

WORK AND ENERGY

Work done by a constant force

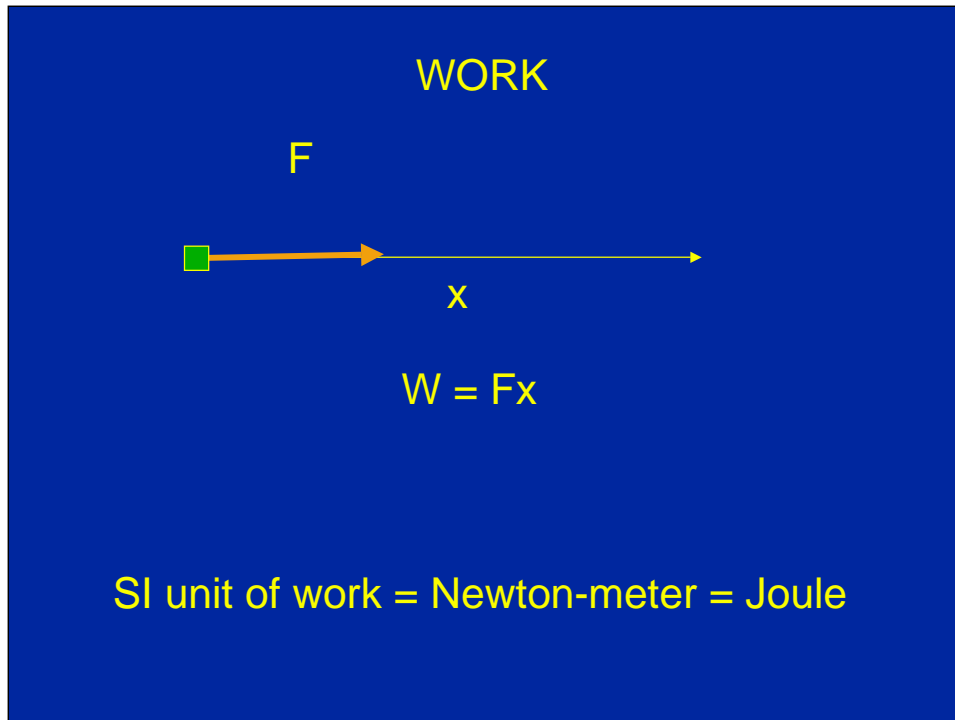
Kinetic Energy

Gravitational Potential Energy

Simple Machines

The ideas we shown above were developed in the generation after Newton. Here we will discuss work and mechanical energy. These are important concepts, developed for mechanics problems that have found wide application in other areas. Today we know that nutritional energy is important, as are electrical energy, nuclear energy, thermal energy, and so on. Our understanding of energy and its usefulness began with mechanical energy.

First we define work done by a constant force. Kinetic energy is energy a body has due to its motion. Gravitational potential energy is energy a body has due to its position relative to a massive body such as the earth. And simple machines are examples of the usefulness of the concept of energy.



The concept of work is very useful in physics. It will allow us to solve a number of problems more easily than if we were to use $F = ma$ alone.

If I drag a weight across the room through a displacement x , using a force F , then the work done is Fx . All that counts is the component of F in the direction x .

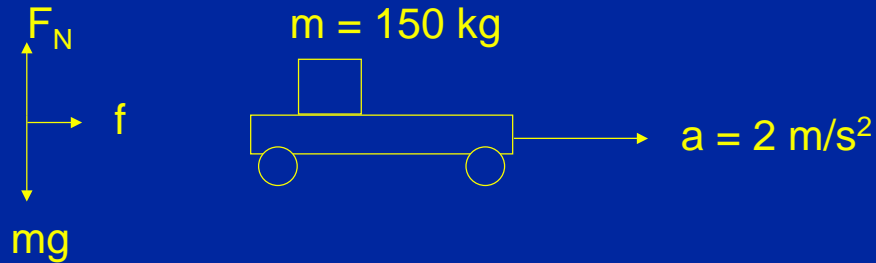
SHOW: Lifting up a weight, lowering it down, carrying it across the room at constant height.

