Lecture 10 Induction and Inductance Ch. 30

- Cartoon - Faraday Induction
- Opening Demo - Thrust bar magnet through coil and measure the current
- Warm-up problem
- Topics
  - Faraday's Law
  - Lenz's Law
  - Motional Emf
  - Eddy Currents
  - Self and mutual induction
- Demos
  - Thrust bar magnet through coil and measure the current in galvanometer. Increase number of coils
  - Compare simple electric circuit- light bulb and battery with bar magnet and coil.
  - Coil connected to AC source will induce current to light up bulb in second coil.
  - Gray magnet, solenoid, and two LED's, push and pull, shows that different LED's light up. Lenz's Law
  - Hanging aluminum ring with gray magnet. Lenz's Law
  - Jumping aluminum ring from core of solenoid powered by an AC source. Press the button.
  - Slowing down of swinging copper pendulum between poles faces of a magnet. Eddy Currents
  - Two large copper disks with two magnets
  - Neodymium magnet swinging over copper strip. Eddy currents
  - Neodymium magnet falling through copper pipe. Cool with liquid nitrogen. Eddy currents
  - Inductive spark after turning off electromagnet. Inductance.

Introduction

- Stationary charges cause electric fields (Coulombs Law, Gauss' Law).
- Moving charges or currents cause magnetic fields (Biot-Savart Law). Therefore, electric fields produce magnetic fields.
- Question: Can changing magnetic fields cause electric fields?

Faraday's Law $Emf = -\frac{d\phi_m}{dt}$

- Discovered in 1830s by Michael Faraday and Joseph Henry. Faraday was a poor boy and worked as a lab assistant and eventually took over the laboratory from his boss.

- Faraday's Law says that when magnetic flux changes in time, an Emf is induced in the environment which is not localized and also is non-conservative.

- Lets look at various ways we can change the magnetic field with time and induce an Emf. If a conductor is present, a current can be induced.

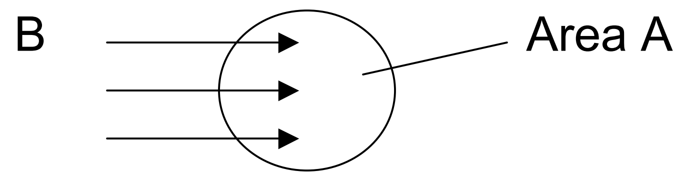
1. What is magnetic flux ϕ_m ?

2. What is an induced Emf ?

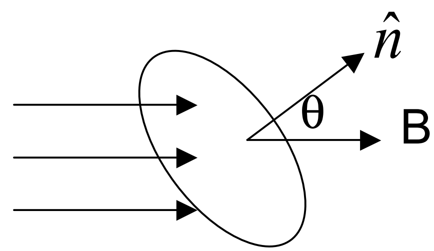
Magnetic flux

First a Reminder in how to find the Magnetic flux across an area

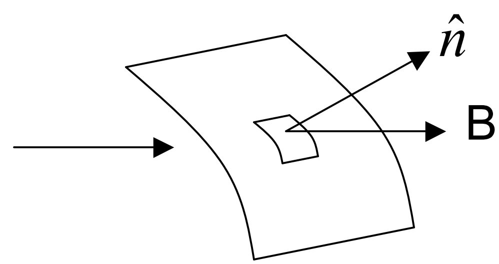
- $\phi_m = BA$



- $\phi_m = \vec{B} \cdot \hat{n} dA$
  $= \vec{B} \cdot d\vec{A}$
  $= B \cos\theta dA$



- $\phi_m = \int \vec{B} \cdot \hat{n} dA$



Units:

B is in T

A is in m^2

ϕ_m is in (Webers) Wb

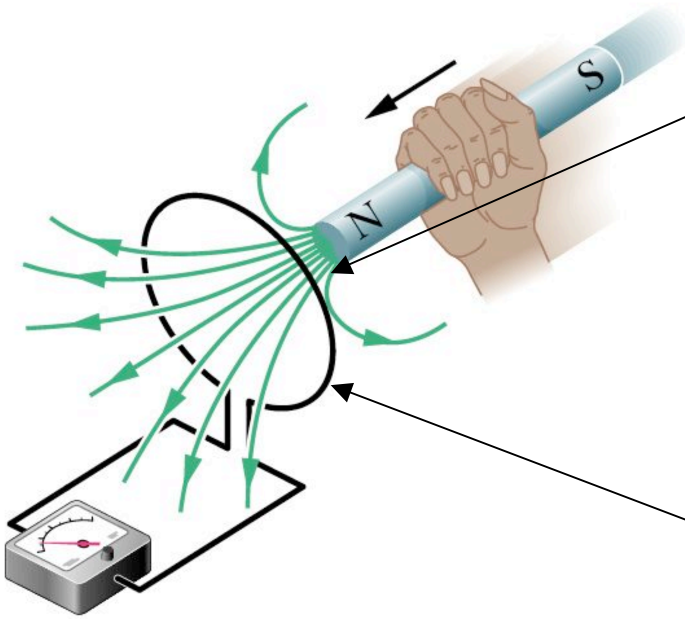
Experiment 1 Thrusting a bar magnet through a loop of wire

