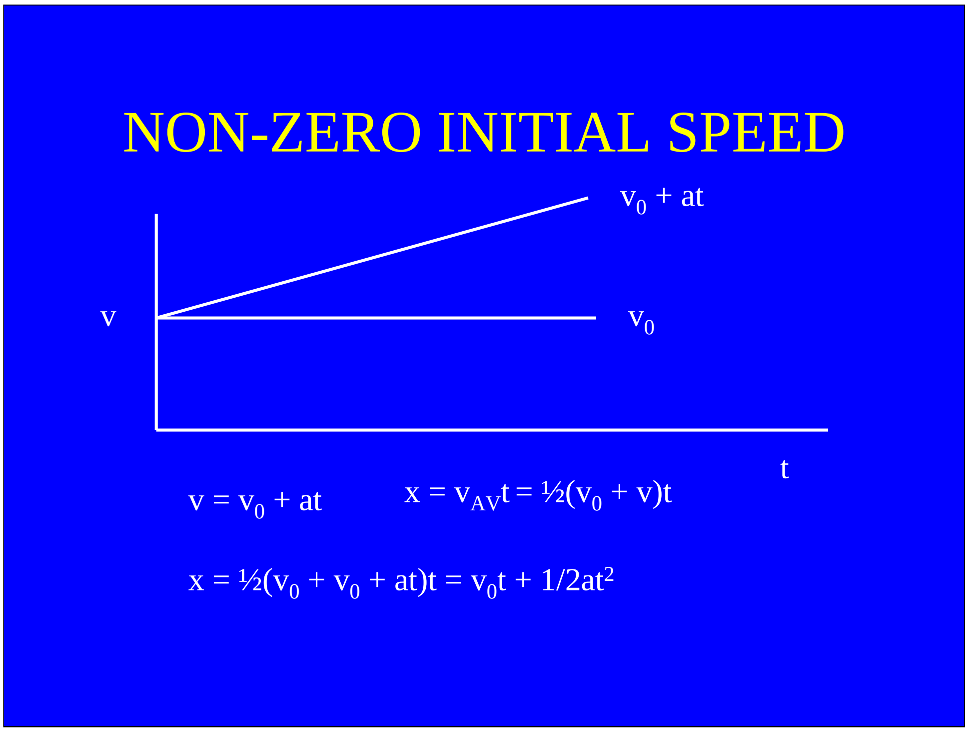
GALILEAN RELATIVITY

Projectile motion

The Principle of Relativity

When we think of the term relativity, the person who comes immediately to mind is of course Einstein. Galileo actually understood what we now call the principle of relativity before Einstein. He explained it carefully in his book Two New Sciences.

Today we will discuss what Galileo understood about motion including projectile motion, and the principle of relativity.



Suppose we are observing a motion in which the initial speed is not zero as we have assumed so far. For example we could throw a rock upwards in the air. How do we handle this?

We describe this graphically above. An initial speed v_0 exists, and acceleration begins at $t = 0$. This just means that the speed increases (or decreases) starting from its initial value.

How can we evaluate the distance traveled? The distance is always given by the average speed times the time. For uniform acceleration, the average speed is just the average between the initial and final values. Inserting our expression for the final speed, we have the result shown above.

A simple interpretation of this result is that the object moves as though its initial speed continued unchanged and in addition the acceleration occurred. It is as though the initial speed and the acceleration are unaware of each other.

EXAMPLE: THROW ROCK UP

Choose upward to be positive displacement and speed. (This choice is arbitrary)

Then $v = v_0 - gt$

Let $v_0 = 5 m/s$ and $g = 10 m/s^2$

When does the rock get to the top of its motion?

$v = 0 = 5 - 10t$ so $5 = 10t$ and $t = 0.5s$

How high did it go?

$y = v_0 t - 1/2 g t^2 = 5(0.5) - 5(0.5)^2 = 2.5 - 1.25 = 1.25m$

Let's look at a simple example illustrating these results. This is something we have all done. Throw a rock in the air. To describe motion we must choose a coordinate system (think Ptolemy and Copernicus). Of course our coordinate system is fixed on the surface of Earth, but we must still choose whether up or down displacements will be labeled positive. This is arbitrary. The initial speed and g are opposed so one has to be negative.

