

## TYCHO BRAHE AND JOHANN KEPLER

An unlikely partnership developed between these two very different people. It turned out to be very important for the history of science. Let us see how this came about.

Tycho Brahe was born in 1546, a member of the royal family of Denmark, but not closely related to the King. At a very young age he was kidnapped by a childless rich uncle and subsequently raised by him. He attended the University of Copenhagen with the intention of studying law. While there he made his first recorded astronomical observation: A conjunction of Jupiter and Saturn, in August 1563. He found that most of the existing almanacs were grossly in error. Even the recent tables of Copernicus were several days off in their prediction of the date of this event. This led him to decide to devote his life to making more accurate observations of astronomical data.

During the 1560's he traveled widely around Europe, learning about astronomy and acquiring instruments. His most notable acquisition was a 19 foot quadrant for accurately locating star positions. Before the telescope measuring stellar angular positions was a matter of lining up cross hairs. The farther apart they were the more accurately this could be done, up to the limit of human eye acuity. So this meant using large size instruments. This was the beginning of big science.

In 1571 he returned home upon the death of his father, where he set up a small observatory. His reputation was made the next year when he observed a new star come into existence – a nova. He showed by careful measurements over a period of time that (due to its lack of parallax) it was farther away than the moon. This was a shocker, because the Aristotelian doctrine was that the heavens are eternal. Stars are not supposed to change at all – in position, brightness, or number. This nova was for a time brighter than Venus, but remained in the same position as stars but not planets do. Now he was no longer a Danish dilettante, he was an astronomer with an international reputation.

Knowing that large instruments were the key to accurate measurements, he proposed building a large observatory in Germany, and began raising money for it. This prompted King Frederick II of Denmark to try to keep him in Denmark by giving him the island of Hven, near Copenhagen, along with an income for construction and expenses.

Tycho built an observatory there which he called Uraniborg. He had an artistic flair and invited craftsmen from all over Europe to help with the finishing touches. It became as much a palace as an observatory. It had its own printing shop so he could publish his results as he saw fit, equipment for building instruments, many fresh water ponds for fish for the chef, and, most unusual for the time, flush toilets (of his own design). Astronomers and others came from all over Europe to see this architectural and scientific achievement. He estimated that it had cost the King a ton of gold, but the King was happy because it brought notoriety to Denmark.

What Tycho did was reduce the angular error of stellar measurements from 10 minutes of arc to about  $\frac{1}{2}$  minute of arc. Then over 20 years he compiled a collection of data on stellar and planetary positions that far exceeded anything the world had seen earlier. Not only were the measurements more accurate, but he measured planetary positions at much more frequent intervals than had been the practice. Ptolemy, for example, only measured planetary locations at certain positions in their orbits, since he was assuming the orbits were circles, he did not need very closely spaced data. Tycho made no such assumptions.

Frederick II died in 1588, and his son was not as generous to Tycho as he had been. Things went from bad to worse, and eventually Tycho left Hven in 1597 to travel, looking for a new patron. He found one in Emperor Rudolf II of Prague, where he became court mathematician in 1599. Shortly after arriving in Prague, Tycho had the good fortune of hiring a bright young man named Johann Kepler as his assistant.

Kepler had been born in 1571 in a small city in the Holy Roman Empire (now Germany) to a poor family. He attended the University of Tübingen on a scholarship, studying to become a Lutheran minister. There he learned about astronomy from Michael Mastlin, who, unusual for that time, was a Copernican. When he had nearly completed his studies, the mathematics teacher at the high school in Graz died, and Kepler was recommended for the job. He quit his theology training and began teaching in Graz.

While teaching a class in 1595 a striking idea came to him. There are six planets orbiting the sun in the Copernican picture, each a known distance from the sun. Why six and not 10 or 100? And why at those distances from the sun? Kepler's new idea purported to answer both of these questions.

Here is the idea. Each of the planets is located on an Aristotelian sphere that rotates. The smallest sphere is that of Mercury. What is needed is five "spacers" to separate the planets' spheres appropriately. He chose as spacers the five regular solids of ancient Greece – the same ones Plato used to represent the elements and the zodiac.

Starting from the sphere of Mercury, construct an octahedron, so the sphere is inscribed within the octahedron. Outside the octahedron construct the sphere of Venus. Outside that construct an icosahedron. Outside that is the sphere of the earth. Construct a dodecahedron around the sphere of the earth, then the sphere of Mars outside it. Around the sphere of Mars construct a tetrahedron, then the sphere of Jupiter. Outside that construct a cube, outside that being the sphere of Saturn.