Magnetic flux

Faraday's Law $$Emf = -\frac{d\phi}{dt}$$

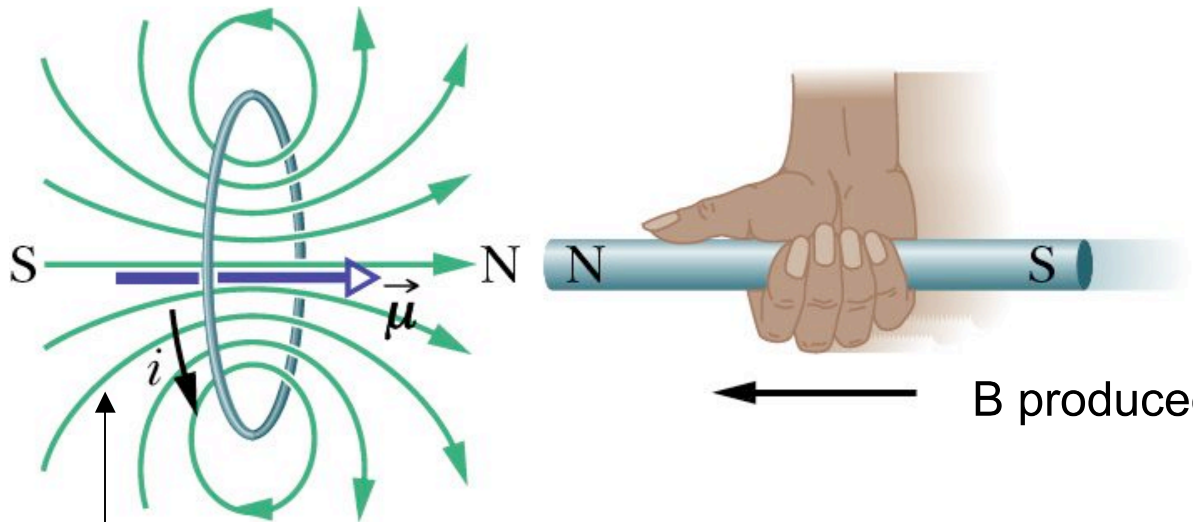
$$\phi = \int \vec{B} \cdot \hat{n} dA$$

Bar magnet field



Current flows in the ring and produces a different B field.

Lenz's Law



B produced by bar magnet

$$B = \frac{\mu_0 i}{2R}$$ at center

Produced by current flowing in the wire.

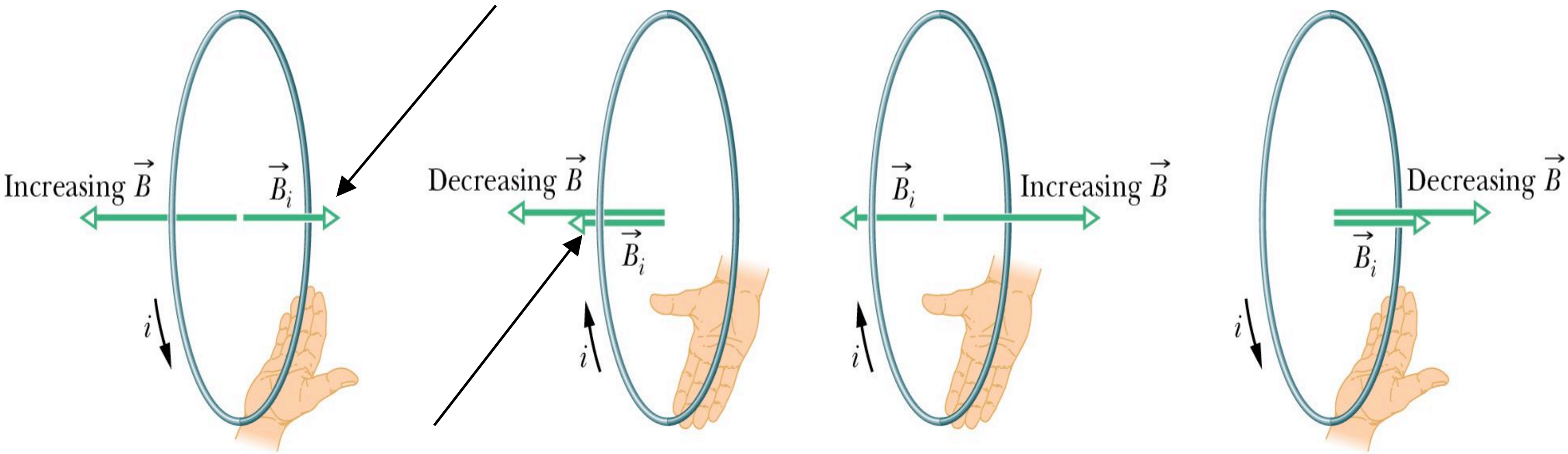
Lenz's Law → The current flows in the wire to produce a magnetic field that opposes the bar magnet. Note North poles repel each other.

More on Lenz's Law:

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current

Question: What is the direction of the current induced in the ring given *B* increasing or decreasing?

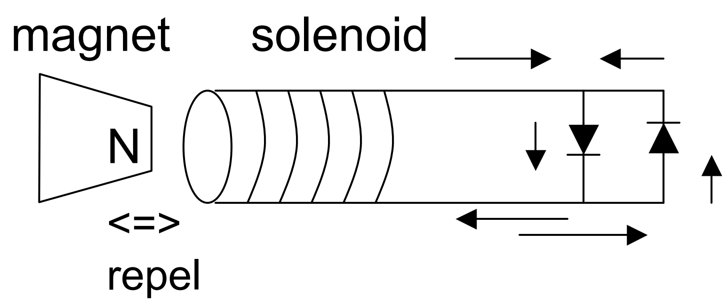
B due to induced current



B due to induced current

(*a*) (*b*) (*c*) (*d*)