When does the rock reach the top of its motion? How do we define the top of the motion. Try tossing your pencil in the air in front of you. What happens at the top? It stops and turns around. When you walk down the hall and turn around because you forgot something, you must stop in the process.

So to find t at the top, set v = 0 and solve for t. To find how high it went, we just use our expression (with correct signs) for displacement including an initial speed.

You could now find how long it stayed in the air by finding the time for it to fall from rest from a height of 1.25 m.

QUESTION

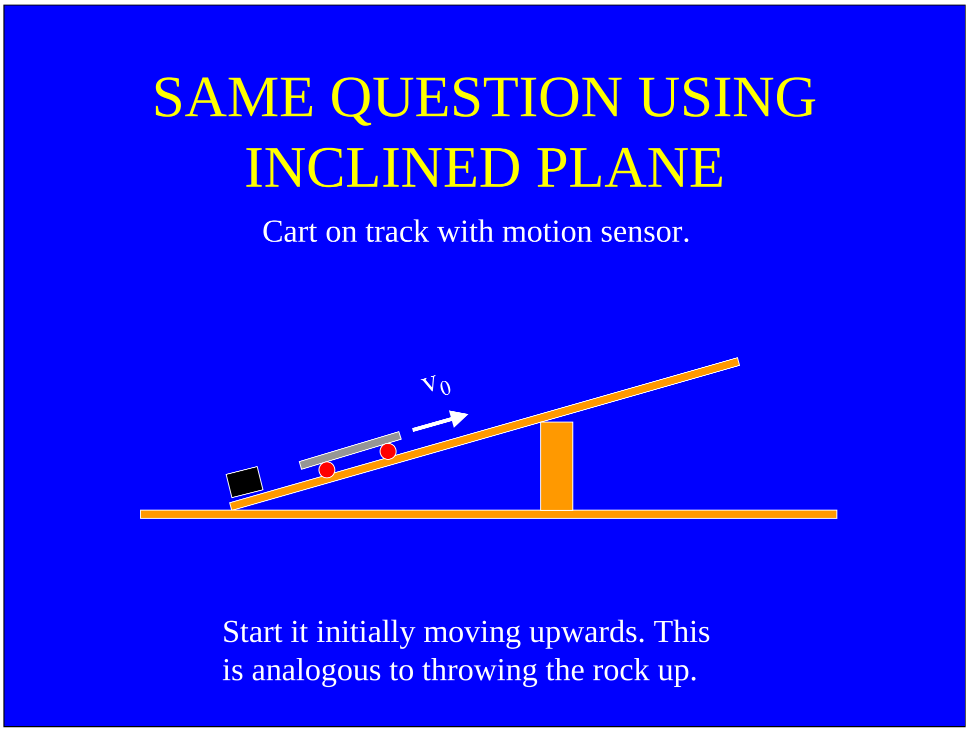
For the rock just thrown, what was its acceleration at the very top of its motion?

A. + 10 m/s^2
B. + 5 m/s^2
C. 0 m/s^2
D. - 5 m/s^2
E. - 10 m/s^2

Here is a short quiz. Think about this for a minute or two and then we will vote.

The question posed here is must the acceleration have any special value when the speed is zero? The answer is no. Acceleration is the rate of change of speed and does not depend on whatever value the speed has at the moment. The only question is whether the speed is changing or not, and of so, at what rate?

In this case, the acceleration due to gravity acts at all times for all unconstrained bodies. They always accelerate downward until they hit the ground.



SHOW cart on inclined track, starting it with an upward push from near the end of the track. Display x vs t. It first decreases, then increases. This is like measuring the position of the rock thrown up from up in a tree.

SHOW speed vs time. Note it goes through zero at the top of the motion as we said for the rock.

SHOW acceleration vs time. It remains constant throughout the motion except at the ends when other forces enter.

DROP ROCK FROM TOP OF BUILDING

Here we extend our earlier equations by adding an initial displacement

$$y = y_0 - 1/2gt^2$$

Let $y_0 = 50$m.

