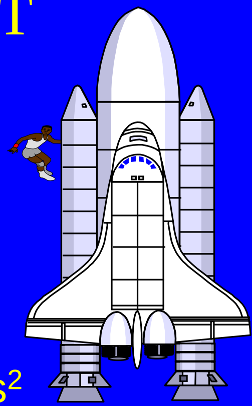
ASTRONAUT PUSHES SPACECRAFT

$F = 40\ N$
$m_a = 80\ kg$
$m_s = 15000\ kg$



$a_s = F/m_s = 40N/15000\ kg$
$= 0.0027\ m/s^2$

$a_a = -F/m_a = -40N/80kg = -0.5\ m/s^2$

If $t_{push} = 0.5$ s, then $v_s = a_s t_{push} = .0014$ m/s, and
$v_a = a_a t_{push} = -\ 0.25$ m/s.

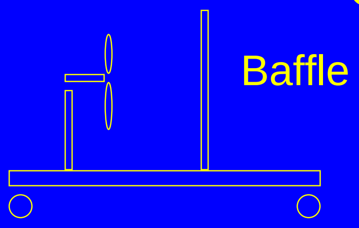
An astronaut is floating around outside a spacecraft and gives it a push. Newton’s third law says that if he pushes on the spacecraft, it will push back on him with an equal and opposite force.

The spacecraft and astronaut will both be accelerated since there is a net force on each, and they will drift apart. They move at different rates, however, since they have different masses.

The velocity of both spacecraft and astronaut after the push is over will equal their accelerations during the push times its duration.

The same thing happens when the astronaut is <u>inside</u> the spacecraft. After pushing on a wall, he/she will float away, and the ship accelerates also. One of the things astronauts must adjust to is to push on walls they encounter gently, since a hard push gives them too much velocity for comfort.

QUALITATIVE QUIZ



If we add a baffle to our propeller-driven cart, the acceleration of the cart will:

a. Increase a lot
b. increase a little
c. not change
d. decrease a little
e. decrease a lot

Here we modify the propeller example we saw earlier by adding a baffle. Does the baffle make a difference, and if so in which direction?

The propeller is pushed forward by the air it pushes back just as before, but now that moving air encounters a baffle. If the baffle simply stops the air, then it must push forward on the moving air, and the air pushes back on the baffle. This reduces the forward thrust of the propeller, reducing the acceleration of the cart.

Newton's third law is invoked twice here, and the forces on the cart are in opposite directions.

NEWTON'S LAWS IN EVERYDAY LIFE

You are standing still, then begin to walk. What was the external forced that caused you to accelerate?

Hint: It is very hard to start walking if you are standing on ice.

What force causes a car to accelerate when a traffic light turns green?

The second law states that whenever a body accelerates, an external force must be present to cause the acceleration. It is not always obvious what this force is even in very ordinary circumstances.

In order for you to accelerate forward when you start walking, something must push you forward. What is it? It must be the sidewalk. You push back with your feet, and the sidewalk pushes you forward by an equal amount according to Newton's third law. This could not happen without friction.

You may think that it is your pushing against the sidewalk that causes you to move forward, but it is really the sidewalk pushing forward on you, otherwise you would not accelerate in that direction.

Same argument for a car accelerating.

NEWTON AND THE APPLE

Newton knew that at the surface of the earth bodies (apples) fall 5 m in the first second, and that this acceleration is due to earth's gravity.

He showed that the gravity force is the same as if all earth's mass were at its center, 4000 mi from the surface. (This required inventing Calculus).

He wondered whether the same force attracts the moon towards earth.

There is a story you have probably heard that Newton was stimulated to think of his Universal law of Gravitation by seeing an apple fall at his mother's farm in Woolsthorpe. This might even have happened. He at least said it had happened years later while having tea there with a friend.

At that time a gravity force was being thought about by several people and commonly thought to decrease inversely as the square of the distance between the two objects attracting each other.

The earth and the apple attract each other resulting in the acceleration studied by Galileo. Does the same force, reduced by the square of the distance, cause the moon to fall towards the earth?

Does the moon fall towards the earth? It remains the same distance away, so at first it does not seem to do so, but let's look more carefully into this.

ACCELERATION OF OBJECT MOVING IN A CIRCLE

Speed is rate of motion without regard for direction. A car goes 60 mph.

But to tell where the car goes, direction must be specified as well as speed.

The term velocity is used to describe both speed and direction.

Acceleration in Newton’s second law, is the rate of change of velocity, not just speed.

