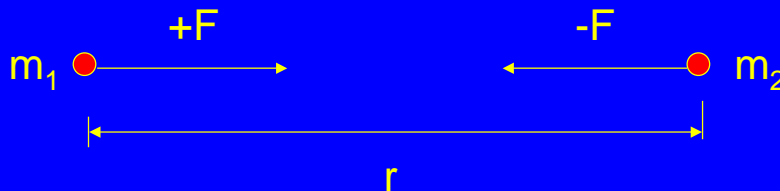


UNIVERSAL GRAVITATION

For any two masses in the universe:

$$F = G m_1 m_2 / r^2$$

G = a constant later evaluated
by Cavendish



Emboldened by his comparison of the Moon's acceleration with that of the apple, Newton proposed a universal law of gravitation. By universal he meant just that. The proposed force is to exist between any two masses whatsoever in the Universe. You cannot build a shield around a mass and prevent it interacting with other masses. And its range is proposed to be infinite.

Note that this gravity law automatically satisfies Newton's third law.

It is proportional to the product of the two masses and in that sense is completely symmetric. The force exerted on you by the Earth equals the force you exert on the Earth. You may not think of yourself as exerting a force on the earth but you do.

Newton could only guess at the value of G, the constant that determines the strength of the gravity force. He did so by guessing the mass of the earth, getting it right within 10%.

NEWTON: G DOES NOT CHANGE WITH MATTER

For masses near the earth, $mg = GMm/r^2$

Therefore, $G = g[r^2/M]$

Newton built pendula of different materials, and measured g at a fixed location, finding it to remain constant.

Therefore he concluded that G is independent of the kind of matter. All that counts is mass.

Newton had no way of directly measuring G , but he could measure g , since it was related in a simple way to the period of a pendulum. According to the above equation, G is proportional to g with proportionality constant r^2/M where r is the distance to the center of the earth and M its mass.

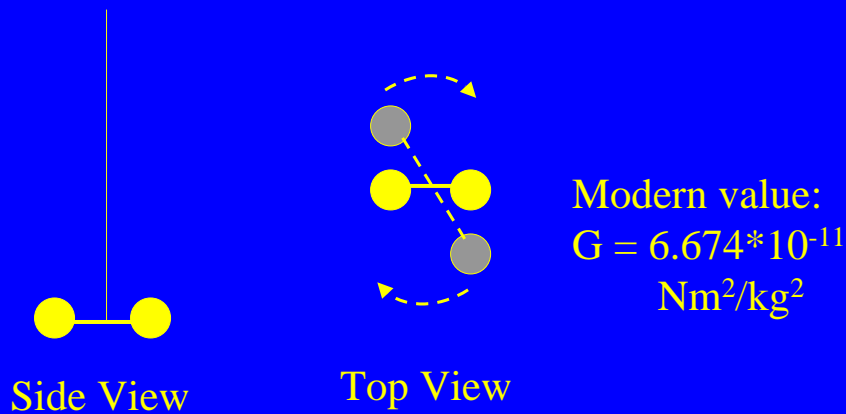
Newton then built pendula made of many different materials and measured their periods accurately. This involved counting the number of swings in many hours. Friction had to be compensated for of course, but ways of doing that had been worked out without interfering with the period of the pendulum in the design of accurate pendulum clocks by Huygens.

Newton found that the value of g was independent of the kind of matter used in making the pendulum (within his experimental errors).

Since all this was done at the same location so the value of r was the same, he concluded that G was independent of the choice of matter which the mass was made of.

CAVENDISH: MEASURING G

Torsion Pendulum



How is a very small force to be measured? A torsion pendulum is the method used by scientists in England and Europe about a century after Newton. The idea is to hang a dumbbell of two equal masses from a fine long fiber. The dumbbell can twist easily because the fiber offers little elastic restoring force.

After letting this system settle down, which may take some time, Cavendish then brought two masses near the two in the pendulum. Their gravitational forces on the two masses were such as to twist the fiber just a little. Then to verify that the effect was due to the two external masses and not some other disturbance, the two masses could be rotated on their own mount so as to reverse their twisting force on the pendulum.

The fiber then had to be calibrated so as to learn its elastic restoring force. This must be done with another known force which could however be considerably larger than the gravitational force being measured.

This method has been improved considerably over time, but measuring G is still one of the most difficult of all fundamental measurements. SHOW Torsion pendulum and Cavendish torsion pendulum.

