# THE TWO PRINCIPLES

The laws of physics are the same in all unaccelerated reference frames.

The speed of light is not affected by the motion of its source.

These are the two principles from which all the consequences of special relativity can be deduced. Neither of these should be new to you. The first principle was stated clearly by Galileo. He wrote that if you are in a closed room in a ship, and have brought with you anything you like to make observations of, including fish, insects, etc, that nothing will take place in the room to indicate whether or not the ship is moving.

The second principle is one we have seen good experimental support for. We saw that gamma rays emitted by pions moving at 0.99c, travel at the same speed as light emitted by a stationary source. One could argue that the second principle is implied by the first since Maxwell’s equations state that the speed of light is c. Stating both principles makes the constancy of c for all observers more explicit.

## SOME SPEEDS

| Example | Speed | v/c |
|---|---|---|
| Earth about Sun | 30km/s | 0.0001 |
| Sound in air | 343 m/s | 0.0000011 |
| 90 mph fastball | 40 m/s | 0.00000013 |
| 10 MeV electron | 300000 km/s | 0.99 |

c = 1 ft/ns

Before pursuing the consequences of the two principles, let us look briefly at some examples of speeds of objects we are familiar with. Everyday objects have speed that are very small compared with the speed of light. In order to see clear examples of the consequences of the principles of relativity, we will need to imagine objects moving at nearly the speed of light, or as is commonly said, at relativistic speeds.

Relativistic speeds are common for electrons or protons emerging from a particle accelerator. We need to understand special relativity in order to analyze the collisions of these particles with their targets.

To discuss the implications of the principles, we will need to imagine whole reference frames moving at relativistic speeds. Einstein called these imaginings “thought experiments”.

Think of our imaginary reference frame as a huge room, big enough that it takes light an appreciable time to go from one side of it to another. There are meter sticks or tape measures available to determine positions in the room, and there are clocks wherever we need them to time events that occur. Our reference frame is unaccelerated, but likely will be moving relative to other reference frames with relativistic speeds.

## Synchronizing Clocks in our Reference Frame

Place clocks in their needed locations (probably far from each other), and use light signals to synchronize them.

Clock A sends a light pulse to clock B where it is detected and reflected back. The two will be synchronized if when the pulse reaches it, clock B reads:

$$t_{B(receive)} = t_{A(sent)} + ½(t_{A(return)} - t_{A(sent)})$$

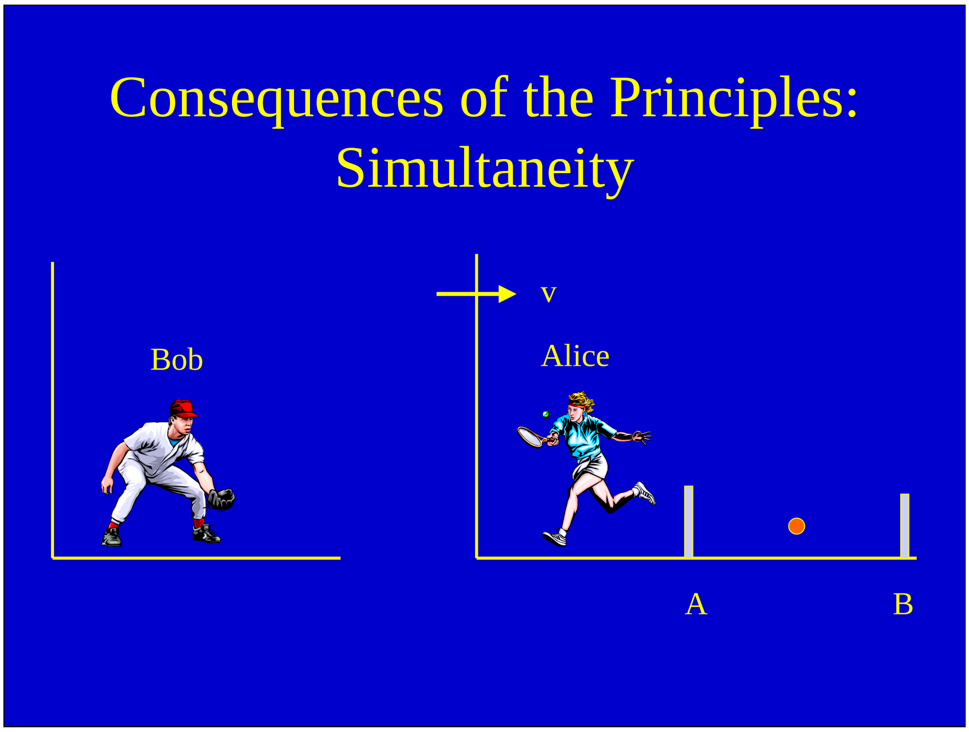
We will be carrying out various thought experiments both in our reference frame and between two reference frames in relative motion with each other. To get started we need to know how to synchronize the clocks in our frame. There is nothing tricky about this or conceptually difficult, it is simply a practical matter we need to straighten out before proceeding.