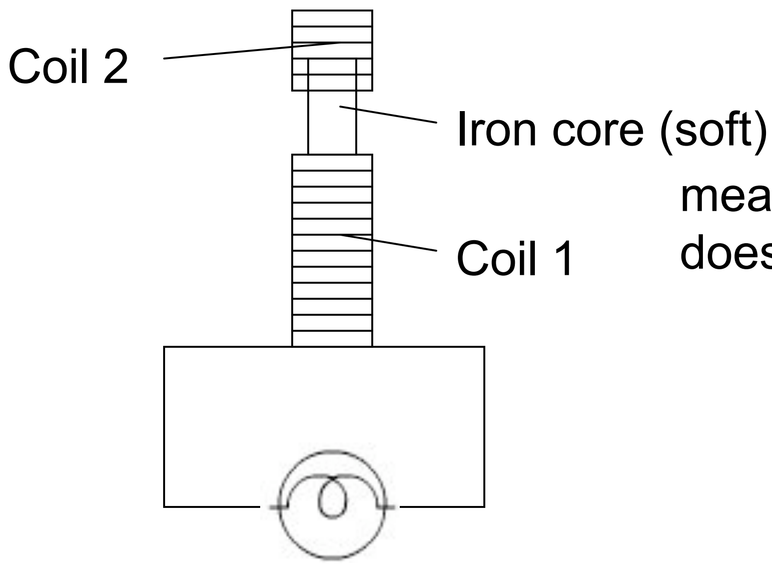
Demo: Gray magnet, solenoid, LEDs



Push magnet in, one LED lights

Pull magnet out, the other LED lights

Demo: Coil connected to AC source
Light bulb connected to second coil
(same as solenoid)



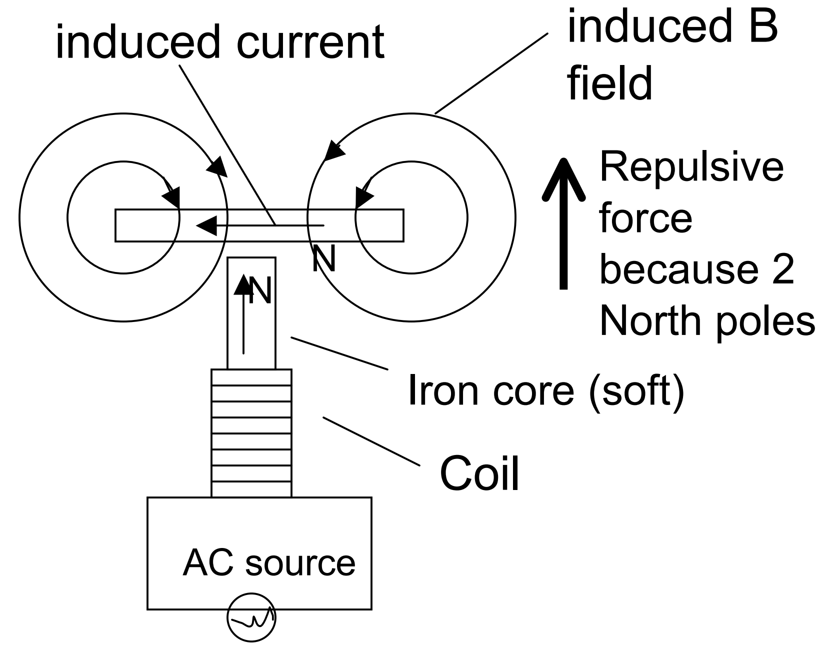
means lots of inductance in wire so AC doesn't heat up wire.

Shows how flux changing through one coil due to alternating current induces current in second coil to light up bulb. Note no mechanical motion here.

Demo: Jumping aluminum ring from core of solenoid powered by an AC source. Press the button.

- When I turn on the current, B is directed upward and momentarily the top of the iron is the North pole. If the ring surrounds the iron, then the flux in it increases in the upward direction. This change in flux increases a current in the ring so as to cause a downward B field opposing that due to the solenoid and iron. This means the ring acts like a magnet with a North pole downward and is repelled from the fixed coil.

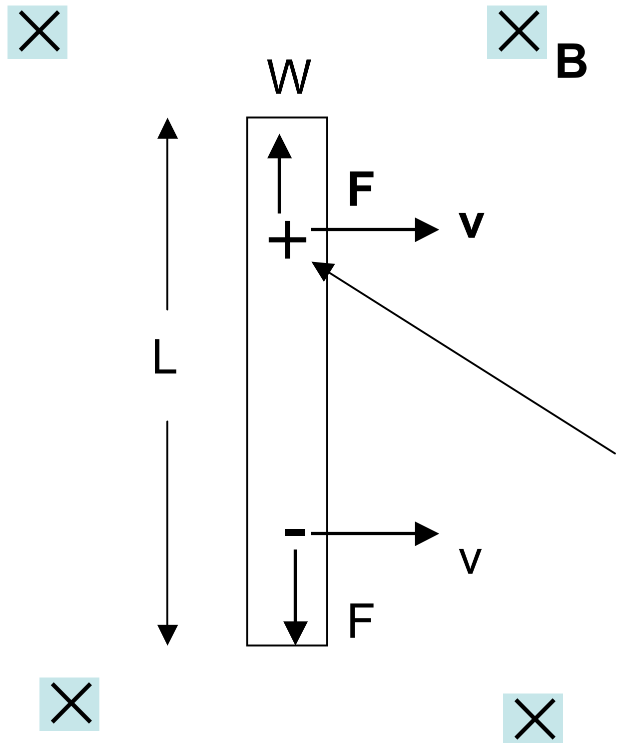
- Try a square-shaped conductor
- Try a ring with a gap in it
- Try a ring cooled down to 78 K



Experiment 3: Motional Emf

Pull a conducting bar in a magnetic field. What happens to the free charges in the material?

Moving bar of length L and width W entirely immersed in a magnetic field B. In this case an Emf is produced but no current flows



$$Work = qvBL$$
$$U = q \times emf = qvBL$$

$$F_m = qv \times B$$

Positive charges pile up at the top and negative charges at the bottom and no current flows, but an Emf is produced. Now let's complete the circuit.