Two people pass in a hall. Find the gravity force between them.

- $m_1 = m_2 = 70 \text{ kg}$
- $r = 1 \text{ m}$
- $F = 6.7 \cdot 10^{-11}(70 \cdot 70) = 3.3 \cdot 10^{-7} \text{ N}$

When two people pass in a hall, there is an attractive force between them according to Newton's law of gravity. How large is it?

Assume each person has a mass of 70 kg, which means they weigh about 155 lb. Let's estimate the force of attraction when they are 1 m apart.

The resulting force is very small. This is less than a ten-millionth of a pound, or about one millionth of an ounce. This is much too small for us to notice during the hurly-burly of everyday life.

The mass of the earth on the other hand is about $6 \cdot 10^{24}$ kg large enough to pay attention to.

NOTE: Large spherical objects, such as the Earth, exert an attractive gravitational force on objects in their environment which is the same as though their entire mass were concentrated at the center of the sphere.

NEWTON'S CANNON II

Assuming a circular orbit:

$$GmM_E/r^2 = mv^2/r$$

$$v = (GM_E/r)^{1/2}$$

For low orbits (few hundred km up) this is about 8 km/s = 17000 mph

Now that Newton had formulated his gravity law, he could rethink the cannon problem in a more straightforward way. This is an example of a centripetal force problem in which gravity provides the force. We find immediately that only one speed in orbit is possible for a given radius (distance from the center of the Earth).

Interestingly, we see that as r increases, v decreases. If we double r , the speed will be 5.7 km/s. And in the limit that r goes to infinity, v goes to zero. This must be right, because infinitely far from earth, the gravity force (the centripetal force) approaches zero also. So the cannon ball will simply drift through space in a straight line with constant speed as Newton's first law says it must.

Kepler's Third Law

$$T^2 = Ka^3$$

Simplify to a circular orbit

$$F = GMm/r^2 = mv^2/r = m/r(2\pi r/T)^2 = 4\pi^2 mr/T^2$$

$$T^2 = [4\pi^2/GM]r^3$$

Newton was of course well aware of Kepler's laws describing the motions of the planets about the sun. An obvious test of whether his gravity law extended to the planets was whether he could explain, or predict, Kepler's laws.

Here we see a simplified derivation of Kepler's third law for the case of a circular orbit (which is actually quite accurate for most of the planets).

The constant K in Kepler's third law now has a meaning. In fact, once G had been measured, knowing K allowed a determination of the mass of the sun. The same can be said of the Earth, knowing the period and distance to the Moon.

In fact, Cavendish's experiment was referred to as "weighing the Earth" since G determined the mass of the Earth in this way.

This result allows us to determine the mass of any planet or star whose satellite (moon or planet) we can observe.

Distance to TV Satellite

$$v = (GM_E/r)^{1/2} = 2\pi r/T \text{ Solving for T:}$$

$$T = 2\pi r^{3/2}/(GM_E)^{1/2} \text{ so } r = [GMT^2/4\pi^2]^{1/3}$$

EXAMPLE: Geosynchronous satellites have
 $T = 24 \text{ hr} = 8.6 \times 10^4 \text{ s}$

$$\begin{aligned} \text{Evaluating, we find } r &= 42,000 \text{ km} \\ &= 26,000 \text{ mi} \end{aligned}$$

We can relate the speed of a satellite to its period very simply. Then we have a relation for the period of the satellite that may be more useful than the speed itself.

This is quite evident in the case of a TV communications satellite which we want to hover over a particular longitude stably for a long time. Then we want the satellite's period to match the rotational period of Earth (actually 23hr, 56 min since what counts is the rotation relative to the stars, not the sun).

Note that these results for satellites do not depend upon their mass. Different size satellites will have the same period at the same radius regardless of their size.

Apparent weightlessness

What is the weight of an astronaut in a satellite?

Apparent weight is the normal force needed to support the astronaut.

Both the satellite and astronaut are in uniform circular motion about Earth. They move together. No normal force is needed.

We have seen an example of apparent weightlessness that lasted momentarily in the case of the roller coaster. For an astronaut in a satellite, however, it lasts as long as they are in orbit.

The satellite and astronaut are in the same orbit. They are falling toward earth with the same centripetal acceleration all the time, and so remain together.