The acceleration of an object is its rate of change of velocity, not just speed. For example a car going around a corner at a constant speed of 25 mph is accelerating, and you as a rider in the car feel that acceleration. The seat of the car must push on your body to make it accelerate.

Turning left, the car seat must push you to the left.

The moon is moving (nearly) in a circle about earth with constant speed. That means its velocity is constantly changing. If it were not, it would continue in a straight line. So it is accelerating.

Let’s see if we can understand the magnitude and direction of that acceleration. This means we want to study objects that move uniformly in a circle.

UNIFORM CIRCULAR MOTION

- Centripetal Acceleration
- Centripetal Force
- Example: The moon

In studying uniform circular motion we will be using the same ideas of kinematics (x, v, a) and Newton's Laws of motion that we have been studying, but we will apply them to situations involving motion in circles.

A new word in involved: Centripetal. This just means directed towards the center. A centripetal acceleration is one directed towards the center of the circle in which an object is moving.

Roller coasters at amusement parks use circular motion to achieve apparent weightlessness momentarily. This can occur near earth where g is still strong, but the object behaves in some ways as if it were far from any star or planet.

The example we are working towards is the moon, in order to understand how Newton compared the fall of an apple with the motion of the moon, taking his first step towards understanding gravity.

Uniform Circular Motion is the motion of an object traveling at constant speed in a circular path.

Examples:

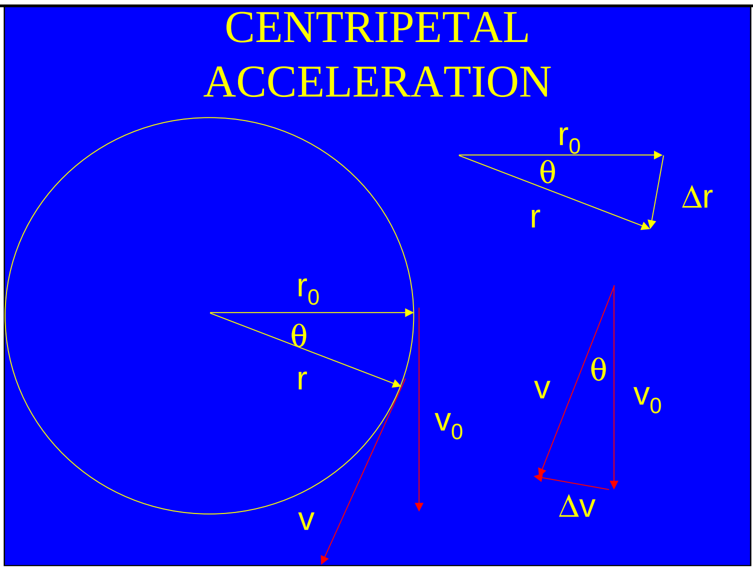
- spot on a phonograph record
- washing machine during spin cycle
- ball whirled around on a string
- car turning a corner
- moon in orbit around Earth

The definition of uniform circular motion is very simple. There are many examples such as these. Many others involve such things as a centrifuge, any motor with a rotating shaft...

Since the same ideas can be used to describe such a wide range of phenomena, they will prove to be very useful.

In all of these examples the speed, but not the velocity is constant. The magnitude of the velocity is constant but its direction is constantly changing.

Another quantity that is constant is the rotation rate (revolutions per second, day, year). For example the earth rotates at a constant rate of once per day, so all objects fixed to the earth are undergoing uniform circular motion.



Here we have an object moving in a circle with a constant speed. Why is there any acceleration? Simply because velocity is a vector quantity, and in this case, its magnitude doesn't change, but its direction does.

Consider our moving object at two times: It has moved from r zero to r. Here I have moved the two r vectors away so we can see them more easily. The change in r is delta r as shown.

Since the position is changing with time, there is a non-zero velocity. The velocity vectors are shown in red. The velocity is perpendicular to the radius vector at all times during uniform circular motion. The velocity is always tangent to the circle describing the motion.

But in that case, the velocity itself is rotating around in a circle just like the radius vector. Here I have moved the two velocity vectors away so we can see them also. Because v is always perpendicular to r, the angle between the two r vectors is the same as that between the two v vectors.