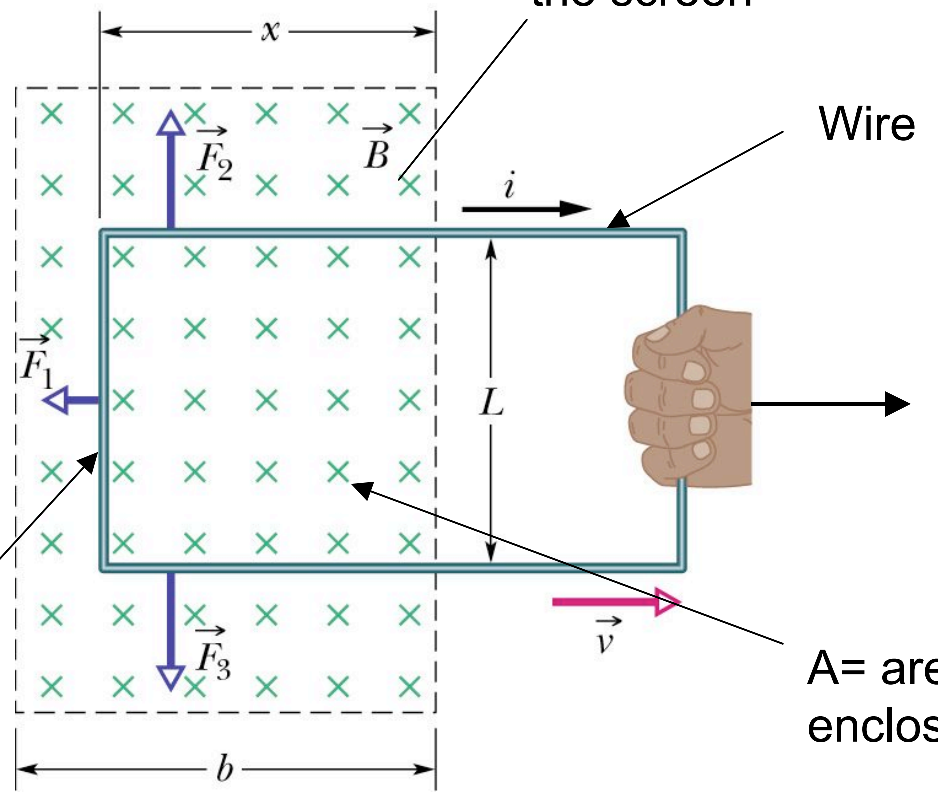
Motional Emf

What force is required to keep current flowing in the circuit?

Pull the rectangular loop out of the magnetic field. A current i will be induced to flow in the loop in the direction shown. It produces a magnetic field that tries to increase the flux through the loop.

Uniform magnetic field into the screen

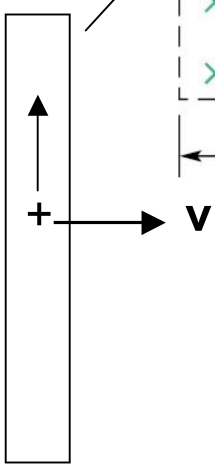
Wire



F_A

A= area of magnetic field enclosed by the wire

Close up of the wire



$$F_m = qv \times B$$

$$F_1 = iL \times B$$

$$emf = -\frac{d\phi_m}{dt} = -B\frac{dA}{dt}$$

Motional Emf Continued

$$F_1 = iL \times B$$

$$\left.\begin{aligned} F_1 &= BiL \\ F_2 &= Bix \\ F_3 &= -F_2 \end{aligned}\right\} \text{cancel}$$

$$emf = -\frac{d\phi_m}{dr} = -B\frac{dA}{dt}$$

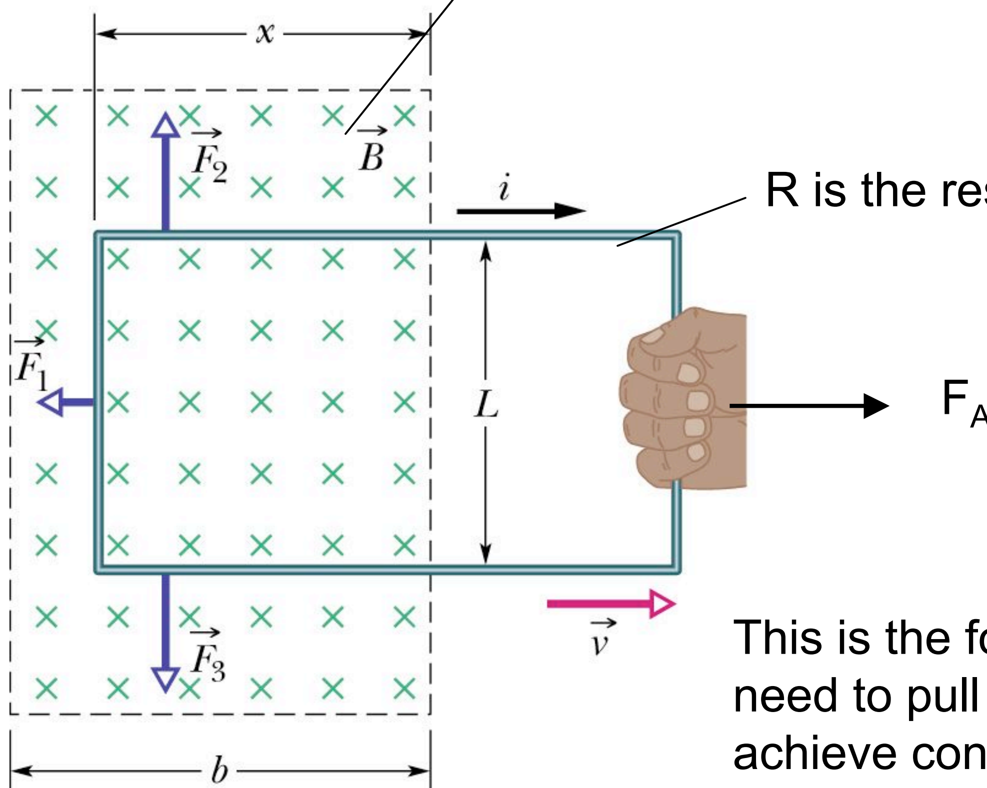
$$= -B\frac{d(Lx)}{dt} = -BL\frac{dx}{dt} = -BLv$$

$$emf = iR$$

$$BLv = iR$$

$$i = \frac{BLv}{R}$$

$$F_1 = \frac{B^2L^2v}{R}$$



This is the force you need to pull at to achieve constant speed v.

