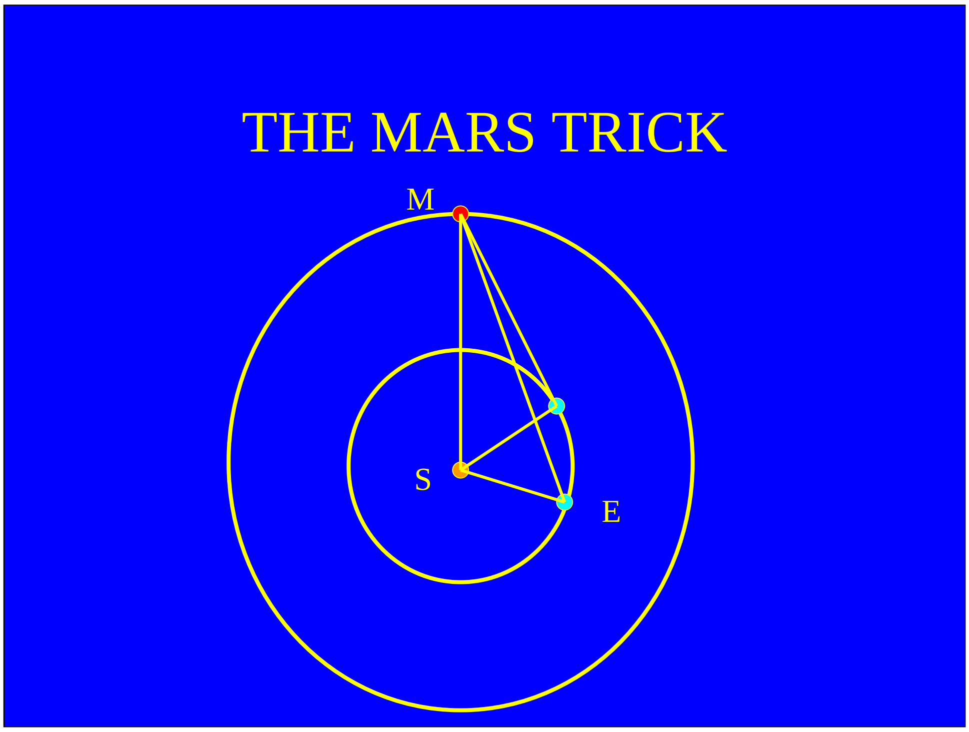
JOHANNES KEPLER
and
TYCHO BRAHE

Tycho: Large instruments, 20 years worth of accurate data.

Tycho hired Kepler as an assistant in Prague.

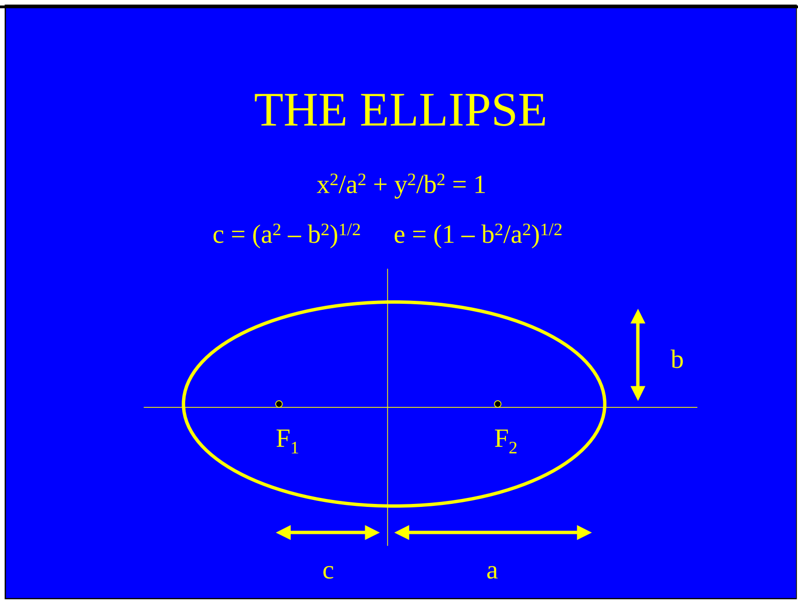
When Tycho died, Kepler began his lengthy study of the data.