This means that the r triangle is similar to the v triangle. They have the same shape. One thing that means is that the ratios of corresponding sides are equal. We also see that as Δt becomes small, Δv is perpendicular to v just as Δr is perpendicular to r.

$$a_c = v^2/r$$

$\Delta r/r = \Delta v/v$

And, $\Delta r = v\Delta t$ so

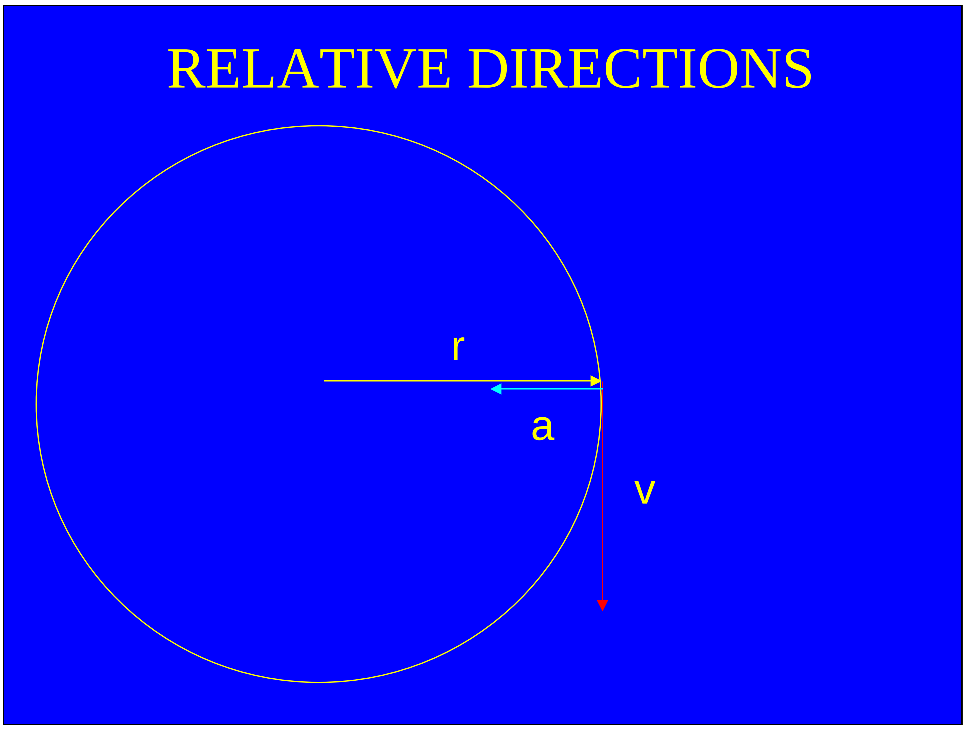
$\Delta v = v(v\Delta t)/r$
$\Delta v/\Delta t = v^2/r$

Centripetal Acceleration = a_c
$a_c = v^2/r$

The centripetal acceleration points radially inward toward the center of the circle.

From the similarity of the triangles we see that the change in r divided by r is equal to the change in v divided by v during the same small time interval Δt. This allows us to solve for the centripetal acceleration magnitude as shown. This is a very simple and useful result: Whenever an object moves in uniform circular motion, it is undergoing an acceleration equal to v^2/r.

But a is a vector also. What direction does it point? Looking at the previous slide we can see that a must be perpendicular to v just as v is perpendicular to r in the limit as theta (delta t) gets very small.



Here we show the relative orientations of r, v, and a that we have just established for uniform circular motion. This is quite different from our linear F = ma problems so far, where the relative orientations of these vectors could be anything depending on the circumstances.

In this way uniform circular motion is simpler than linear motion problems.

SHOW rotating table with accelerometer on it. This clearly shows that there is an acceleration whose magnitude can be maintained nearly constant, but whose direction rotates around with the table.

BALL ON STRING

- $r = 0.5$ m, $T = 2$ s. What is a_c?
- $v = 2\pi r/T = 3.14/2 = 1.6$ m/s
- $a_c = v^2/r = 2.5/0.5 = 5$ m/s^2
- What if we cut the period in half?
- a_c quadruples to 20 m/s^2

Here is a game we have all played: Whirl a ball around on a string. Let us define T to be the period of the motion ie the time for the ball to go around one time. As long as we keep it moving with a constant period and radius, it is exhibiting uniform circular motion. For the given radius and period, what is the magnitude of the centripetal acceleration?

We know the radius, so all we need is the speed. How do we find that? The period is the time for one round trip, ie one time around the circle. How far does the ball go during one period? Just the circumference of the circle. So the speed (magnitude of the velocity) is 2 pi r divided by T which is 1.6 m/s in this case.

Then the centripetal acceleration is just the velocity squared divided by the radius, or 5 m/ss.