$F_1 = F_A$

Motional Emf : Work done

How much work am I doing in pulling the circuit? W =Force x d

Note that the magnetic field does do any work,

What is the rate at which I am doing work? P=Fv

$$P = F_1 v = \frac{B^2 L^2 v^2}{R}$$

What is the thermal energy dissipated in the loop?

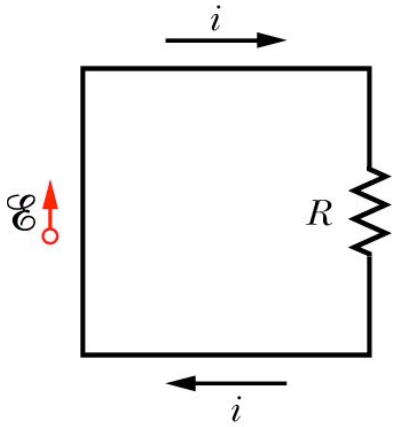
$$P = i^2 R$$

$$i = \frac{BLv}{R}$$

$$P = \frac{B^2 L^2 v^2}{R}$$

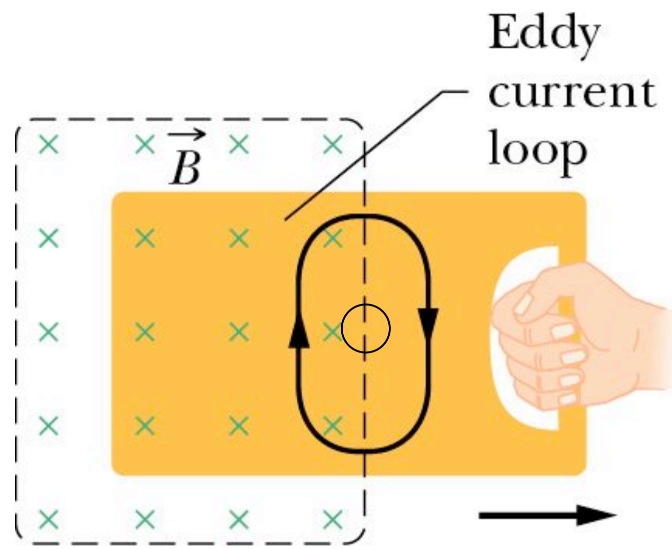
Note that the rate at which I do work in pulling the loop appears totally as thermal energy.

Circuit diagram for motional Emf. R is the resistance of the wire



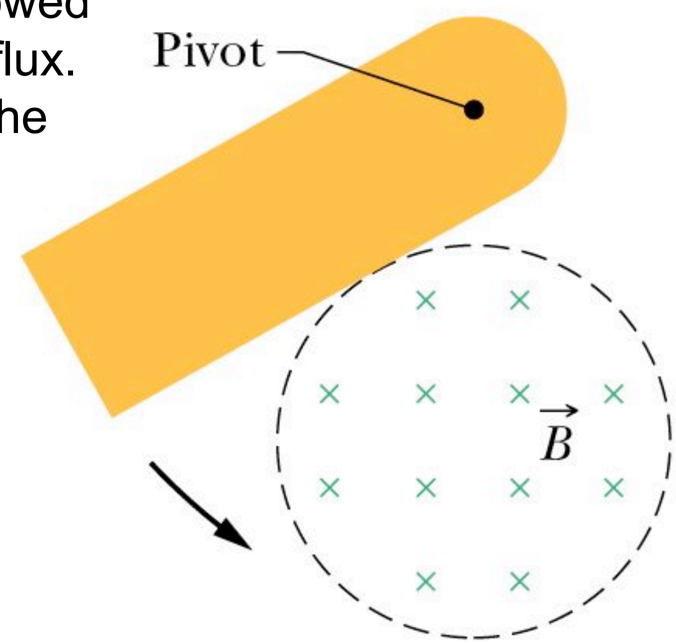
Eddy Currents

A solid piece of copper is moving out of a magnetic field. While it is moving out, an emf is generated forming millions of current loops as shown.



(a)

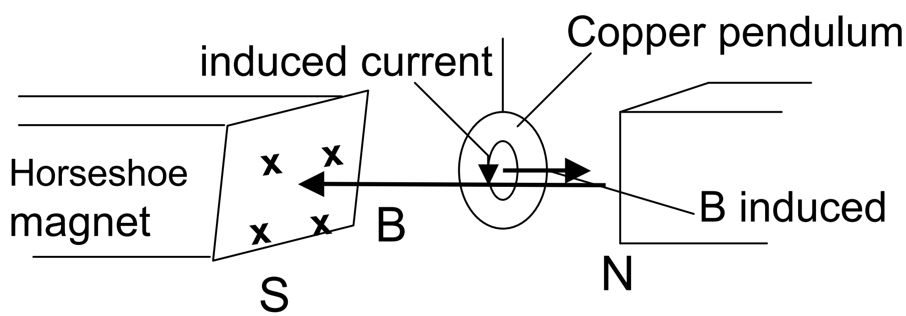
Eddy currents are also formed in a copper pendulum allowed to swing across a magnet gap cutting magnetic lines of flux. Note that when the copper plate is immersed entirely in the magnet no eddy currents form.