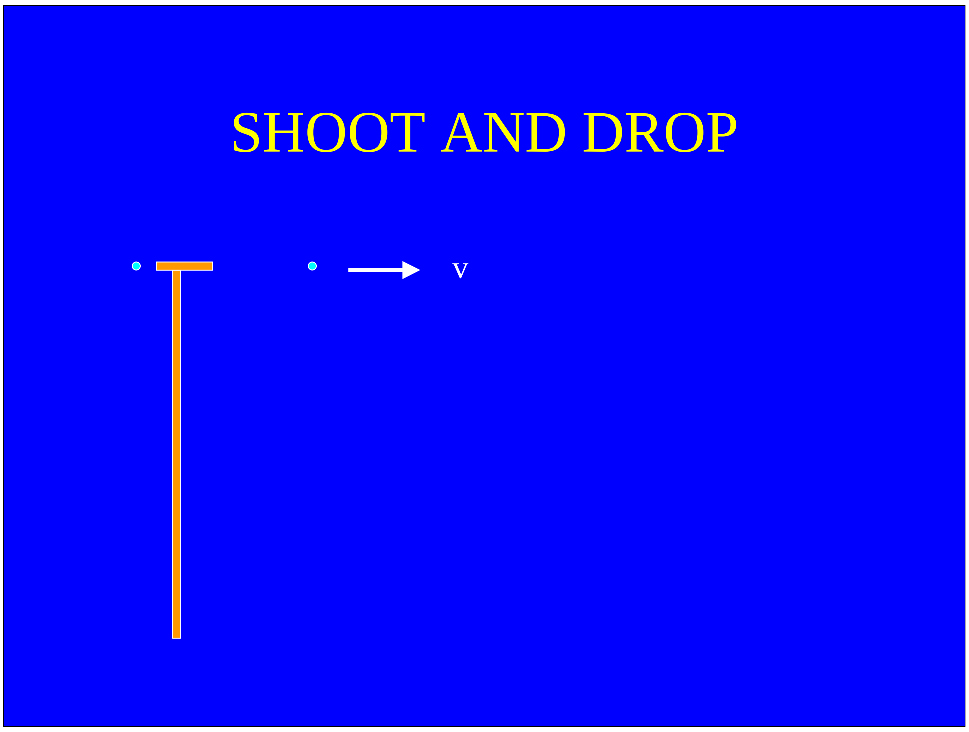
$$y = 50 - 5t^2$$

When does it hit? Answer: when $y = 0$

$0 = 50 - 5t^2$ so $5t^2 = 50$ and $t^2 = 10$ so $t = 3.2$ s

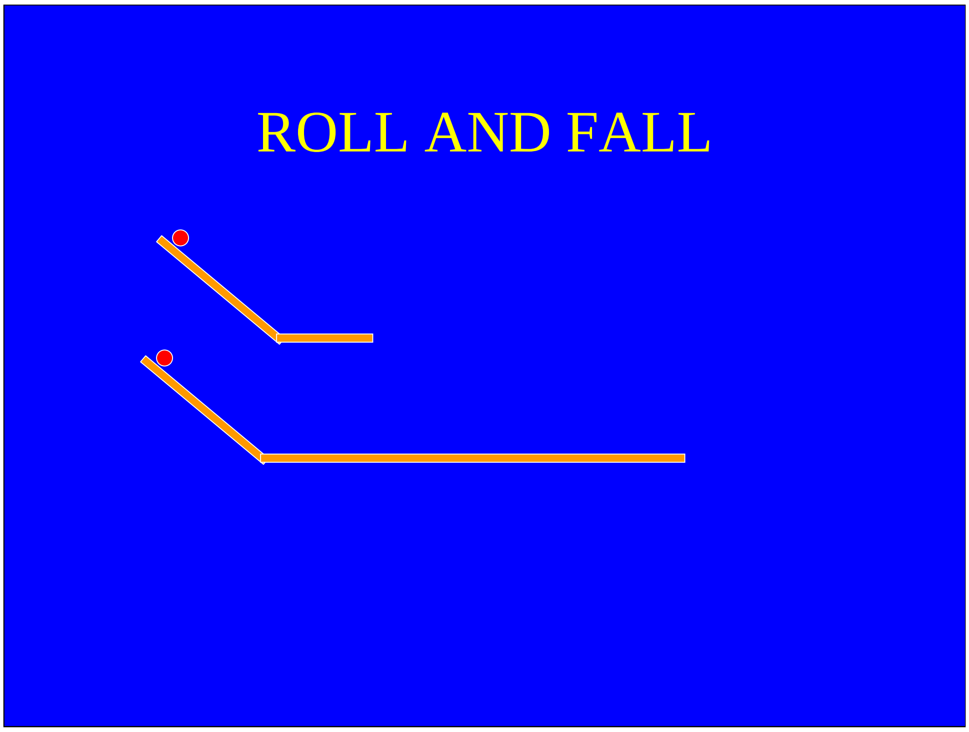
Instead of adding an initial speed, let's add an initial displacement. We drop a rock from the top of a building 50m high. We could use a coordinate system with $y = 0$ at the top of the building, but instead let's choose ground level as our zero of height. Then clearly we just add y_0 to our equation for displacement.