See word document KEPLER-TYCHO.



Using Tycho's data at 687 day intervals, Kepler was able to locate the Earth in its orbit by triangulation with Mars and the Sun always at the same fixed points. He found the Earth's orbit to be closer to a circle than that of Mars, with a mean radius of 93 million miles, and the Sun located 1.5 million miles from the center. He then used a circular orbit with an equant to paramaterize the position of the Earth at all times in its orbit. Now he was ready to approach the question of the orbit of Mars able to make full use of Tycho's data.

After several years' work he eventually tried a radical new idea: **The ellipse.** Two thousand years before Plato had defined the problem in terms of presumed circles: "What combination of uniform circular paths accounts for the motions of the planets?" Since then all astronomers had tried to answer Plato's question until Kepler allowed himself to think of another orbital shape.



The ellipse is a figure that had been known to students of geometry since the second Century BCE. In the long direction, the diameter, or length of an ellipse is called the major axis, equal to 2a in the above standard notation. The minor axis is the perpendicular diameter equal to 2b. What we see is a plot of the shape of an ellipse in the x-y plane with the x coordinate plotted horizontally and the y coordinate plotted vertically.

An ellipse has two foci F_1 and F_2 located on either side of the center along the major axis. One of the properties of an ellipse often used to draw them is that the distance from one focus to any point on the ellipse and then to the other focus is a constant. If two pins hold the ends of the string down at the two foci, and a pencil used to stretch the string tight, an ellipse will be created as the pencil is moved around the two foci.

Two equivalent parameters are often used to characterize the shape of an ellipse: the ellipticity e, and the distance from the center to the foci, c. It is clear that e = c/a. When b = a, the ellipse reduces to a circle with equation $x^2 + y^2 = a^2$, and a is the radius of the circle. When e = 1, the ellipse becomes a straight line along the x axis.

The ellipse is one of those geometrical figures that looks graceful whatever its ellipticity. It is used much in art and design, and has the advantage that it can easily be generated by computers.

Kepler found that he could describe the orbit of Mars very well with an ellipse. No epicycles, no equants, just an ellipse with the right values of a and b oriented correctly in space, with the sun located at one focus. He found the same was the case for the other planets.

For all the planets e is small. This means the ellipse is close to a circle, so the two foci are close the center and to each other.

KEPLER'S FIRST LAW

The planets move in ellipses with the sun at one focus.

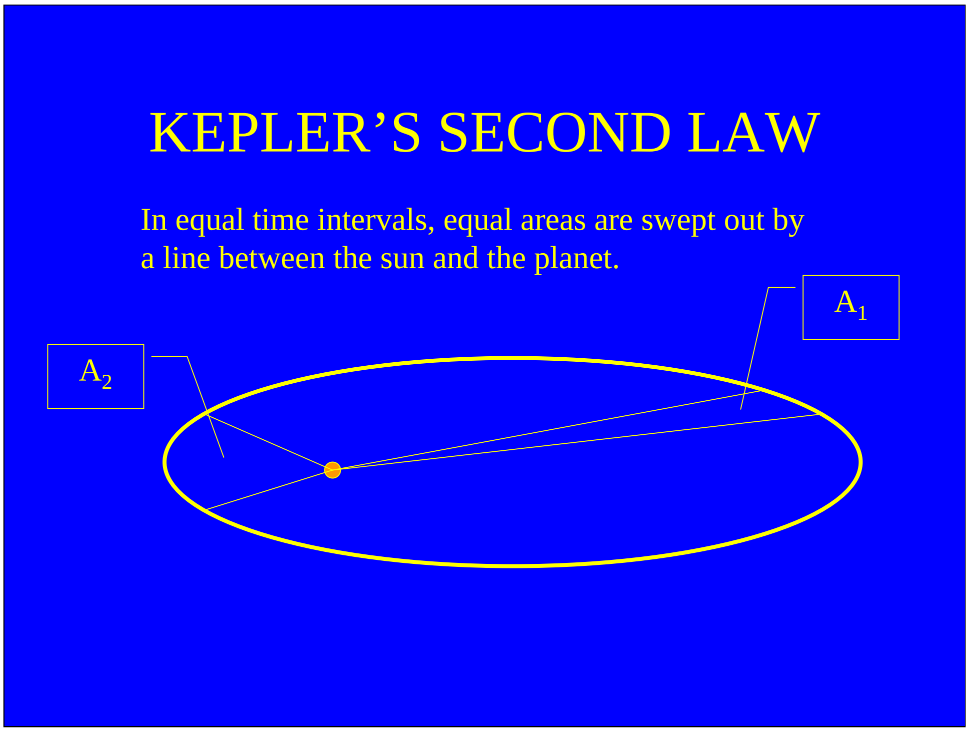
Body	a	e
Moon	0.24	0.05
Mercury	36.0	0.21
Venus	67.3	0.01
Earth	93.0	0.02
Mars	141.7	0.09