(b)

Eddy Currents Demo

- If a bulk conductor is present, we can induce currents to flow in the bulk conductor. Such currents are called eddy currents since they flow in circles.
- *Demo*: Try to place a copper sheet in between a pole faces of a magnet and/or try to pull it out. For example, in pulling it out, that part of the plate that was in the B field experiences a decrease in B and hence a change in magnetic flux in any loop drawn in that part of the copper. An emf is developed around such loops by Faraday's Law and in such a direction so as to oppose the change.
- Also try copper plate with slits.

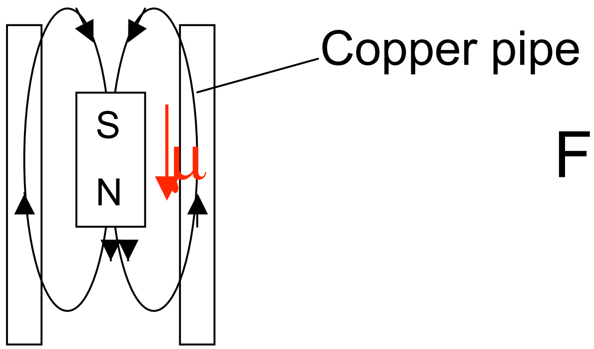


Pull back pendulum and release. Pendulum dampens quickly. Force acts to slow down the pendulum.

•Application: locomotive breaks operate on this principle. Magnetic dampening on balances. This is like a friction force that is linear with velocity.

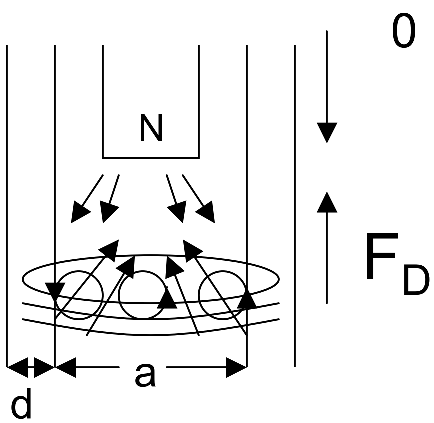
•*Demo*: Show neodymium magnet swinging over copper strip.

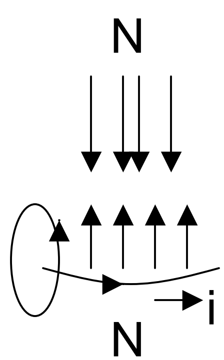
Demo: Copper pipe and neodymium-iron-boron magnet with magnetic dipole moment $\vec{\mu}$



F_D=Magnetic Drag Force ~ $\dfrac{\sigma\mu^2 d}{a^4}$

σ is the conductivity of copper



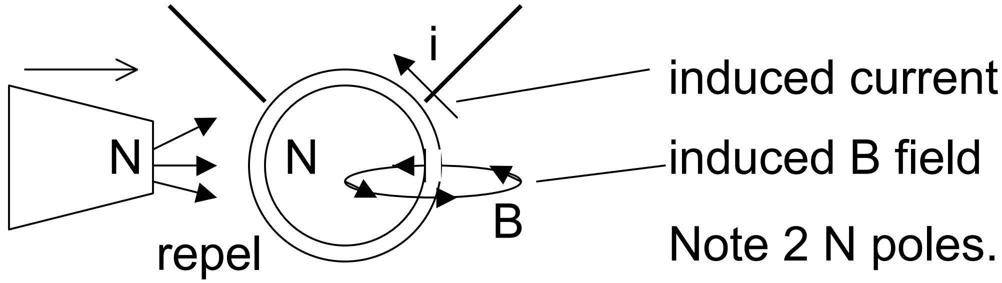


Two Norths repel so the magnet drops more slowly.

Cool down the copper pipe with liquid nitrogen 28 K. This will increase conductivity by about a factor of 5.

W.M. Saslow Am. J. Phys. 60(8)1977

Demo: Hanging aluminum ring with gray magnet

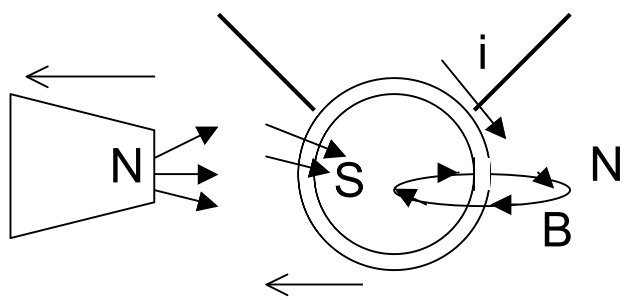


- Move magnet toward ring – they repel

Current induced in ring so that the B field produced by the current in the ring opposes original B field.

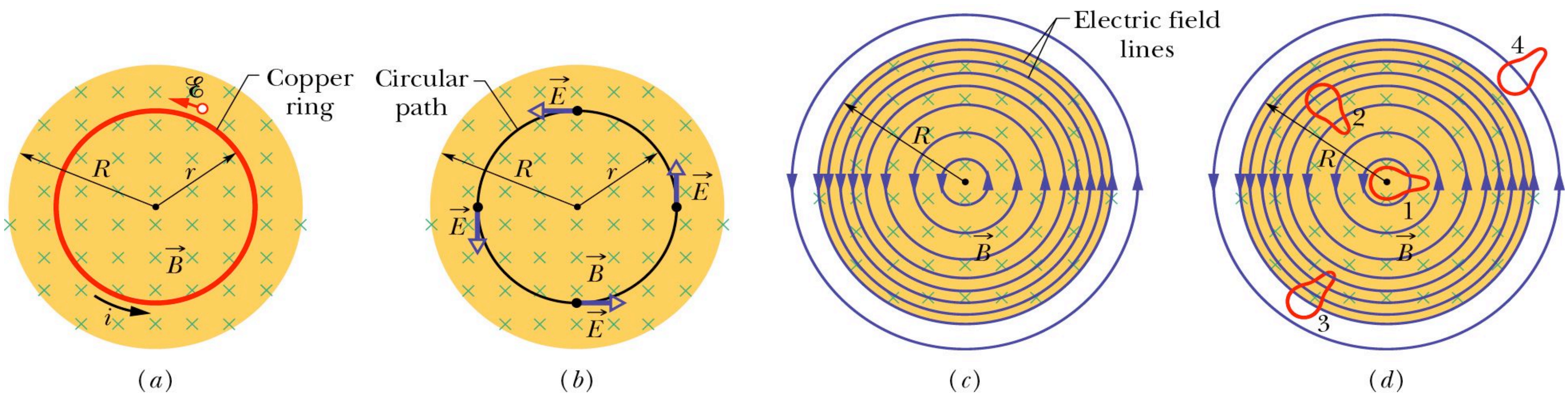
This means the ring current produces a N pole to push away the N pole of the permanent magnet.

