the explanation of planetary motions, it allows the outer sphere of fixed stars to be at rest. Copernicus apparently did not foresee that this apparently trivial change could lead to significant consequences: If the stars no longer have to rotate together they do not all need to be the same distance from Earth. Perhaps they are scattered through space at varying distances. Suddenly we have the possibility of an infinite universe.

# COPERNICUS

Copernicus dedicated his book, *Revolutions*, to Pope Paul III, and suggested among other things that it might be of help in establishing a more accurate calendar, a project the Church had been discussing for some time. In fact, when the Gregorian calendar was established in 1582, the Copernican system was used in doing the calculations because it was easier than the Ptolemaic system. The Church recognized it as a more efficient calculational device, but did not accept it as a correct picture of the universe. So the Church took an instrumentalist view of the Copernican model.

The Gregorian calendar, which we now use, replaced the Julian calendar with its leap days added every fourth year. In the Gregorian calendar leap days are not added if the year is divisible by 100 unless it is also divisible by 400.

## DISTANCES OF THE PLANETS TO THE SUN



One feature of the Copernican system that was not present in the Ptolemaic model, is that the relative distances of the planets from the sun could be found using geometry. The example above shows Venus and Earth in their orbits about the Sun. Epicycles, needed to make the orbits more accurate, have been left off for simplicity. When we observe Venus at its greatest angular distance (or elongation) from the Sun, the angle between the Sun and Venus is  $46^\circ$ . The line of sight between Earth and Venus at that time is tangent to Venus's circular orbit. This means that the angle between the line of sight and the radius from Venus to the Sun is a right angle.

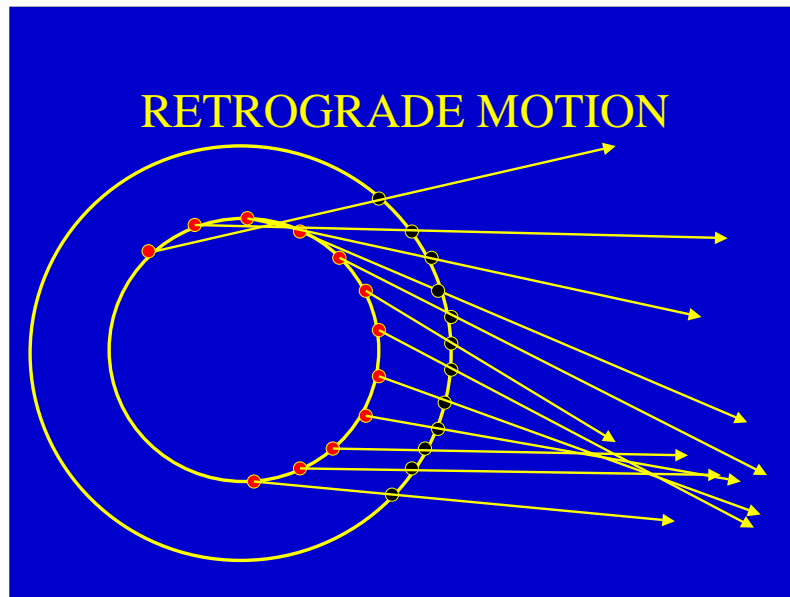
So now we can find the distance from Venus to the Sun in terms of the distance from Earth to the Sun. If we call the distance from Earth to the Sun 1 AU (Astronomical Unit), then this distance is the hypotenuse of the right triangle in the above diagram. In trigonometry, the sine of an angle in a right triangle is equal to the ratio of the side opposite to the hypotenuse. So the distance from Venus to the Sun,  $R_V = (1\text{AU}) \cdot \sin(46^\circ) = 0.72 \text{ AU}$ . Similar arguments can be made for the other planets. So the Copernican model immediately gives us distances from the sun to the various planets. Information not contained in the Ptolemaic model.

In Ptolemy's model, the deferent of each planet was set equal to 60 units by convention (he used the sexagesimal system). Although the sizes of the deferents meant nothing, the sizes of the epicycles relative to the deferents is meaningful in Ptolemy's model.