Is a satellite in orbit falling towards Earth? It depends on what you mean by falling. It isn't getting any closer, but it is accelerating towards Earth at all times.

Definition of Weight

- The weight of an object on the earth is the gravitational force the earth exerts on the object.
- $W = GM_E m / R_E^2$
- $R_E = 6.4 \times 10^6 \text{ m} = 6400 \text{ km}$

The weight of an object on the moon is the gravitational force exerted by the moon on the object. On the moon you would weigh about one sixth what you weigh here on earth. So the weight of a mass depends on its location. Mass remains the same as an object is moved from Earth to space, to the Moon, etc.

Weight of Satellite

- A geosynchronous communication satellite is in orbit a distance 42,000 km from the center of the earth.
- If it weighs 1000 N on earth, how much does it weigh at that distance?
- $1000\text{N} * (6400/42000)^2 = 1000\text{N} * 0.023 = 23\text{N}$
- Note: Its mass is still the same.

An object near earth but above its surface will weigh less than it would here on the surface of the earth. This is just because of the inverse r squared dependence of the gravity force.

By weight of the satellite, we mean that somehow it is stopped in its orbit and held stationary, and then weighed with a spring scale. While it is in orbit, it is of course weightless.

Distinction between Mass and Weight

- $W = GM_E m/r^2$
- Near the surface of Earth, we often replace this with $W = mg$
- $g = \text{acceleration due to gravity} = GM_E/R_E^2 = 9.8 \text{ m/s}^2$

When we write $W = mg$, with $g = 9.8 \text{ m/s}^2$, don't forget that this is only accurate on or very close to the surface of the earth.

An undergraduate student working with me some years ago measured the decrease in g on going up from floor to floor in one of the dormitories here at UVA. He then used that data to estimate the radius of the earth, and got a value that was accurate to about 5%.

Variation of g on Earth's Surface

Location	$g(\text{m/s}^2)$
Charlottesville	9.80
Latitude 0° (sea level)	9.78
Latitude 90° (sea level)	9.83

What is a_c at equator due to Earth's rotation?

$$a_c = v^2/r \quad v = 2\pi r/T = 467 \text{ m/s}; \quad r = 6400 \text{ km} \\ = 0.034 \text{ m/s}^2$$

Does not account for difference: Earth is oblate

Once Newton's law of gravity had been established, it could be used to pursue numerous questions beyond the understanding of Kepler's Laws. For example, of g is caused by a gravity force at the surface of Earth, why does it vary as it does?

The numbers above show that g is smaller by about 0.05 m/s^2 at the equator than at the poles. An obvious candidate to explain this is the rotation of the earth which requires a centripetal acceleration at the equator but not on the earth's axis (ie at the poles).

We see that a_c , which must be subtracted from g , is not large enough to account for the observed difference. What else is taking place? The Earth, although it seems solid to us, acts, over long times like a fluid. Rocks slowly deform and flow like a stiff fluid. A spinning earth will not be spherical, but oblate, with a larger diameter around the equator than around the poles. These two effects together account for the observed variation of g .

Note: The upper Missouri river, headwaters of the Mississippi, is actually closer to the center of the earth than New Orleans, where the Mississippi flows into the gulf. How can the river flow uphill?

Note: Local variations in g are used to learn about rock and mineral formations below the surface. A sensitive device is carried around by geologists capable of measuring parts per million changes in g . Maps of these differences are used to understand the geology of a region.



Tides have been a fact of life for much of the human population going back countless millennia since many of our ancestors lived by the sea. What causes tides? This is a question many tried to answer, including Galileo, and always unsuccessfully until Newton.

The moon has the greatest effect in causing tides. The moon and earth pull on each other with a gravity force calculated as though all the mass of each were located at the center. But the near side of earth is closer to the moon and so feels a greater force, and the far side farther with a weaker force. If the earth were all water, then the larger force on the near side would raise the water level there, creating a wave that would travel around the earth as it rotates. The far side of earth, feeling a weaker force, also builds up a wave maximum. These are where high tides occur. In between, the tide is low.

When the sun and moon line up, either at full or new moon, their tidal effects reinforce and we have a spring tide. When the moon is half full their tidal effects subtract from each other and we have a neap tide.