How long until the rock hits ground? This occurs when $y = 0$. Solving for t we find 3.2s.



So far we have dealt with motion in one dimension only. A car is driven down the street, a rock is dropped or rolled on an inclined plane, etc. How do we handle situations in which more than one dimension is involved? For example when a baseball is thrown and caught, it moves both horizontally and vertically. Our whole understanding of this field hinges on an important experimental discovery: The observed motion of the baseball is the result of two *separate* motions followed simultaneously by the ball, one horizontal and the other vertical. Furthermore, these two components do not interfere with each other in any way. They each proceed as though the other were not taking place. Galileo discovered these things and describes them in TNS.

We begin with a simple and perhaps surprising experiment. We have a spring driven device that can shoot one sphere horizontally and simultaneously drop another. The question is, which one reaches the floor first? SHOW shoot and drop demo. We can hear that they both land at the same time. What this experiment shows is that the vertical motion is not influenced by a ball’s horizontal motion. The ball shot sideways falls just as fast as the dropped ball. If we were to do the same thing with a high powered rifle, the result would be the same. The shot bullet would land at at the same time as one that was dropped when the trigger was pulled.

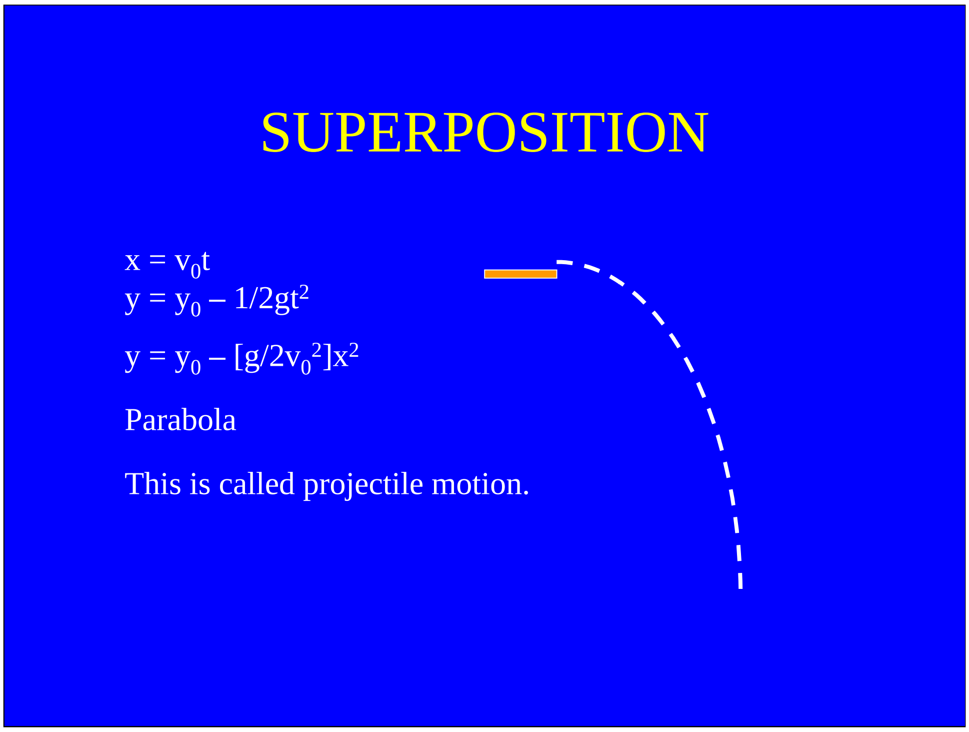


This experiment is similar to the one we just saw except vertical and horizontal motion are interchanged. Two balls roll down identical inclines gaining the same speed. The lower one then rolls along a horizontal track. The upper one rolls along a shorter horizontal plane and then falls towards the lower track. It lands on top of the lower rolling ball showing that the two balls continue to have the same horizontal motion. The horizontal motion of the upper ball is not influenced by its vertical motion.