This scheme reproduced the orbital radii within an accuracy of about 5%, and also "explained" why there are six planets, one more than there are regular solids in geometry. This was an ingenious idea, and it demonstrated Kepler's ability in three dimensional geometry. It is no trivial task to inscribe these solids within spheres.

This is a Pythagorean-Platonic mystical idea. Much like Pythagoras' idea that number is all in the universe, extended to the three dimensional solids of Plato. In that sense it is a throw-back to ancient ideas rather than a step forward. But it got him a job. He sent copies of his work to several scientists, including Tycho and Galileo. This became one of the first published endorsements of the Copernican system, Tycho was impressed by his mathematical strength and offered him a job as his assistant, which Kepler accepted, arriving there in 1600.

Right away Kepler became very popular for his astrological predictions, made for various court members. And right away, Tycho and Kepler did not get along well. Tycho was a laboratory scientist who could build things and make them work. Kepler was an idea man, younger, bright, and very self-confident. Kepler found he had difficulty gaining access to Tycho's huge collection with 20 years worth of collected astronomical data – the most accurate in the world.

Things were going downhill for Tycho. He had always enjoyed partying. One evening he slipped going downstairs and fell, bursting his bladder. He died a few days later. The lawyers from Copenhagen were on their way to claim Tycho's possessions and divide them among the members of the Royal Family (Tycho had married a commoner).

Kepler recognized the importance of Tycho's data, and believed it might all be lost if it went back to Copenhagen. So he went to Tycho's apartment, stole the catalog of data carrying to his room, and described this as being the normal state of affairs when the lawyers arrived. Kepler was then appointed court mathematician (at one-sixth the salary Tycho had received) and was able to continue studying Tycho's data.

Kepler now had a stable job, and some assistants. He chose to study first the orbit of Mars. But before he felt he could do this, he needed to understand the effect of the refraction of the earth's atmosphere, since this could alter some of the data. He succeeded in doing so, and then went on to show how a lens focuses light. In particular he provided the first clear explanation of how the human eye focuses images on the retina, and what near-sightedness and far-sightedness are caused by (he was very near-sighted). He also provided the first explanation for how telescopes work.

Kepler's preliminary analysis of the orbit of Mars showed it to be close to that of a circle of radius 142 million miles, with the sun 13 million miles off center. Furthermore, Mars varied in speed as it went around, going faster when it was nearer the sun. The way Ptolemy handled this problem was with an equant, which Kepler tried. He found, however, that this left an error of 8 minutes of arc even after all the parameters had been optimally adjusted. Recall that the data Copernicus had to work with had errors of at least 10 minutes of arc, whereas Tycho's data was accurate to  $\frac{1}{2}$  minute of arc. So Copernicus would not have noticed this discrepancy, but Kepler knew it was real.

Kepler could have used an epicycle – that is what Copernicus would have done. Kepler, however, had taken a new conceptual approach to thinking about the planets. He imagined that they were driven around their orbits by a force from the sun. He was the first to think of a force from the sun playing a role. At one point he proposed that this force was due to magnetism. In any case, thinking like that made it difficult to understand why a planet would move around in a little circle with nothing at its center. Kepler insisted on taking a **realist** approach to modeling the planetary motions.

This discrepancy caused Kepler to throw out the Copernican model entirely. Note that this is a voluntary choice based on his attitude. He refused to add epicycles. But now, of course, he had no model of the motions of the planets.

Kepler realized that to get the most out of Tycho's data, he first needed to determine the Earth's orbit, since all planetary observations are made from Earth. The only way to do this is by triangulation, using two fixed objects in space which, together with Earth, would form a triangle. The sun is an obvious choice for one fixed object but what could he use for the other? The stars are too far away and the planets are all moving. He decided to use Mars. Here is the trick he used to get rid of the motion of Mars.

He did not yet know the orbit of Mars precisely, but he did know that its period was 687.1 days. Using Tycho's voluminous data he could locate Mars at a particular time and again 687.1 days later when the Earth was in another position. This creates a fixed Mars-Sun baseline along with different Earth positions. Just what is needed for triangulation. Doing this repeatedly allowed him to locate the Earth's position in its orbit many times and thus to map out the orbit.