SHOW pushing on the wall. No work is done.

So our everyday idea of doing physical work differs from the definition we use in physics. I could carry bags of fertilizer across the room, put them back down, and I have done no work at all according to our definition, but I would be exhausted and have sore muscles the next day.

Work does not have a direction associated with it like force and displacement. Technically this means that work is a scalar quantity. The same amount of work is done whether a weight is dragged North, South, East, or West. Scalars are easier to work with than vectors, such as force and displacement, so any time we can approach a problem using work rather than forces, it is probably a good idea to do so.

EXAMPLE Accelerating a crate on a truck



$$f = ma = (150)(2) = 300\text{N}$$

If the truck accelerates for $x = 50 \text{ m}$, the work done on the crate is:

$$W = (f)x = 300(50) = 15000 \text{ J}$$

In the previous example, the person dragging the box was doing work against an opposing force, namely friction. So the work was being done against friction.

Here is an example where static friction plays a role, but we ignore air friction, so work is being done, but not against an opposing force. The crate is being accelerated. Its velocity is increasing.

The friction force needed to accelerate the crate is given by Newton's second law. Here we assume the static friction force is large enough that the crate does not slip.

In this example, work is done on the crate and the result is its velocity has been increased.

KINETIC ENERGY

- The work done on the crate is $W = \max$
- Use $x = 1/2at^2$
- $W = 1/2m(at)^2 = 1/2mv^2$
- Kinetic Energy = $KE = 1/2mv^2$
- SI unit of kinetic energy = Joule
- Work-Energy Theorem: $W = KE_f - KE_i$

In this example, work was done on the crate. What was the result? It speeded up.

It is useful to define a new quantity, the Kinetic Energy, that measures the work done in producing motion.

Here we assume the forces doing the work are constant, so the acceleration is also constant. Then the work done is mass times acceleration times distance. Now we use our expression for the distance covered under constant acceleration to relate the work done to the change in square of the velocity.

By defining KE to be $1/2mv^2$ we find that the work done is equal to the change in the kinetic energy of the crate. This is called the Work-Energy Theorem. When there is a net force on an object so that it accelerates, the work done is equal to the change in its kinetic energy.

At this point all we have done is define a new quantity so that it has this property. We would not be doing this however if it were not an idea with usefulness far beyond this accounting neatness.

Crate Example Backwards

$W = 15000\text{J}$. What is v ?

$\frac{1}{2}mv^2 = 15000$, so

$v^2 = 30000/m = 30000/150 = 200(\text{m/s})^2$

$v = 14.1 \text{ m/s}$

In our crate on a truck example, we found that the work done on the crate in accelerating it was 15000 J. Let's use this to find the velocity.

We could have found the velocity by finding the time and knowing the acceleration, but this is easier. If we know the work done in accelerating a mass, the kinetic energy gives us the speed immediately.

The work-energy theorem is more general than the derivation we used indicates. It also applies when the forces and accelerations are not constant.

Example: Space Ship

- $m = 50000\text{kg}$, $v_0 = 10,000\text{ m/s}$
- Engine force = 500,000 N, $x = 3,000,000\text{m}$. What is final speed?
- $W = (5 \cdot 10^5\text{N})(3 \cdot 10^6\text{m}) = 1.5 \cdot 10^{12}\text{ J}$
- $KE_f = KE_i + W = 2.5 \cdot 10^{12} + 1.5 \cdot 10^{12} = 4 \cdot 10^{12}\text{ J}$
- $v_f = (2KE_f/m)^{1/2} = 12,600\text{ m/s}$

Here is a space ship with given mass and initial velocity drifting through deep space. It is far from stars and planets so we can ignore gravity.

Its rocket engine is now fired, providing a force of 500,000N in the direction of motion. The engine continues firing over a distance of 3 million meters. What is its final speed?