- When magnet is pulled back, it attracts the ring.



Current in ring is opposite to that above

The orange represents a magnetic field pointing into the screen and let say it is increasing at a steady rate like 100 gauss per sec. Then we put a copper ring In the field as shown below. What does Faradays Law say will happen?



Current will flow in the ring. What will happen If there is no ring present?

Now consider a hypothetical path Without any copper ring.There will be an induced Emf with electric field lines as shown above.

In fact there will be many concentric circles everywhere in space.

The red circuits have equal areas. Emf is the same in 1 and 2, less in 3 and 0 in 4. Note no current flows. Therefore, no thermal energy is dissipated

We can now say that a changing magnetic field produces an electric field not just an Emf. For example:

Work done in moving a test charge around the loop in one revolution of induced Emf is

$$Work = emf \times q_0$$

Work done is also $\int F \cdot ds = q_0 \oint E \cdot ds = q_0 E(2\pi r)$

Hence, Emf = 2πrE or more generally for any path

$$Emf = \oint E \cdot ds \qquad \oint E \cdot ds = -\frac{d\Phi_B}{dt}$$

Faraday’s Law rewritten

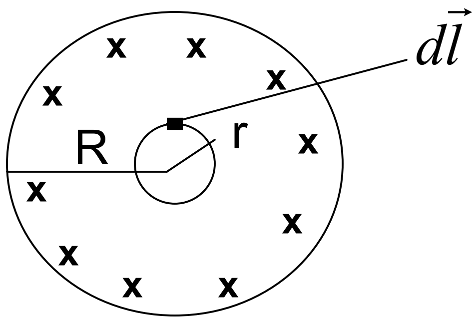
But we can not say $V_f - V_i = -\int_i^f E \cdot ds$ **because it would be 0.**

Electric potential has no meaning for induced electric fields

Summary
Characteristics of the induced emf

- The induced emf is not localized such as at the terminals of a battery.
- It is distributed throughout the circuit.
- It can be thought of as an electric field circulating around a circuit such that the line integral of the electric field taken around a closed loop is the emf.
- Since the line integral is not 0, the field is non-conservative.
- There are no equipotential surfaces.
- If there is a conductor present, a current will flow in the conductor.
- If no conductor is present, there is no current flow, only emf.
- Energy is dissipated only if charges are present.

Example: A magnetic field is ⊥ to the board (screen) and uniform inside a radius R. What is the magnitude of the induced field at a distance r from the center?

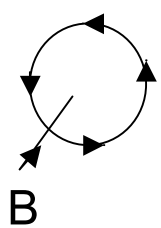


*E is parallel to d**l***

$$\oint E dl = E 2\pi r = -\frac{d\phi_m}{dt}$$

$$\phi_m = BA = B\pi r^2$$

$$\frac{d\phi_m}{dt} = \frac{d}{dt}(B\pi r^2) = \pi r^2 \frac{dB}{dt}$$



Field circulates around B field

$$E 2\pi r = -\pi r^2 \frac{dB}{dt}$$

$$\boxed{E = -\frac{r}{2}\frac{dB}{dt} \qquad r < R}$$

Notice that there is no wire or loop of wire. To find E use Faraday's Law.

$$2\pi r E = \pi R^2 \frac{dB}{dt} \qquad E = -\frac{R^2}{2r}\frac{dB}{dt} \qquad r > R$$

Example with numbers

Suppose dB/dt = - 1300 Gauss per sec and R= 8.5 cm

Find E at r = 5.2 cm

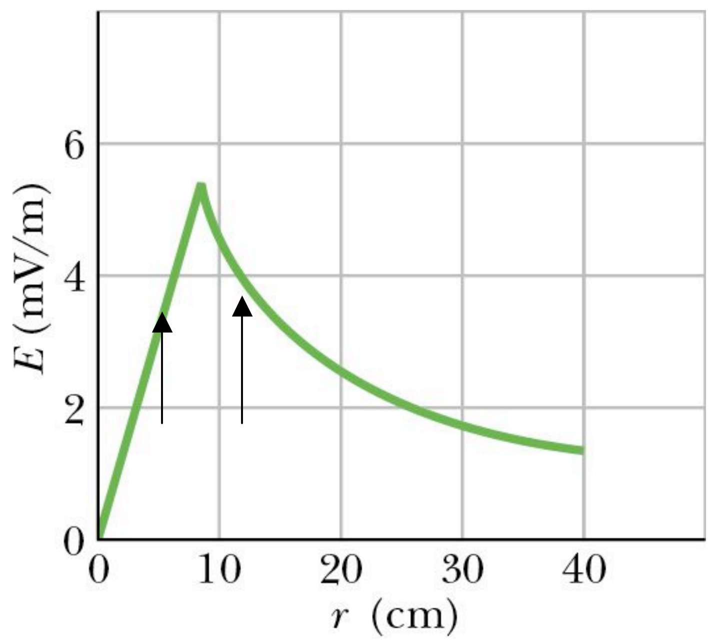
$$E = -\frac{r}{2}\frac{dB}{dt} \qquad r < R$$

$$E = \frac{(0.052m)}{2} 0.13T = 0.0034 \tfrac{\text{V}}{\text{m}} = 3.4 \tfrac{mV}{m}$$