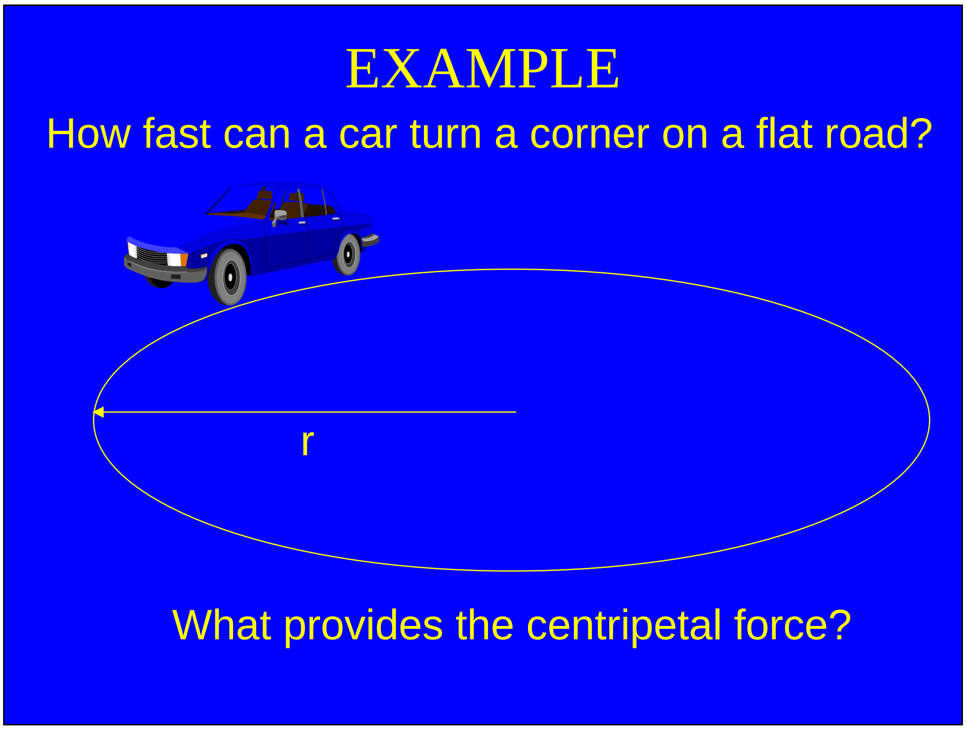
Question: What if I speed the ball up so as to cut the period in half. What is the centripetal acceleration now?

Centripetal Force

- The name given to the net force needed to keep a mass m moving with speed v in a circle of radius r.
- Magnitude: $F_c = mv^2/r$
- Toward the center of the circle

Newton tells us that when there is an a there will be an F. The centripetal force is not a new special kind of force, like friction, or gravity. It is simply the name we give to whatever force exists in a situation where uniform motion in a circle is taking place, that causes the object to move in that way. It must always point toward the center of the circle, and have a constant magnitude as long as v and r remain constant.

In the example of the ball on the string we just saw, the centripetal force is provided by the string tension.