## PLANETARY DISTANCES AND PERIODS

PLANET	RADIUS	PERIOD
Mercury	0.387	80 days
Venus	0.723	9 mo
Earth	1.00	1 yr
Mars	1.52	2 yr
Jupiter	5.20	12 yrs
Saturn	9.54	30 yrs

Copernicus now had an orbital radius and period (time to go once around the Sun) for each planet. These radii and periods are shown in the table above. It is clear that the period increases uniformly and rapidly with the radius. This regularity of the model is a hint that it is physically meaningful. If the period is going to vary with some other orbital characteristic, it is reasonable that it be the radius.



The Copernican model had a simple explanation of retrograde motion. As an example consider Earth and Mars. The Earth moves faster around its orbit than does Mars, since Mars has a larger radius orbit. So when Mars is in opposition, Earth moves past it. As we look toward Mars, we see it seeming to move backwards, even though it is moving forwards in its orbit as usual.

This is just like passing a slower car on the highway. The other car appears to move backward even though we know it is moving forward in its lane. We are never confused about this on the highway because we can see the trees, buildings, etc that are stationary, and can see that the slower car is moving past them. But if we could not see those stationary markers, we would not be so sure that the other car was moving at all.

Of course the speedometer in the faster car might be reading 70 mph, and we can see that we are passing the other car at a relative speed of about 10 mph, so we know it is moving forward. But without the speedometer, or the stationary markers, we would not know at all whether it was moving or how fast.

The above diagram shows a series of positions for Earth in the smaller orbit, and one of the outer planets, e.g. Mars. The arrows indicate the direction we need to look to see the other planet. We judge the direction of those arrows by seeing which stars are behind Mars. As Earth passes Mars, it clearly moves backwards for a time, then resumes its usual ccw rotation.

## SUPPORTING ARGUMENTS

He followed Platonic principles

Lots of quantitative analysis (comparable to Ptolemy)

We speak of the Copernican Revolution, and deservedly so. The eventual acceptance of the Copernican model of the universe changed our conception of the place of humans in creation heretofore thought, and our presence correspondingly reduced. It is remarkable that – that we were no longer at the center, and that it was much larger than this great change was brought about by a person who was no revolutionary. He was indeed very conservative. He was bothered by the fact that the Ptolemaic system could be constructed in different ways – either with equants, or with eccentric circles (with the Earth positioned away from the center of the circle on which the celestial body moves), along with deferents and epicycles, so it was difficult for him to believe in the reality of any of these arrangements.

Copernicus did not introduce any new mathematical methods in his analysis. He did make a few observations, but mostly relied on the same data Ptolemy had assembled. All he did was suggest the seemingly trivial change of moving the origin of coordinates from the Earth to the Sun. He would be appalled to know that today we regard him as having started a revolution.

Knowing that many would regard his proposal as repugnant, Copernicus not only carried out countless calculations to show that his system worked as well as that of Ptolemy, but he also supported his idea with careful arguments:

**a. He** argued that his methods coincided with Platonic principles at least as well as Ptolemy's. He argued that his own system was more orderly and pleasing. To him, finding symmetry and order in the apparent chaos of the world of the senses was an act of reverence, proof of the beauty of Creation.

**b. He** prepared enough quantitative work to put his book on the same footing as Ptolemy's. This is in striking contrast to the proposal of Aristarchus. Both models could describe planetary data within the then current angular uncertainty of about 1/6 degree of arc (or 10 minutes of arc).



## SUPPORTING ARGUMENTS

Specific responses to several objections  
(spinning Earth should burst, clouds  
left behind, stellar parallax)

Geometric simplicity

**c. Copernicus** tried to answer several objections that he knew would be raised against his proposal, as they had long ago when Aristarchus made the same suggestion. To the question of why the earth, rotating so rapidly, does not burst apart, he answered with another question: “Why does not the defender of the geocentric theory fear the same fate for his rotating celestial sphere – so much faster because so much larger?”

To the argument that clouds and birds in the air would be left behind by the rapidly rotating earth, he answered that the air was dragged along by the surface of the earth.

As to the absence of stellar parallax, Copernicus could only argue that the starry vault is much farther away than had been thought.

**d. Probably** to us the greatest success of the Copernican system is its geometric simplicity. For example, by eliminating the epicycles needed to describe retrograde motion in the Ptolemaic system, he had indeed arrived at a simpler model. Fewer parameters were needed in solving practical problems such as constructing a calendar. And indeed, the Church did use the Copernican system in constructing the Gregorian calendar in 1582.