We could use many different schemes to synchronize out clocks, such as having a messenger travel between them, but the simplest is to use light flashes. Light flashes involve a shorter time delay than any other method, and we are guaranteed that light travels at a fixed speed c.

If it takes two seconds for light to go from clock A to clock B, and the flash is sent out at exactly 4:00, then when the flash gets to clock B, it should read 2 seconds past 4:00. If it does, then the two are synchronized.

Of course the surest way of knowing that light takes two seconds to go from A to B is to determine using clock A that the reflected pulse arrives back at A after four seconds. The above equation expresses this way of synchronizing the clocks.

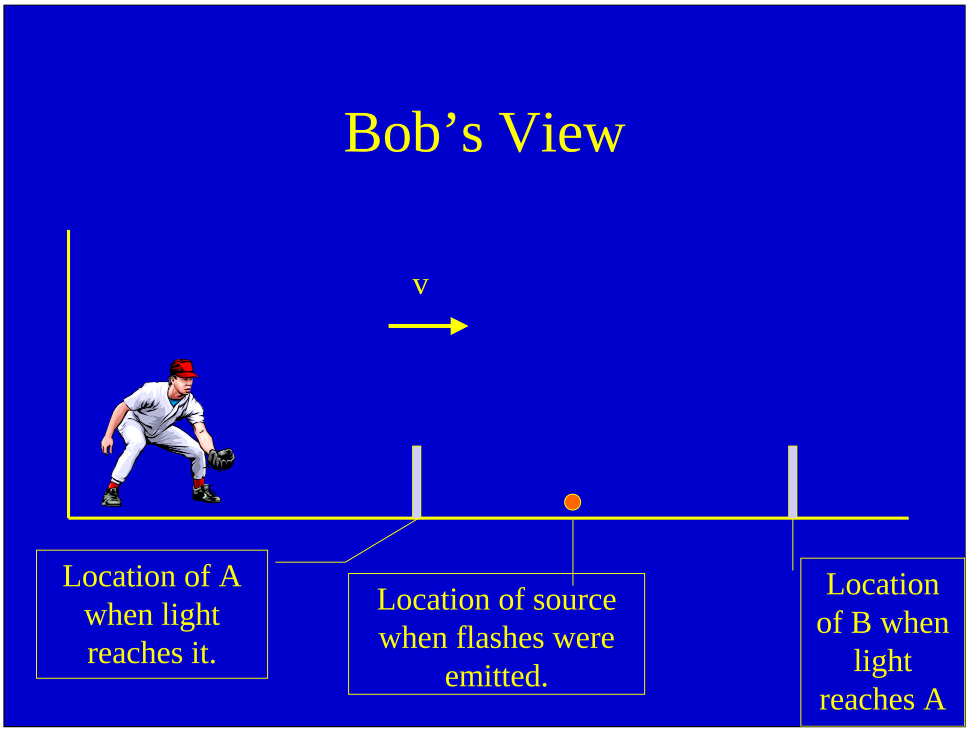
Examples: The Sun 8 light min away, alpha centauri 4 light years away. Using a worm as messenger taking 5 min to crawl from A to B




A flash bulb creates a pulse of light. It is located exactly halfway between markers A and B in Alice's reference frame which is moving with speed v relative to Bob's frame. She sees the flashes of light arrive at the same time at markers A and B, and so says that those two events were simultaneous. Note: She may have assistants with clocks at A and B. They record the arrival time of the flash at their marker and report it to Alice.