Tidal forces and distortions occur for the solid earth as well, but the size is small (a few inches). Io, Jupiter's moon with smallest orbital radius, is so churned by tidal forces that it remains very hot. When two stars get too close together, tidal forces can rip one or both apart. Curiously, it is sometimes the smaller star that gains mass, eventually becoming the larger star.

HALLEY'S COMET

He observed it in 1682, predicting that, if it obeyed Kepler's laws, it would return in 1759.

When it did, (after Halley's death) it was regarded as a triumph of Newton's laws.

Edmund Halley was one of Newton's friends who, in fact helped encourage him to publish the Principia. He was also an astronomer who carefully observed "the great comet" of 1682. He predicted it would return in about 75 years if it obeyed Kepler's laws.

Its return at that time was regarded as a triumph of Newton's laws, not Kepler's. Why?

Comets had always been held in reverence and fear. Their sight is stunning in the clear skies of the ancient world, and their sudden appearance and disappearance added to their mystery. Pope Calixtus III actually excommunicated Halley's comet when its appearance in 1456 coincided with a Turkish attack on SE Europe.

Newton's laws provided a regularity to the appearance of comets, and a rational underlying that regularity. People need no longer fear them. The laws of nature are not malevolent, although nature can be violent. The giant meteor or comet impact 65 million years ago on the Yucatan peninsula is thought to have contributed to the extinction of thousands of species of living creatures on earth at the time, including the dinosaurs. They were unlucky, but not the victims of malevolent design.

ABSOLUTE SIZE OF SOLAR SYSTEM:

From the work of Copernicus and Kepler, the relative sizes of the planetary orbits was known but not their absolute sizes.

Example: $R_{\text{mars}}/R_{\text{earth}} = 1.52$

In 1672 French astronomers triangulated Mars when in opposition, observing it from Paris and Cayenne at the same time.

Result: Earth-Sun distance = 87 million miles (modern value = 93 million miles)

In order for a quantitative understanding of the solar system to be created, its absolute size, ie distances from planets to sun in meters or miles, was needed.

This was first achieved in 1672. Since Copernicus the relative planetary distances had been known, and Tycho-Kepler improved those ratios. Of course the ancient Greek estimate of the distance to the Sun was available, but it was very uncertain.

French astronomers observing simultaneously from Paris (0° Longitude, 49° N Latitude) and Cayenne (53° W. Longitude, 5° N Latitude) (where the pepper came from, now part of French Guiana), measured the angular position of Mars when it was in opposition. This means Mars was as close to Earth as it gets, enhancing the size of the angular position difference between the two observers.

Knowing the absolute distance from Earth to Mars, and the ratio of their distances to the Sun, the distance scale of the Solar System could be found.

This is just how Thales measured the distance to a ship at sea, only now it was done with telescopes and trigonometry instead of naked eye measurements and similar triangles.

DISCOVERY OF NEW PLANETS

Small departures from elliptical orbits occur due to the gravitational forces of other planets.

Deviations in the orbit of Uranus led two astronomers to predict the position of another unobserved planet.

This is how Neptune was added to the Solar System in 1846.

Deviations in the orbits of Uranus and Neptune led to the discovery of Pluto in 1930

Most discoveries of new things are made by accident, aided by training and hard work on the part of the discoverer. Newton's law of gravity describes the motions of the planets so precisely that new planets were discovered by means of it.

If only a single planet orbited the sun, it would follow an elliptical orbit precisely. The presence of other planets, attracting each other, causes slight departures from this result. The effects of other planets are small because the sun is so massive in comparison, having 99.9% of the mass of the solar system.