The work-energy theorem applies directly here. The work done is the force times distance, and since they are in the same direction, the speed and KE increase. The work done is huge: $1.5 \cdot 10^{12}$ Joules. But these big numbers are no problem using exponential notation.

Now the final KE is equal to the initial value plus the work done. And the final velocity can be found from the final KE.

Gravitational Potential Energy

- The gravity force can do positive or negative work on an object.
- $W = mg(h_0 - h)$
- All that counts is the vertical height change.
- $PE = mgh$

SHOW mass. Lowering it, the gravity force and the displacement are in the same direction, so gravity is doing work on the mass.

If I raise it, gravity is doing negative work on the mass.

The work done is just the force times the change in vertical height.

What if we don't move vertically, but at some other angle? If I carry the mass horizontally its gravitational energy does not change. Its gravitational energy remains the same.

EXAMPLE: PILE DRIVER

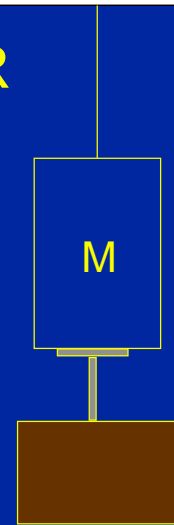
Mass is dropped on a nail from a height h .

$$W_g = mgh = 1/2mv^2$$

It exerts force F on nail, pushing it into the wood a distance d , and coming to a stop.

$$W_n = -Fd = -1/2mv^2$$

$$F = mg(h/d)$$



Pile drivers are used to pound large pilings into the ground. Huge driving forces are needed. Here we have a small version of a pile driver that can illustrate how they work.

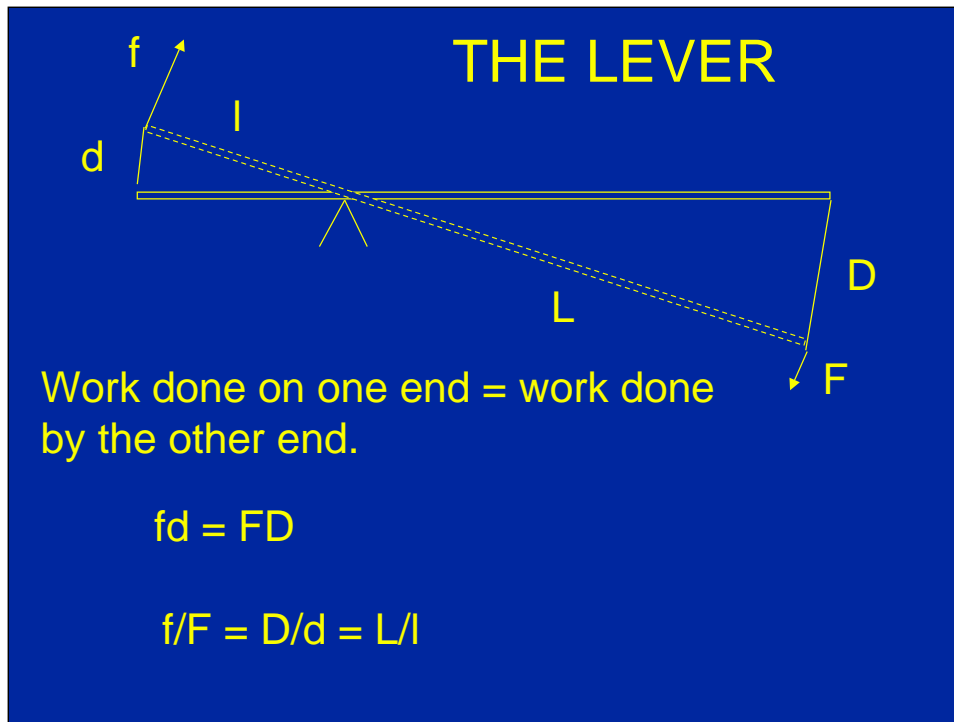
The mass falls through a height h . Gravity does work on it, increasing its kinetic energy.