Here we are talking about two events – the arrival of a light flash at marker A and at marker B. We know the locations of the markers and Alice knows their arrival times. In this case the arrival times are the same, so the events are simultaneous.

Bob observes the same two events. How do they appear to him?



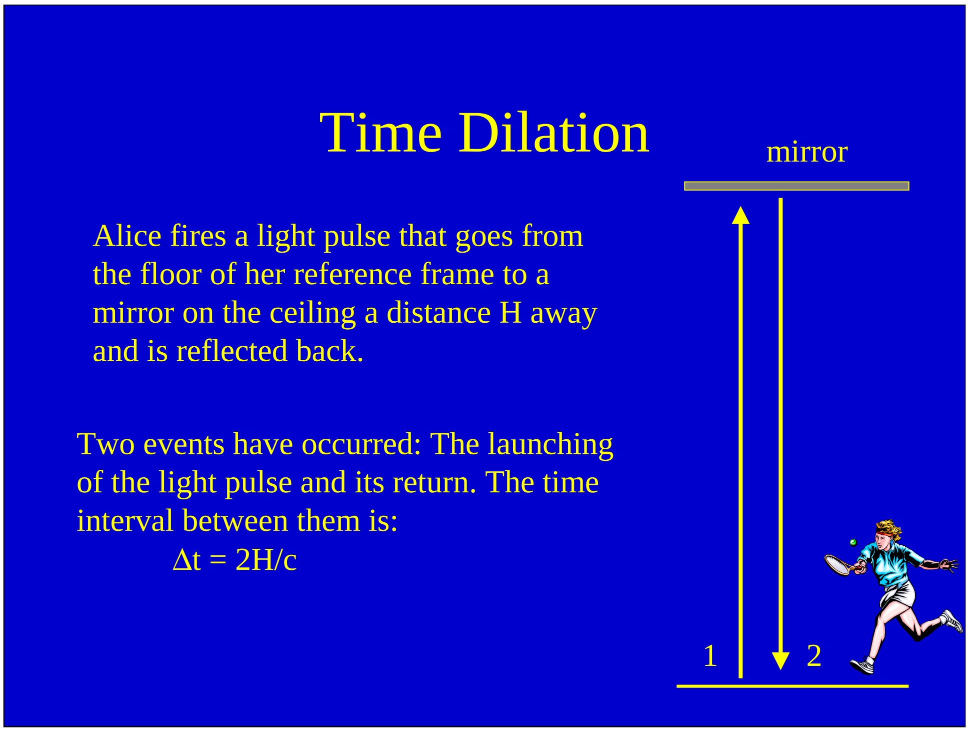

Bob sees light emitted midway between markers A and B. As the light pulse travels out in both directions, A moves towards the source and B moves away. So the distance from source to A decreases, the distance from source to B increases. So the light reaches A before it reaches B according to Bob.

As observed in Bob's reference frame, these two events are not simultaneous.

Conclusion: Spatially separated events that are simultaneous in one frame are not in general simultaneous in another reference frame.

Suppose v = 0. Then if Bob is using synchronized clocks, he will report that the arrival of the light at A and B are simultaneous events. When relative motion occurs, this is no longer the case.