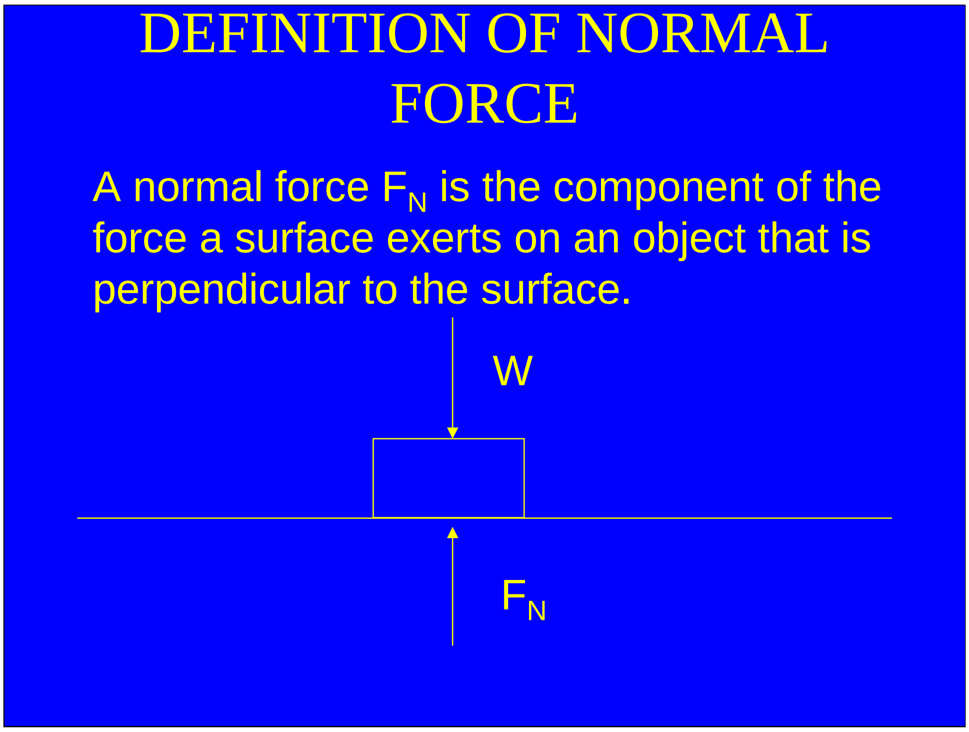


A car goes around a corner on a flat road. During the time it is turning, we can approximate its path as an arc of a circle, so for a time it is undergoing uniform circular motion.

What provides the necessary centripetal force, and with what maximum speed can it go around the corner? Friction force between the tires and the road.

Racing cars can experience as much as 4 g's of horizontal acceleration due to their special tires and large downward force on the car provided by the air. Production cars are limited to 0.5 – 1 g of horizontal acceleration.

If you turn a corner with $v = 10$ m/s (about 25 mph). and $R = 10$ m (30 ft), then $a_c = 10\ \text{m/s}^2 = g$. This is too fast for such a sharp corner for a production car.



When you stand still on a sidewalk, you are not accelerating along the sidewalk, so there is no force on you in that direction. In that case the only force the sidewalk exerts on you is upward, opposing gravity. This force is perpendicular to the sidewalk surface. This is called a normal force. Normal is a term that refers to something being perpendicular to a surface or line.

APPARENT WEIGHT

In a vertically accelerated reference frame, eg an elevator, your apparent weight is just the normal force exerted on you by the floor.

Applying Newton's second law:

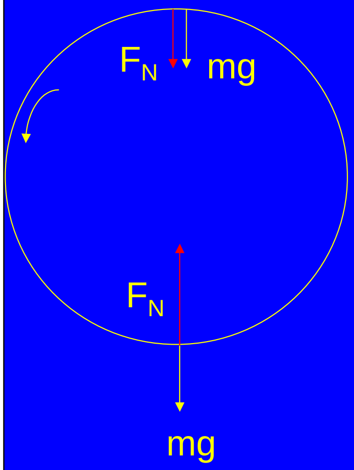
F_N - mg = ma, or

$$F_N = mg + ma$$

Apparent weight = true weight + ma
where a = upward acceleration

When you are in an elevator and it begins to move upwards, you feel heavier than usual for a moment. As the elevator slows and stops, you feel lighter for a moment. What happens in these examples, is that the normal force the elevator exerts on you increases or decreases as the elevator accelerates. We can understand this using Newton's second law.

VERTICAL CIRCULAR MOTION



At the top:
Minimum v for $F_N = 0$:
(apparent weightlessness)

$$F_c = mg$$

$$mv^2/r = mg$$

$$v = (rg)^{1/2}$$

At the bottom:
$F_N = F_c + mg$

SHOW Wine glass on tray.

SHOW Loop the Loop

This resembles the roller coaster rides that have become so much fancier in recent years. With this equipment we can observe that it takes a minimum speed for the ball to make it over the top without leaving the track. When it just leaves the track momentarily at the top, what does that mean? The normal force between the track and ball has become zero. Any slower than this and we're in trouble.

Here is a diagram showing the forces involved. At the bottom the normal force and gravity oppose each other as we are accustomed to. At the top, the normal force and gravity act in the same direction. The minimum speed for weightlessness corresponds to normal force = 0. In that case the only centripetal force is mg. This gives us a condition for v which is independent of m. So for a given r, all objects will become weightless at that v just at the top.

At the bottom, the situation is different: Now the normal force must provide the centripetal force needed for the object to move in a circle, as well as mg. If your roller coaster is going the same speed at the bottom as the top, you will feel twice as heavy as usual at that point.