Now the mass encounters the nail. The nail pushes up on it as it drives the nail into the wood. The nail does negative work on the mass, slowing it down and stopping it.

Using the work-energy theorem twice, we can relate the nail force to the gravity force.

SHOW the pile driver pounding the nail into wood.

The same thing happens when we use a hammer to pound in a nail. In this case it is not gravity but the force of our hand that gives the hammerhead its KE. We accomplish the same thing: We can multiply the force our hand can exert by giving the hammerhead KE and letting it give that energy to the nail, quickly pounding it in.



The lever is one of the greatest inventions of the ancient world. Archimedes understood levers and explained the lever law. We use levers all the time. Every time we wave our arms or walk, we are using muscles connected to our bones in such a way as to create levers.

A lever as shown here is just a stick. When work is done **on** one end, it is also done **by** the other end. The lever cannot store up work to be done later. It is a passive object. So the two amounts of work must be the same.

Here we have a lever which has been rotated through an angle by pushing on one end. The other end also exerts a force. The two forces will not be equal if the two sides of the lever are of different lengths, as shown here.

This leads us to the lever law: $f/F = L/l$. We can use the lever to multiply force.

SHOW lever. A teeter totter is a simple example of a lever.

WORK-ENERGY THEOREM: GRAVITY DOING THE WORK

$$W = 1/2mv_f^2 - 1/2mv_i^2 = \Delta KE = - \Delta PE$$

$$W = -\Delta PE$$

$$\Delta KE + \Delta PE = 0$$

Mechanical Energy = E = KE + PE = CONSTANT

When friction can be ignored

Our work-energy theorem says that the net work done by external forces equals the change in KE of an object.

When gravity does work on a mass that is moving downward, its potential energy, mgh , has been decreased. So the work done is equal to the negative of the change in potential energy.

The total mechanical energy is the sum of kinetic and potential energy. If friction is present, the work it does is usually negative, since it opposes the motion, in which case the total mechanical energy decreases.

Principle of Conservation of Mechanical Energy

- E remains constant as an object moves provided that no work is done on it by external friction forces.

This statement of conservation of mechanical energy follows immediately from the work-energy theorem.

So we usually are saying that the mechanical energy remains constant as long as friction forces are not present, or are not large, or if they act perpendicular to the displacement and so do no work on the object.

EXAMPLE: PENDULUM

Forces: Gravity

Tension (does no work)

$E = KE + PE$ remains constant as
pendulum swings

The pendulum is an excellent example of a simple mechanical system for which the mechanical energy remains nearly constant as it moves. The string tension force does no work on the mass since the motion is always perpendicular to that force. So we only have to contend with air friction which is not large as long as the velocity is small.

If I pull the pendulum off to one side and release it from rest, its initial KE is zero. Its gravitational PE has been increased from its initial value at the bottom of the swing, which we can call zero as a reference point.

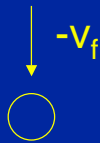
As the mass swings, its KE increases as its PE decreases. At the bottom of the swing, KE is a maximum, and PE is back to zero. Then it begins slowing down as it rises. The height to which it rises on the other side should be the same as that from which it was released.

Galileo intuitively understood all this in his work with pendulums and inclined planes.

BOUNCING BALL

○ $E = PE = mgh$

h



○ $E = KE = 1/2mv^2$

Initial

Before
bounce

After
bounce

Drop a ball from height h . Its initial energy is all potential, equal to mgh . Just before it collides with the floor, its energy is all kinetic. Just after the bounce, if no friction forces have acted, its energy is the same as before, and is all kinetic, therefore its speed must be the same.

As it rises it converts its kinetic energy to potential energy, doing the reverse of what it did during the fall.