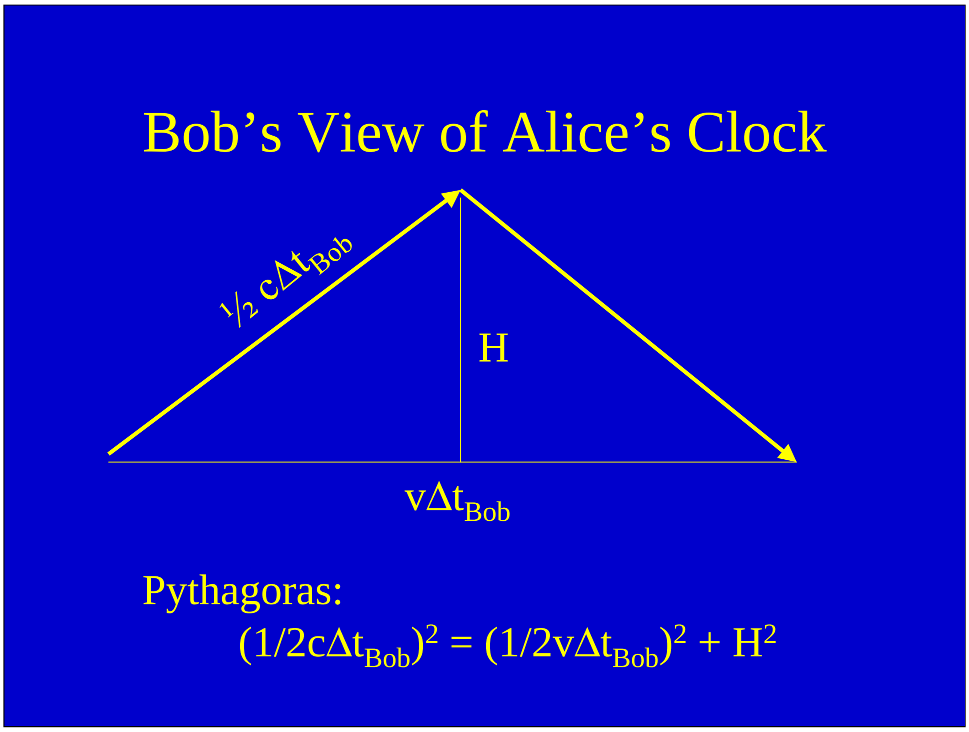
So thinking carefully about these events forces us to give up the intuitive sense we have of the existence of a universal, global "now". Simultaneity is a relative notion – relative to the reference frame from which the events are observed.




We have just seen that two observers in different frames can disagree about the time interval between two events, such as the arrival of the light pulses at markers A and B.  One could say it is zero, and the other that it is not zero. Let's make this comparison of time intervals more quantitative. Alice wants to time how fast she can run across the tennis court in her reference frame and back. She decides to use light to make a clock since she knows its speed is always c. She mounts a mirror near the ceiling. She fires a light pulse as she starts running and finds that it reflects back just as she returns to the starting point. She adjusts the height of the mirror H to make this happen.

So the time it takes Alice to run across the court and back is Dt = 2H/c. She lists this time on her CV as an indication of how fast she is on the tennis court.

Bob objects. He says she is not this fast. Here is how Bob sees these same two events.




Bob observes Alice timing her running and decides to verify her results. Since Alice’s frame moves with speed v relative to Bob’s he sees the light pulse move in a diagonal path from tennis court to mirror and along a similar diagonal path back down to the court. The light traveled farther from Bob’s point of view, and since the speed of light is exactly the same for Bob and Alice, he thinks it took her longer to run across the court and back than she does.

We can use Pythagoras to make this difference quantitative. Using the light path and tennis court path for the first half of the light pulse journey, we have a right triangle as shown above. Pythagoras gives us a relation between the lengths of the sides of the triangle.

## Quantifying the Comparison

From Pythagoras:

$$(c^2 - v^2)(1/2\Delta t_{Bob})^2 = H^2$$

$$(1 - v^2/c^2)(\Delta t_{Bob})^2 = 4H^2/c^2$$

$$(1 - v^2/c^2)^{1/2}(\Delta t_{Bob}) = 2H/c = \Delta t_{Alice}$$

$$\Delta t_{Bob} = [1/(1 - v^2/c^2)^{1/2}](\Delta t_{Alice})$$

$$\Delta t_{\text{Frame with speed v}} = [1/(1 - v^2/c^2)^{1/2}](\Delta t_{\text{Frame with events at same place}})$$

A few steps of algebra allow us to solve for the time interval observed by Bob in terms of that measured by Alice. We see that since the factor $(1\text{-}v^2/c^2)^{1/2}$ is less than 1, Bob does indeed measure a longer time interval than Alice. He thinks she is slower on the tennis court than she does.