The conclusion we draw from these two experiments is that the vertical and horizontal components of the motion of an object are independent of each other. It is as though these two dimensions exist in different universes.

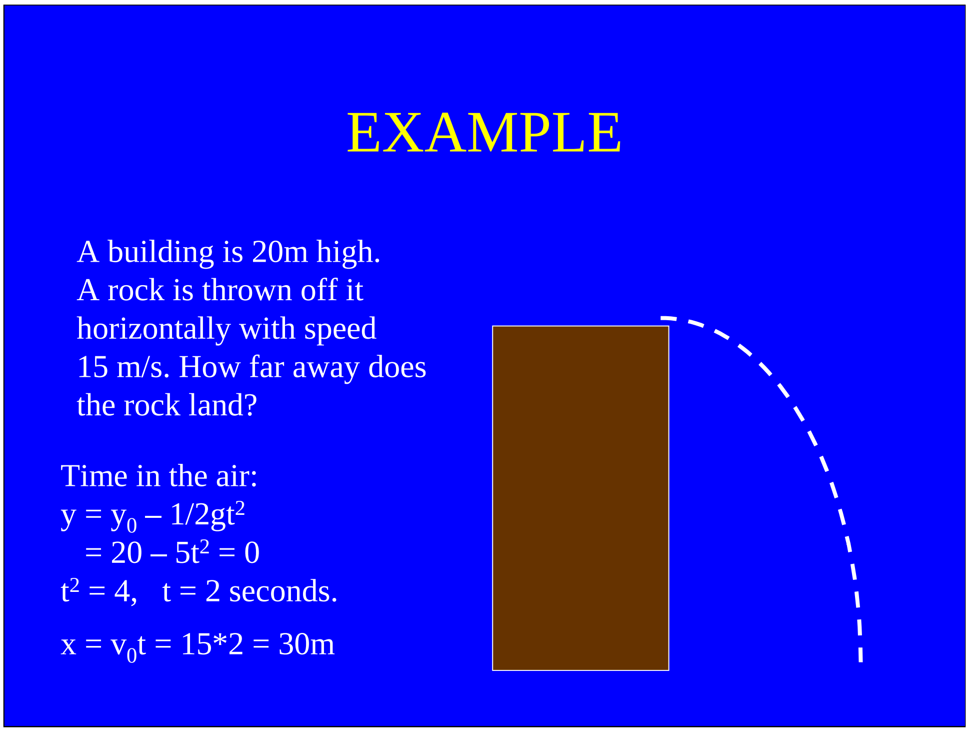
In the above experiment the horizontal motion of the upper ball remains what it was as it left the track. This is consistent with Galileo’s principle of inertia. No change in its horizontal speed occurs because no horizontal forces act on it. Another way of saying this is that the principle of of inertia applies separately in each dimension of the motion.

The overall trajectory of the upper ball is a superposition of its horizontal and vertical motions.



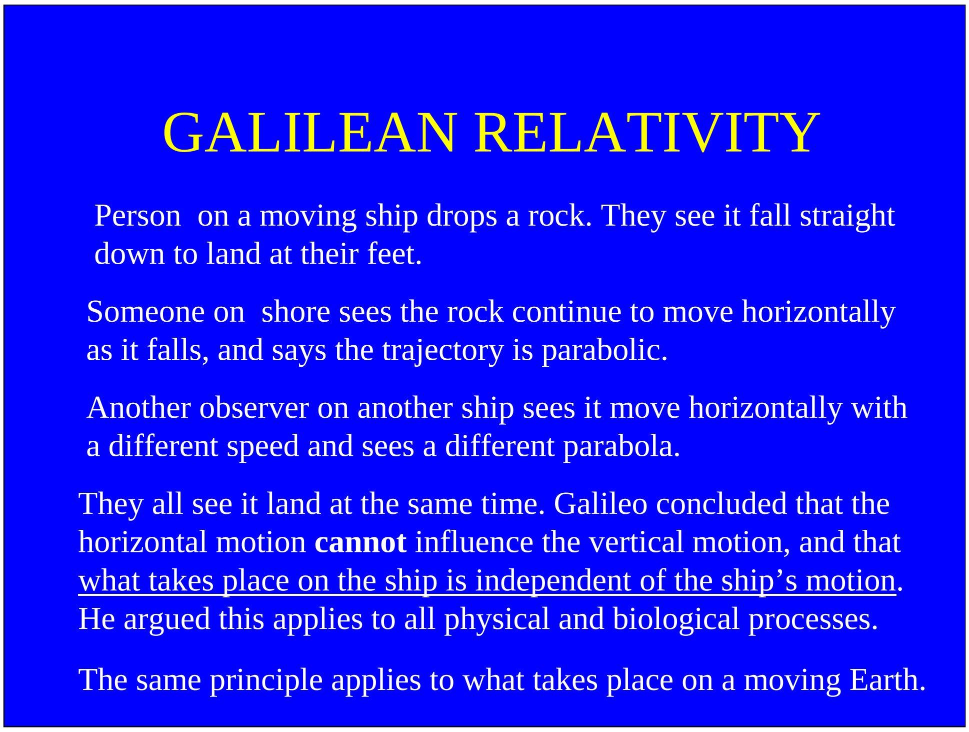
The equations that describe the x and y components of the motion of the upper ball in the previous experiment are familiar to us. In the x direction the velocity is constant, so x increases in proportion to t. In the y direction we have a falling body dropped from a height y_0. We can eliminate t from the two equations by solving the first one for t and substituting it in the second. The result is a parabola.