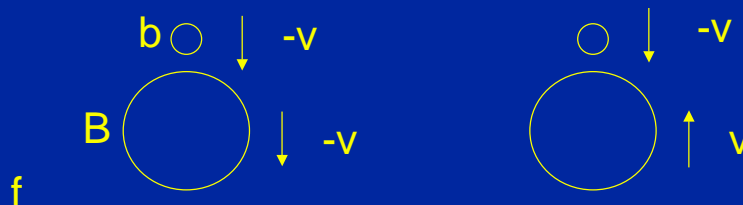
The ball rises to the same height it had initially. This is called an elastic bounce, or elastic collision. In reality no ball is perfectly elastic. Some energy is lost to friction during the collision with the floor when the ball, and also the floor are distorted by the force between them needed to accelerate the ball.

SHOW balls of varying elasticity bouncing.

SHOW double ball bounce. Does this violate energy conservation? What is happening here?

DOUBLE BALL BOUNCE

A problem in relative motion



Just after big ball hits floor, $v_{bB} = -2v$

Just after little ball hits big ball, $v_{bB} = 2v$

and $v_{bf} = v_{bB} + v_{Bf} = 3v$. How high will it rise?

$$h = v_{bf}^2/2g = 9v^2/2g = 9h_0$$

Here are the two balls falling towards the floor just before any collisions occur. On the right we see the two balls just after the big one bounces but before the little one bounces off the big one. Since the big ball reversed its velocity in the bounce, the relative speed between the balls is now $-2v$.

If the collision between the two balls is elastic, then this relative velocity will just be reversed in the collision. Then the relative velocity will be $+2v$.

If the big ball is massive compared with the little ball, its velocity relative to the floor is still nearly the same. Then the velocity of the little ball relative to the floor is given by following our notation and adding to the relative velocity, the velocity of the big ball relative to the floor.

So the little ball is moving up at 3 times the speed it would have if we dropped it alone. It then will rise 9 times as high as the point from which it was dropped.

Using the Conservation of Mechanical Energy

- Identify important forces. Friction forces must be absent or small.
- Choose height where gravitational PE is zero.
- Set initial and final $KE + PE$ equal to each other

Here is a prescription for solving problems using the conservation of mechanical energy. Here we assume gravity is the principle force in the problem.

Other forces that act perpendicular to the displacement do not invalidate the conservation of mechanical energy. Examples: String tension force for a pendulum. Normal force for a ball rolling down an incline.

The choice of where the gravitational PE is zero is arbitrary, so do this for maximum convenience. Once this choice has been made, don't change it.

Then it is just a matter of writing down the expressions for initial and final $KE + PE$ and solving the resulting equation for the unknown as we did for the pile driver and bouncing ball.

Roller Coaster

- After a vertical drop of 60 m, how fast are the riders going?
- Neglecting friction, mechanical energy will be conserved.
- $E_i = mgh$ $E_f = 1/2mv^2$
- $v = (2*9.8*60)^{1/2} = 34.3 \text{ m/s (76 mph)}$

A roller coaster swoops down for a vertical drop of 60 m. Assume it started with zero initial velocity. What is the speed of the riders at the bottom?

There are two forces present besides gravity. Friction and the normal force the track exerts on the wheels of the coaster. Here we simply ignore friction. That means of course that the actual speed will be less than we calculate here. The normal force on the wheels does no work since it is perpendicular to the motion at all times.

If we choose the gravitational PE to be zero at the bottom, then we have only PE initially, and only KE at the bottom, simplifying the equation a bit.

Note that the mass cancels out so all roller coasters will move with the same speed after the same vertical drop. As we have seen, Galileo came to this same conclusion using a pendulum and careful reasoning.

Roller Coaster Again

- If the final speed is 32m/s, how much work was done by friction on a 60 kg rider?
- $W_{nc} = E_f - E_i = 1/2mv^2 - mgh$
- $= 1/2*60*(32)^2 - 60*9.8*60$
- $= 30700 - 35300 = - 4600 \text{ J}$

Given this measured final speed, how much work did friction do on a 60 kg rider?