This difference is not because either have faulty clocks. Indeed they are using the same clock. Note that it is crucial to this result that the speed of light is exactly the same for Bob and Alice. Alice's relative motion does not affect the speed of light as we have seen experimentally, and as postulated by Einstein.

The intrinsic difference between the two observers is that the two events occurred at the same place according to Alice, while for Bob they did not because her frame is moving relative to his. So we can express this result making this difference explicit.

The above difference in clock speed is referred to as *time dilation*. It is an intrinsic property of space and time and relative motion. The fact that time passes at different rates for different observers seems strange to us because our intuition is based on low-speed events. Time dilation applies to all kinds of clocks: biological clocks such as aging, rates of radioactive decay, the performance of a wristwatch,…

# Example

Suppose $v/c = 3/5$

Then $v^2/c^2 = 9/25$, and $1 - v^2/c^2 = 16/25$

and $1/(1 - v^2/c^2)^{1/2} = 5/4$

So $\Delta t_{Bob} = 5/4 \Delta t_{Alice}$

Here is a numerical example in which we suppose Alice’s speed relative to Bob is 3/5 of the speed of light. Then the numerical factor that always enters these problems, one divided by the square root of one minus v squared over c squared is just 5/4, or 1.25.

## Qualitative Summary

An observer for whom two events occur at the same place measures the least elapsed time between them.

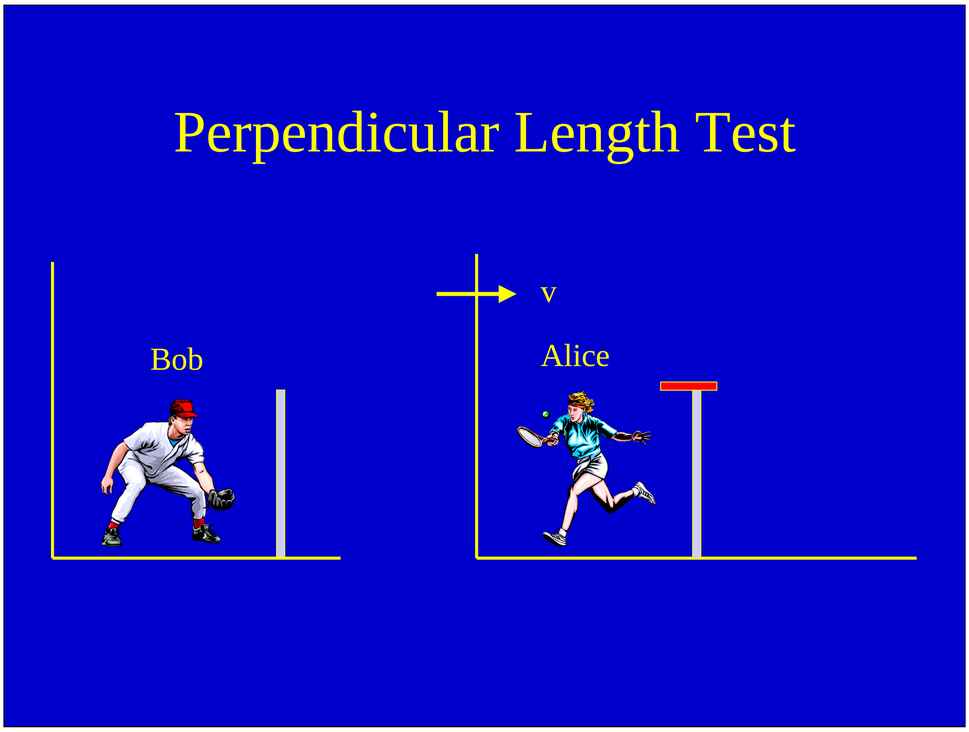
Here is a qualitative summary of our results for time dilation. It is useful to keep this in mind just to make sure you have the dilation factor on the right side of the equation.