The trajectory of a body moving with a constant horizontal velocity and constant vertical acceleration is a parabola. This is referred to as <u>projectile motion</u>. Examples are: baseball, football, tennis, shooting a bullet, etc. In our treatment we are ignoring the force of the air on the moving body. As we have seen, this is only accurate when the speed of motion is small enough.



Here is a simple example of projectile motion. The rock thrown from the building continues moving with the same horizontal speed throughout its motion. All we need to know is how long was it in the air?

This can be found using its vertical motion which is the same as if it had just been dropped. From this we find that it lands after 2 seconds. And now we know how far it moved horizontally during this time.

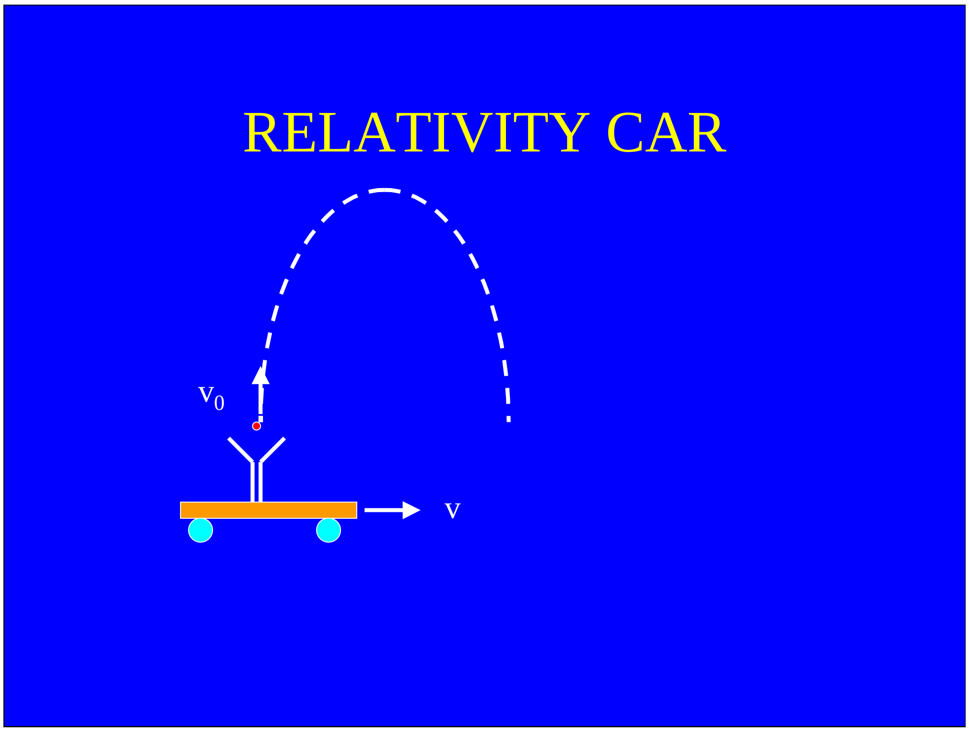


Galileo thought about the question of a moving Earth. If the Earth moves, why do we **not** notice this motion, as Aristotle claimed we would. Here is the thought experiment he carried out in answer to this puzzle.