The work-energy theorem tells us that the net work done by friction forces is just equal to the final mechanical energy minus the initial mechanical energy. We can evaluate both giving -4600 J as the work done by friction.

The work done by friction is negative because the friction forces act to oppose the motion. As the coaster moved down the slope, friction forces are directed up the slope, acting to slow down the coaster.

Power

- $P = \text{Work/Time} = W/t$
- SI unit = J/s = watt (W)
- 1 horsepower (hp) = 746 W
- If a force F is needed to move an object with average speed v_{av} , then the power required is $P_{av} = Fv_{av}$

Power is a measure of how quickly work is being done. A more powerful engine in your car can make it accelerate at a greater rate.

Power is the rate at which work is being done, or in SI units, J/s. This unit is given the name Watt after James Watt who developed the steam engine.

A commonly used measure of power is the horsepower, originally chosen to represent how rapidly one horse could do work, which is equal to 746 W.

If when an object is moving, and we know the force producing the motion and the average speed, then the average power is $P = Fv$.

Accelerating a Car

- A 1500 kg car accelerates with $a = 5\text{m/s}^2$ for 6 s. What power is needed?
- $F = ma = 7500\text{ N}$
- $v_f = at = 30\text{ m/s}$ so $v_{av} = 15\text{m/s}$
- $P_{av} = Fv_{av} = 1.1 \cdot 10^5\text{ W}$ (151 hp)

We are given a car that accelerates at 5m/ss for 6 s, and we know its mass. Then the power is easy to obtain using newton's second law, and kinematics to find the average velocity.

Car at constant speed

- Car going 60 mph (27 m/s) requires $F = 200 \text{ N}$ to overcome friction.
- What power is required from the engine?
- $P = Fv = 200 * 27 = 5400 \text{ W} = 7.2 \text{ hp}$

Now consider a car at constant speed on a level road. The engine must provide a forward force to overcome friction.

Given these numbers we see that the power required is 5400 W , or about 7.2 hp . As we can see from these two examples, large engine power is needed for acceleration, not for maintaining reasonable highway speeds.

When you drive a car around town, the average power needed is quite small, much less than the typical engine powers in today's cars.

Engineers are developing hybrid cars that use a small gasoline engine running at its most efficient speed all the time running a generator, in combination with electric motors to power the car. This results in better gas mileage, and considerably less air pollution.

TOUR DE FRANCE

What power does cyclist need?

$$\text{Air friction force} = f = kv^2$$

$$P = fv = kv^3$$

$$P = 1 \text{ kW for } v = 25 \text{ mph}$$

What power does Superman need to go
50 mph?

$$P = 1 \text{ kW} (v_2/v_1)^3 = 8 \text{ kW}$$

A cyclist does work against air friction in pedaling a bicycle. Here we assume the road is level and there is no wind. At typical cycling speeds, the friction force is proportional to the square of the velocity. That means the power needed to maintain a fixed speed is proportional to the cube of the speed. That is why increasing your speed significantly is so difficult.

Professional tour riders go about 25 mph, and must be doing work at the rate of about 1 kW to maintain that speed.

If Superman existed, what power would he need to generate to cycle 50 mph?

The power scales as the cube of the speed, so to double the speed, the power must be multiplied by 8.

Superman would not be able to maintain that speed, however, he would have to sweat 8 times as fast as a normal cyclist, and since water does not diffuse through steel nearly as fast as it does through human skin, superman would overheat and eventually melt.

Principle of Energy Conservation

- Energy can be neither created nor destroyed, but only converted from one form to another.

Our use of the conservation of mechanical energy ignores the possible existence of other forms of energy. Even in the case of the car we just discussed, other forms are important. For example, the gasoline in the tank represents a chemical energy which can be converted to mechanical energy with a certain efficiency.

The principle of conservation of energy has no known exceptions. All processes we are aware of obey this principle. Mechanical energy is a good example of this broader principle.

This is one of the grand principles in all the sciences. It can be a slippery one to apply however since energy can be changed from one form to another.