Roller Coaster Numbers

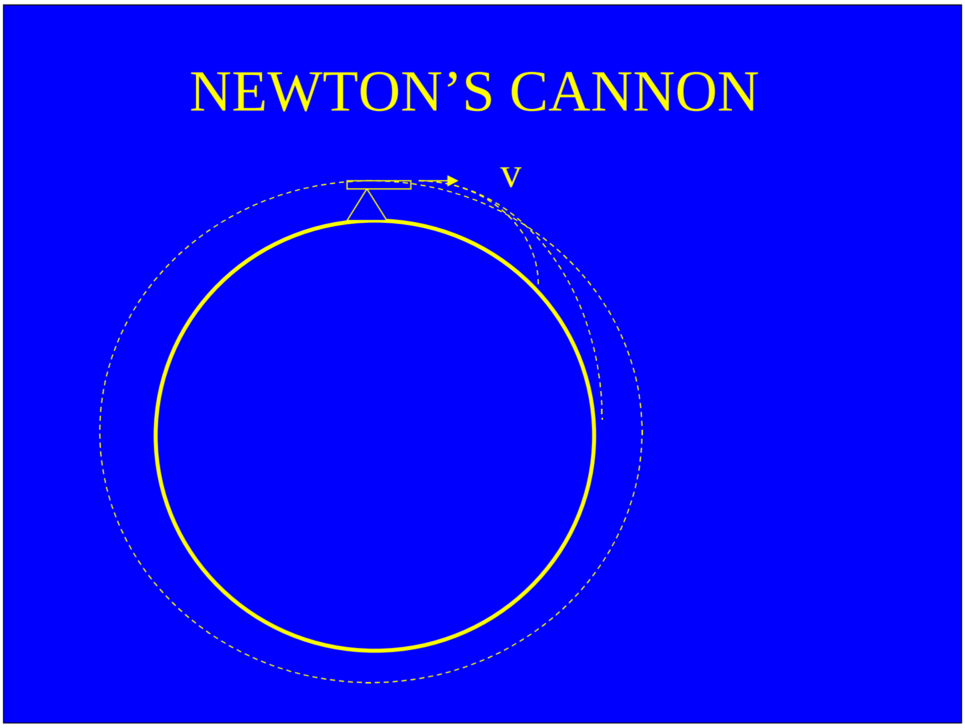
r = 10 m for vertical loop

For apparent weightlessness at top:

$v = (rg)^{1/2} = (10*9.8)^{1/2} = 10$ m/s

Here are some numbers for a roller coaster doing a loop-the loop using reasonable dimensions. 10 m/s (about 25 mph) is not a very great speed. This is made possible by the rather small radius of 10 m.

At this speed the roller coaster is in free fall at the top of its motion.



Newton imagined mounting a cannon on a tall mountain so it would be above the earth's atmosphere. Then shooting it horizontally with a large muzzle velocity. Would it be possible for the cannon ball to go completely around the earth, so it could hit the cannon from behind?

This was purely an imaginative game for Newton. Today we would say the cannon ball is a satellite in low orbit.

The speed needed can be found from our equation for centripetal acceleration. We want a_c to equal g. The earth's radius is about 6000 km, so we have

$g = v^2/R$ so $v^2 = 10*6*10^6$ and $v = 8000$ m/s.

ACCELERATION OF MOON

Does a gravity force cause the moon's acceleration toward earth? If so, does it vary inversely as the square of the distance?

$a_{c\ moon} = v^2/r = (2\pi r/T)^2/r$

$r = 3.85*10^8$ m, T = 27.3 days

$a_{c\ moon} = 2.79*10^{-3}$ m/s^2

$a_{c\ moon}/g = 2.79*10^{-4}$

$(r_{earth}/r_{moon})^2 = (4000\text{mi}/240000\text{mi})^2 = 2.78*10^{-4}$

Back to the apple and the moon question. We know the apple falls with acceleration g = 10 m/s^2 near the surface of the earth. Newton showed that the earth's gravity behaved as if all the mass of the earth were at its center, so the apple is 4000 mi from the center of the earth.

The acceleration of the moon towards earth is given by our equation for centripetal acceleration. So Newton was able to evaluate the ratio of the moon's centripetal acceleration to g.

Now the question is, does this acceleration decrease as the square of the distance? He found that the ratio of distances squared was in fact about equal to the ratio of the accelerations.

This led him to propose that the same gravity force that causes the apple to fall, also causes the moon to accelerate towards earth.

The force we are familiar with in everyday life holds the moon in its orbit about earth. This idea completely turns about the old Greek view that celestial objects behave differently from things here on earth.

Encouraged by this result, he then went on to show that this same force causes the planets to move in their orbits about the sun.