The semimajor axes above are given in millions of miles.

Note that with the sun at one focus, nothing is at the other focus.

The elliptical shape immediately creates a mystery. Circles had been used up until Kepler, but that was a man-made choice: Plato had said so. Who was saying the ellipse? Not Kepler, he was just fitting data. Nature said so. Why did Nature prefer the ellipse to some other shape? Kepler had no explanation. His result is empirical.

This is the first of three "Laws" Kepler put forward during the first two decades of the 17th Century. The use of the term Law is not well defined in science. These three probably should be called something else, like Observation, or Conjecture, or Relation. Law sounds like it is founded on some deep principle we all have confidence in, and this is not the case. Conjecture sounds much too tentative – these statements are based on years of work with excellent data, he did not make them up.



Kepler arrived at this law by using an incorrect model to deduce the result, and then checking it against the data. He found his model's prediction was correct, but that does not mean the model is correct. His model was:

1. Planets are driven around the sun by a force from it.
2. The force is inversely proportional to the distance from the sun. His idea was that the force is spread equally over the circumference of the orbit, so at twice the radius, the force per unit length of the orbit would be half as much.
3. The speed of a planet is proportional to the force pushing it. This is an Aristotelian idea that we now know to be incorrect, except for objects moving slowly through viscous liquids. Here we have frictionless outer space.

Putting these ideas together Kepler deduced that the orbital speed should vary inversely with the radius. Then for small time intervals, the above areas become triangles. The area of a triangle is ½ the base times the height. The base is just how far the planet went in the time interval, = vt. The height is the distance to the sun r. but if v varies as 1/r, then vr is a constant.

Kepler's second law is correct for reasons completely different from those he started out with. He was wised enough to finally conclude that his empirical result should stand alone and not be used as a justification for his model.

Despite his model being incorrect, Kepler was the first person to discuss the idea that planetary motion is caused by a force from the sun. A major conceptual step forward.

KEPLER'S THIRD LAW

$$T^2 = Ka^3$$

Where T = period of planet's orbit,
a = semimajor axis of orbit.

Planet	T	a	T^2/a^3
Mercury	0.241	0.387	1.00
Venus	0.615	0.723	1.00
Earth	1.00	1.00	1.00
Mars	1.88	1.52	1.00

After obtaining his first two laws in 1609, Kepler worked another decade before discovering his third law stated above. This result connects the motions of all the planets in contrast to the first two laws. The semimajor axis a is also equal to the mean distance from the planet to the sun during one orbit.

Kepler's third law can be used to connect the orbits and periods of any two or more satellites in orbit about another body. For example the moons of Jupiter, or communications satellites in orbit about Earth.

Kepler created a simpler and more accurate description of planetary motions than had been known earlier. For this alone he is famous. In addition he grew during his lifetime, from a mystic trying to describe the planets' orbits in terms of the Platonic solids, into a scientist dedicated to allowing the **observed facts** to be the ultimate arbiters in comparing theories. In addition, **mathematics, used appropriately**, emerges with Kepler as the natural language of science.