# OPPOSITION

Central position of Earth in Creation

Celestial Objects made of aether

More than a century passed before the heliocentric model was generally accepted by astronomers. Opposition to it came from several sources:

**a. The** old argument from dogma about the central position of the Earth in Creation was probably foremost in many people's minds. Copernicus was in general not successful in persuading his readers that the heliocentric model was at least as close as the geocentric model to the mind of the Creator. All major religious faiths in Europe found enough biblical quotes to convince themselves that God used a Ptolemaic plan. Indeed the Roman Catholic Church eventually put the *Revolutions* on the Index of forbidden books.

**b. Aristotelian** physics held that all the celestial objects are made of aether. This element was supposed to have no weight, and its natural motion was circular. From that point of view, causing the massive Earth to rotate seemed a greater task than allowing the aetherial spheres to go their natural way.

## OPPOSITION

No stellar parallax (too much empty space)

No smoking gun

c. **Lack** of observation of a stellar parallax remained a problem. It was widely argued that empty space was a repulsive idea. Why was it created if not for some use? The idea of greatly increasing the emptiness of our Universe was not a popular one among those thinking about this problem. Even a positive observation of a stellar parallax would not likely have been decisive. Building this in to the Ptolemaic model would simply have required adding an epicycle to the motion of the starry vault. The observation of several different stellar parallaxes among different stars would have been a problem for this model, since it assumed all the stars are the same distance from earth.

d. **No Smoking Gun.** There was then no important astronomical observation that could be accounted for by only one competing model and not the other. As Francis Bacon wrote in the early 17<sup>th</sup> century, “Now it is easy to see that both they who think the earth revolves and they who hold the primum mobile and the old construction, are about equally and indifferently supported by the phenomena.”

Today we can detect the rotation of the earth easily. We have gyroscopes so precise they can measure the rotation of this building when the sun rises and heats one side more than the other. But this slow rotation of the earth is a subtle effect, difficult to notice. Today we interpret many things in terms of the rotation, such as the direction of rotation of hurricanes and cyclones, and the direction of the trade winds. Understanding these things requires having a well established theory of motion. What is it that causes things to move? Without such a theory, it was not possible to detect and demonstrate the rotation of the Earth. Copernicus’ model had a rocky start, but he was moving in the right direction to lead other scientists toward the development of just such a theory.

## CRITERIA FOR A GOOD THEORY IN SCIENCE

Correlates many separate facts

Suggests new relations and stimulates research

Makes new testable predictions

Hypotheses are few and plausible

Flexible enough to grow. When it dies, it leaves an even better descendant

We are comparing two well-defined and distinct models or theories of the universe. It is appropriate that we pause briefly to discuss what good theories should do in science, using these two as examples.

1. A good theory correlates many separate facts, such as observations gathered over time, in a logical structure of thought. Aristotle wrote that we should progress from the complex to the simple as we work. Both models satisfy this criterion.
2. As it is used, a good theory suggests new relations and stimulates research in science. Neither model obeys this criterion very well. They are both geometric descriptions, about equally accurate. The Copernican model did influence thinking in other areas: It restated man's place in the Universe and so caused a re-thinking of philosophical and religious ideas.
3. A good theory permits us to predict new things that can be tested by measurement. At the time of Copernicus, neither model was able to do this, but this would change as time went on.
4. A good theory will have few hypotheses that are plausible. The major difference between the two models we are comparing is that one uses the Earth as origin of coordinates, the other uses the Sun. These are well defined, simple, and indeed few in number. Plausibility is often not achieved since often great ideas appear at first to be absurd. In our examples, the assumption of Copernicus that the Sun is the center of the universe certainly was not plausible to most of the people who reacted to it.
5. A good theory is flexible enough to grow if needed. And if it eventually dies, it leaves an even better descendant. The Ptolemaic model certainly was able to grow over time as observations became more accurate, adding epicycles as needed. And when that model finally died, it left the Copernican picture to replace it. And when the Copernican model died, it was replaced by Kepler's even better description of planetary motions.

The above list is reasonable and consistent with history. It is, of course, just conventional wisdom. Finally, however, there are no rules. New theories are created by creative people, and there is no set of guidelines that leads to a Newton, a Galileo, or an Einstein.