A person on a ship drops a rock. To that person its trajectory is a straight line. An observer on shore sees the trajectory as a parabola. Someone on another ship sees yet a different parabola. All of them are making valid observations. They all see it land at the same time.

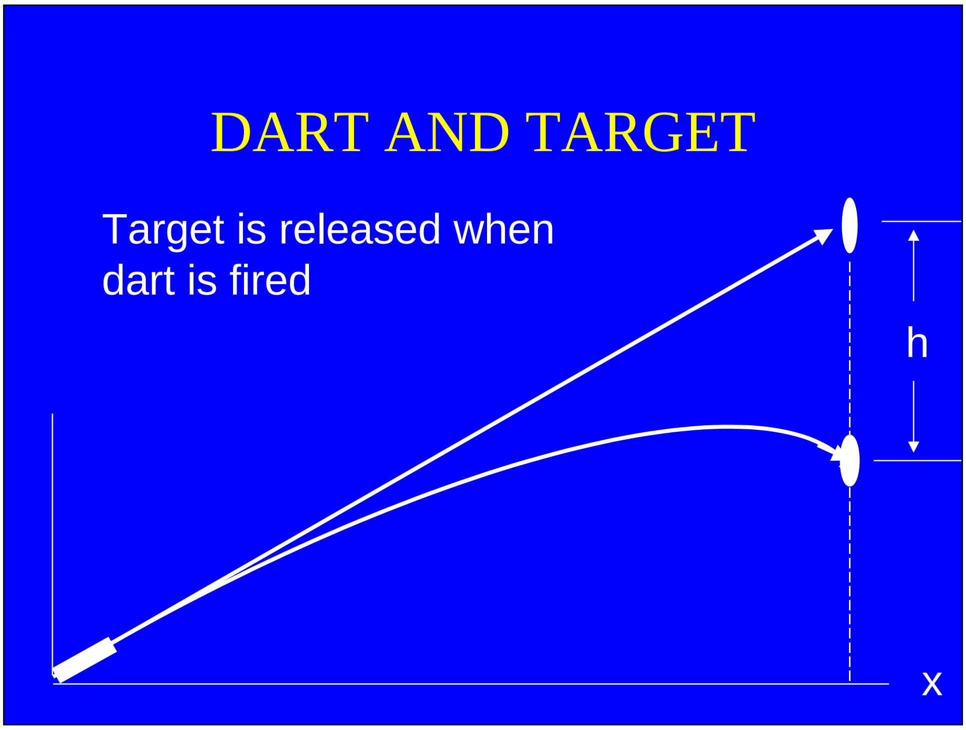
Galileo concluded that the vertical motion of the rock must in principle be independent of its horizontal motion. Furthermore he concluded that everything that happens on board the ship must be independent of its motion. He argued that whatever you take with you on the ship, insects, fish, physics experiments, etc will be unable to detect the horizontal motion of the ship. He was assuming, of course, that this motion is perfectly uniform and smooth.

This is called the Galilean principle of relativity and explains immediately why we do not sense the motion of the Earth about the Sun.



This car with spring cannon illustrates Galileo's principle of relativity. The cannon shoots a ball straight up. As it returns it is caught by the funnel.

When the car is stationary in the room, we see the ball go straight up and down. When the car moves relative to us, we see a parabolic trajectory because now the ball has a constant horizontal velocity along with its vertical free-fall motion. But the ball is still caught by the funnel because the ball and car have the same horizontal velocity. The car and ball move along together and the result is the same as if it were stationary in the room. Its uniform horizontal motion is unimportant, even undetectable by any experiment carried out on the cart.



Here is a dart gun aimed directly at a target. The dart gun and target are so arranged that when the dart is fired, the target is released and falls straight down along the dashed line.

If there were no gravity, the dart would follow the straight line to the target, the target would not fall, and the dart would hit the target.

In the presence of gravity, what happens? Both the dart and target fall. The target falls straight down, and the dart falls below the straight line to the target. When the dart gets to the x coordinate of the target, they have both been falling the same length of time, hence the same distance, so the dart still hits the target.

As the initial velocity of the dart is reduced, dart and target fall farther and farther, but they will still collide as long as they don't hit the ground first.

SHOW Dart and target demo.