## Perpendicular Lengths

Is H the same for Alice and Bob?

Do lengths perpendicular to the relative motion appear different for different observers?

So far we have assumed that the distance H between the two mirrors remains the same for Alice and Bob. We need to check that assumption.



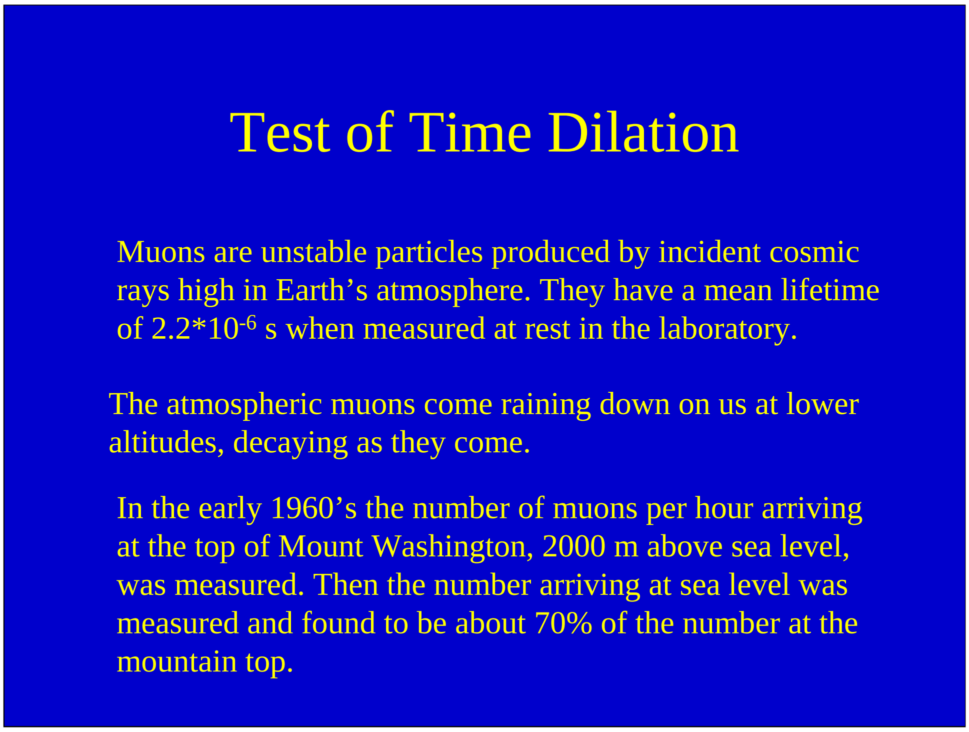

Alice and Bob each have a board mounted perpendicular to their relative motion. The two boards were cut to the same length when they were at rest relative to each other. Alice puts a red crayon on top of her board. The two boards pass very close to each other.

Let’s try “the moving board appears shorter” as a possibility. Then Bob will see Alice’s board as being shorter than his because of its motion relative to him, and the red crayon will mark a stripe on it as they pass each other. As seen by Alice, Bob’s board appears shorter, and her crayon will not mark it.

Whether or not a mark is made cannot depend on the reference frame from which observations are made. Slowing Alice down and stopping her relative motion cannot make the red stripe Bob saw on his board disappear. So the proposition that “the moving board appears shorter” leads to a contradiction and must be wrong.

The same argument rules out “the moving board appears longer”. So we are left with “the moving board appears the same length” as the only possibility, and our time dilation calculations are shown to be valid.

This applies only to lengths measured perpendicular to the relative motion.




In order to make an experimental test of time dilation we need to find a clock we can observe at rest and also at high speed. Once again we find our best example among the elementary particles simply because they are often found moving at relativistic speeds.

But what about the clock? Muons do carry a kind of statistical clock with them, namely their mean lifetime. When measured at rest, their mean lifetime is 2.2 microseconds.

Event one in this experiment is the measurement of the number of muons per hour arriving at the top of Mt. Washington. Event two was a subsequent measurement of the number per hour arriving at sea level.