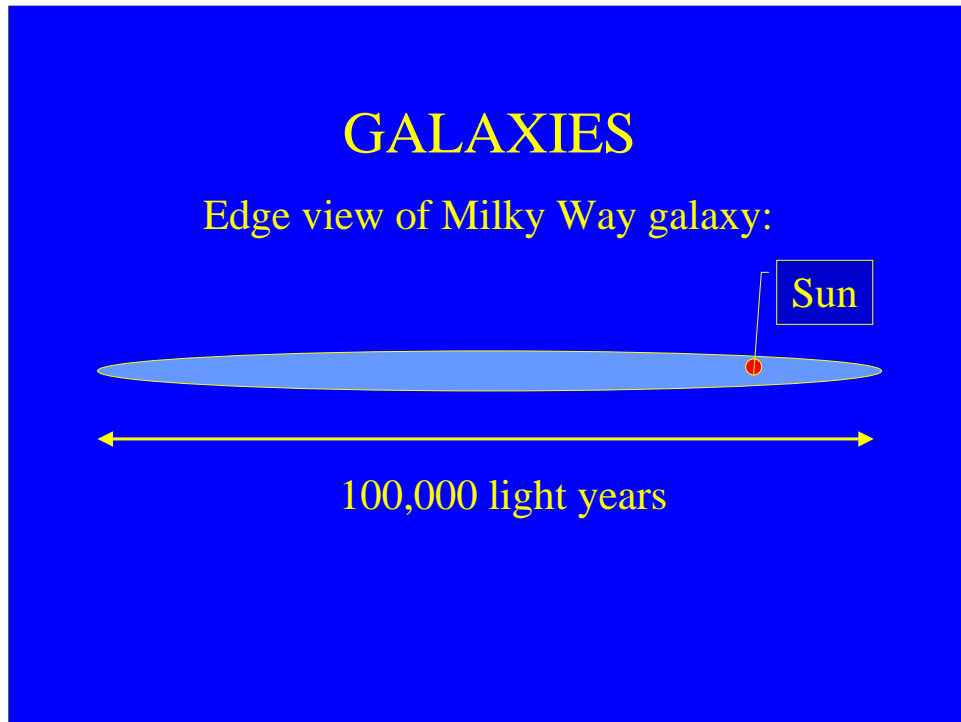
Nevertheless these effects can be observed. Small departures from the expected orbit of Uranus led two astronomers to use these departures to predict where a new planet must be, if it were causing them. This is how Neptune, and later Pluto were discovered. Calculations told the astronomers where to point their telescopes.

This is remarkable evidence for the completeness of Newton's laws for the description of our Solar System.

Earlier we talked about criteria for a good theory in physical science. One of them was that a good theory makes new predictions. This is a truly remarkable example.

GALAXIES

Edge view of Milky Way galaxy:



Our Sun exists in the Milky Way galaxy among about 100 billion other stars. This is a typical size galaxy in a universe of about 100 billion galaxies.

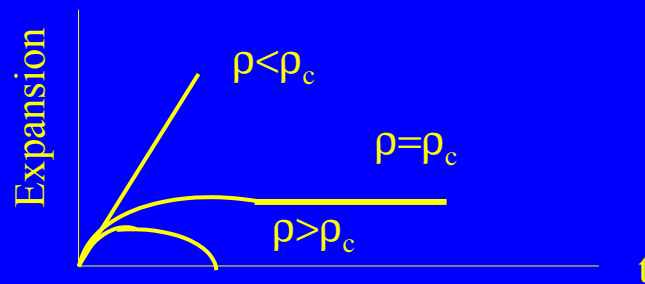
Several mysteries exist regarding the galaxies and the universe as a whole, and Newton's law of gravitation is central in the studies of these things at the edge of our understanding.

For example: Our galaxy is rotating at a certain rate, our sun along with it. Question: Does the gravity force of the other stars in the galaxy cause the centripetal force needed for this circular motion? We can calculate how much mass is needed to hold the galaxy together as it rotates, and compare with estimates of mass based on counting stars.

There is a discrepancy. Not enough visible mass is present to provide the centripetal force. There must be some kind of invisible mass, sometimes called dark matter, that is present since astronomers assume that gravity is the force that holds the galaxy together.

UNIVERSE

Is expanding. Will it continue forever?



$$\rho_c = 3H_0^2/(8\pi G) = 10^{-29} \text{ g/cc}$$

We know the universe is expanding because galaxies around us are all moving away. The same would be observed from any other galaxy. Will this expansion slow and stop, or continue forever?

Another way to ask the question is, whether gravity is strong enough to slow down the expansion. This question is a direct extension of the question Newton asked about our solar system and we discussed regarding the rotation of galaxies: Does gravity hold the system together? It will if the mass density is great enough as shown in the above graphs.

In this case it is not a centripetal force being provided by gravity, but simply a deceleration force to slow the expansion.

The observed mass density of the universe is only about 4% of the critical value shown above, so the universe appears to be open – ie it will go on expanding forever.

There are many mysteries here: Is there dark matter in the universe we can't see? There is also evidence of a repulsive force causing the expansion rate to increase with time rather than slow down. This is a rapidly developing field due to new technologies such as the space telescope that are making new observations possible. Stay tuned.