Since the muon creation rate high in the atmosphere remains constant, this is a way of measuring how many muons decayed on descending 2000 m in the atmosphere.

## Numbers for Muon Experiment

Speed of descending muons = 0.995c (known from energy of the production reaction high in the atmosphere). This corresponds to $(1 - v^2/c^2)^{1/2} = 0.1$

Time to travel 2000 m = $2000\text{m}/(3*10^8 \text{ m/s}) = 6.7*10^{-6}$ s

This is 3 times the mean lifetime, so only 5% should survive to sea level, while 70% did so.

$$\Delta t_{\text{observed by us}} = [1/(1 - v^2/c^2)^{1/2}](\Delta t_{\text{muon}})$$

$$\Delta t_{\text{muon}} = (1 - v^2/c^2)^{1/2}(\Delta t_{\text{observed by us}}) = (0.1)(6.7*10^{-6}) = 0.67*10^{-6} \text{ s}$$

This is 1/3 of a mean lifetime, and corresponds to 70% survival

To make this experiment quantitative, we need to know how fast the muons are moving. We know that because we know the energy of the reaction that produces them. Then we can calculate how long it takes them to go 2000 m. This time is three times their mean lifetime, so only a few should survive at sea level. In fact 70% were observed to do so. How can this be?

What does time dilation have to say about this situation? A muon is at rest in a reference frame moving with it.The two events in the experiment, the measurements at 2000m and sea level, occurred at the same location in that frame, namely the muon’s position. Therefore the time interval between the events measured by us is longer than that measured by the muons by the time dilation factor.

Putting in the known speed, we see that the muon’s clock only ticked off 0.67 microseconds between 2000 m and sea level. This corresponds to a 70% survival rate as observed.

Here we have seen a time dilation factor equal to 10. The muons lived 10 times as long as do muons at rest. You could live 10 times as long as well, if you could move that fast.

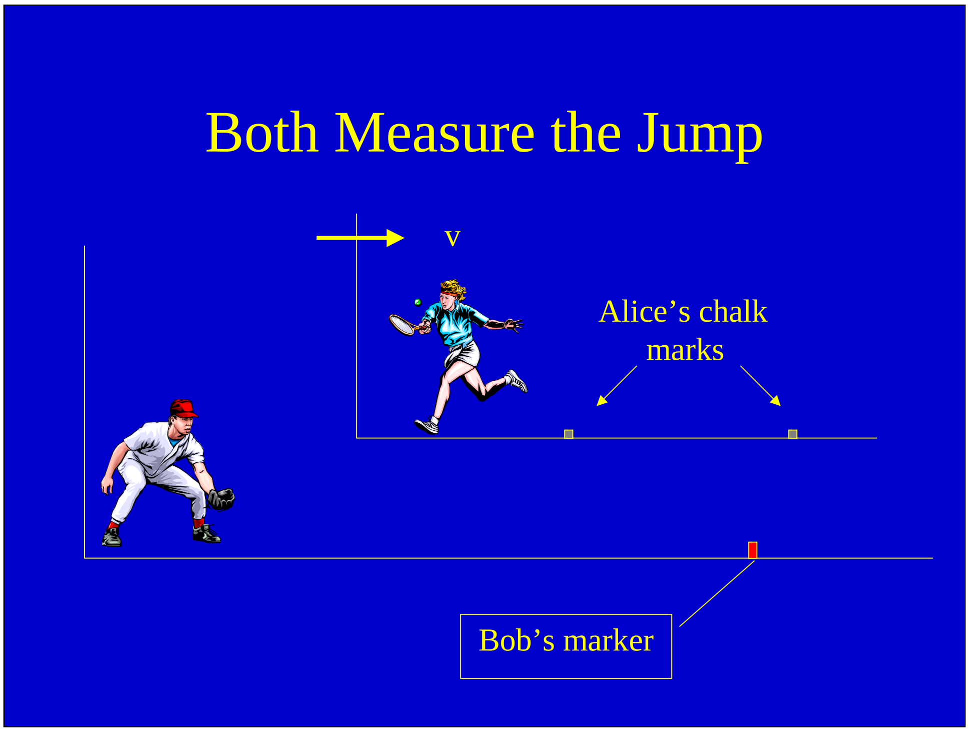
# Length Contraction

Alice leaps across the net after winning another tennis game and marks the take-off and landing spots on the court. She is so impressed by her distance that she adds this to her CV as another athletic achievement.

Once again, Bob disputes her claim, measuring a shorter distance.

What is a good method for them both to measure this distance?

Alice and Bob are destined for disputation. Alice measures her running broad jump distance across the net on her tennis court, and Bob, in his frame, measures a different result. How should they both carry out this measurement?




They agree on the following method for them both to measure the distance Alice jumped. Bob places a marker on the floor of his reference frame. He then measures how long it takes the two chalk marks on Alice’s tennis court to pass his marker. Since he knows the speed of Alice’s frame, this gives him the distance, denoted by d.

Alice makes the corresponding measurements from her frame. For her it is the marker in Bob’s frame that is moving. She measures the time it takes for Bob’s marker to pass between her chalk marks.

## Length Contraction Results

Bob's measured length: $d = v\Delta t_{Bob}$

Alice's measured length: $d_0 = v\Delta t_{Alice}$

The two events occur at the same place in Bob's frame, so

$$\Delta t_{Alice} = [1/(1 - v^2/c^2)^{1/2}](\Delta t_{Bob})$$

Combining the above results we have:

$$d = (1 - v^2/c^2)^{1/2} d_0$$

Length along the direction of motion is shorter when measured from moving frame than when measured in its own rest frame.

Now we can summarize the results of these measurements. Bob's measured length is expressed in terms of the time interval he measures. Same with Alice. In this case the two events occur at the same place in Bob's frame (at the position of his marker), so we know how to relate his time interval to Alice's.

This then gives us the relation between the two measured lengths. Bob does measure a smaller jump distance for Alice than she does.

Distance intervals, or lengths, measured along the direction of relative motion between two frames are shorter when measured from the moving frame. An object has its greatest length when measured in its own rest frame.

This result is general and is an intrinsic property of space and time and relative motion.

## MUONS REVISITED

Time dilation provided an explanation for how 70% of the atmospheric muons observed at 2000 ft altitude survive to sea level. In that explanation the reference frame used was that of the Earth.

Instead let's view the situation from the frame of the muons. Now it is the mountain that moves at 0.995c. It's height is therefore contracted from 2000 to 200m. The time for the muons to go 200 m is $0.67 * 10^{-6}$ s, just what we found using the Earth as a reference frame. The two explanations are consistent.

Just as there is usually more than one way to solve a problem involving Newton's laws, there is also in special relativity.

We have seen that muons produced at high altitude manage to survive in large numbers to sea level even though their lifetime is only $2.2 * 10^{-6}$ s and the journey, measured by us, takes longer than that. Time dilation provided a quantitative explanation for the high survival rate. They really do live longer when moving fast.

Now let's view the situation from the reference frame of the muons. In that frame they are not moving at all, so there is no time dilation. Now it is Mount Washington that moves, going upward at a speed of 0.995c relative to the muons. So the mountain undergoes length contraction. We found before that $(1 - v^2/c^2)^{1/2} = 0.1$. This is the crucial factor in both time dilation and length contraction.

So Mount Washington is only 200m tall as seen by the muons. And it takes them one-tenth as long to speed past it, giving us an alternative explanation for the survival of the muons.

## TWO BASIC RESULTS

Time dilation

$$\Delta t(v) = [1/(1 - v^2/c^2)^{1/2}]\Delta t(0)$$

Length Contraction

$$d(v) = (1 - v^2/c^2)^{1/2} d(0)$$

These two basic results are at the core of special relativity: Moving clocks run slow, and moving meter sticks are shorter along the direction of motion. As we have seen, length contraction is a direct result of time dilation, and time dilation depends directly on the fact that all observers see light moving at the same speed c.

Earlier we noted that events that are simultaneous for one observer may not be for another who is moving relative to the first. We noted qualitatively that this must be the case. Now we can be quantitative about it.

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