Find E at 12.5 cm

$$E = -\frac{R^2}{2r}\frac{dB}{dt} \qquad r > R$$

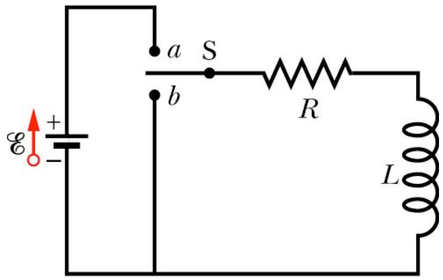
$$E = \frac{(0.085m)^2}{2(0.125m)} 0.13T = 0.0038 \tfrac{V}{m} = 3.8 \tfrac{mV}{m}$$

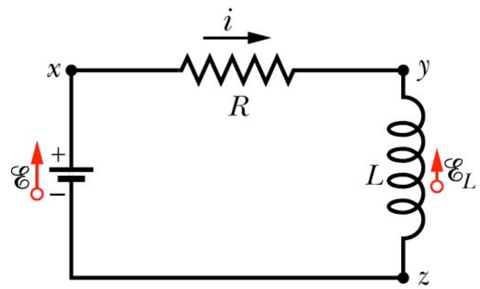


What is an inductor?

An inductor is a piece of wire twisted into a coil. It is also called a solenoid. If the current is constant in time, the inductor behaves like a wire with resistance. The current has to vary with time to make it behave as an inductor. When the current varies the magnetic field or flux varies with time inducing an Emf in the coil in a direction that opposes the original change.

Suppose I move the switch to position a, then current starts to increase through the coil. An Emf is induced to make current flow in the opposite direction.

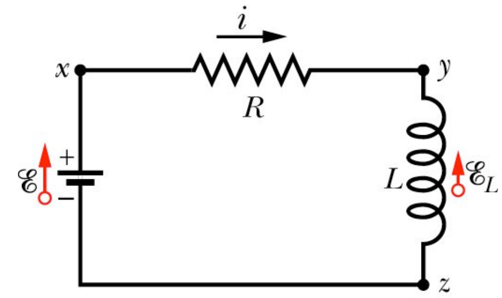




Now suppose I move the switch to position b

What is inductance? What is a Henry?

Start with Faraday's Law

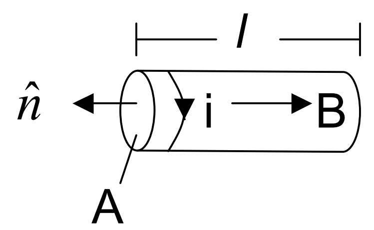


$$Emf = -\frac{d\Phi}{dt} \qquad \Phi = NBA$$

$$= -\frac{NAdB}{dt} \qquad B = \mu_0 \text{ni for a solenoid}$$

$$= -\frac{NAd(\mu_0 in)}{dt}$$

$$= -\mu_0 nNA\frac{di}{dt} \qquad \text{N=nl}$$



$$Emf = -\mu_0 n^2 lA\frac{di}{dt} = -L\frac{di}{dt}$$

$$L = \mu_0 n^2 lA$$ (H=Henry) 1 H=1 T.m²/A ← Amp

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$$

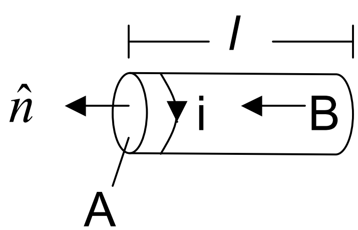
$$\frac{L}{l} = \mu_0 n^2 A$$ (Henry/m)

Numerical example – how many turns do you need to make a L = 4.25 mh solenoid with l = 15 cm and radius r = 2.25 cm?

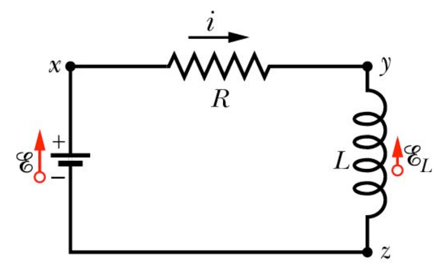
$$L = \mu_0 n^2 l A \qquad n = \sqrt{\frac{L}{\mu_0 l A}} \qquad N = nl$$

$$N = nl = \sqrt{\frac{Ll}{\mu_0 A}} = \sqrt{\frac{(.00425)(.15)}{4\pi \times 10^{-7} \pi (.0225)^2}} = 565\, Turns$$

Area



N turns



Show demo: Inductive spark after turning off electromagnet

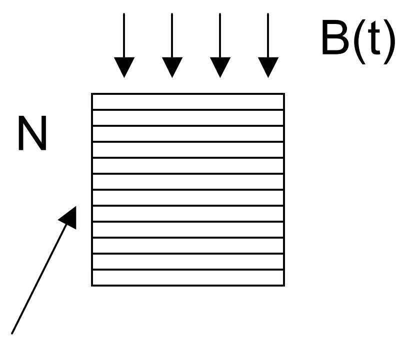
Numerical Example

You have a 100 turn coil with radius 5 cm with a resistance of 10 Ω. At what rate must a perpendicular B field change to produce a current of 4 A in the coil?

Emf = IR = (4A)(10Ω) = 40 Volts

$$Emf = -N\frac{d\phi_m}{dt} = -NA\frac{dB}{dt} = -N\pi r^2\frac{dB}{dt} = 40$$

$$\frac{dB}{dt} = \frac{40}{N\pi r^2} = \frac{40V}{100\cdot 3.15\cdot(.05m)^2} = 51\,T/s$$



N coils so $emf = -N\frac{d\phi_m}{dt}$

Multiply by N

N = 100 turns

R = 5 cm

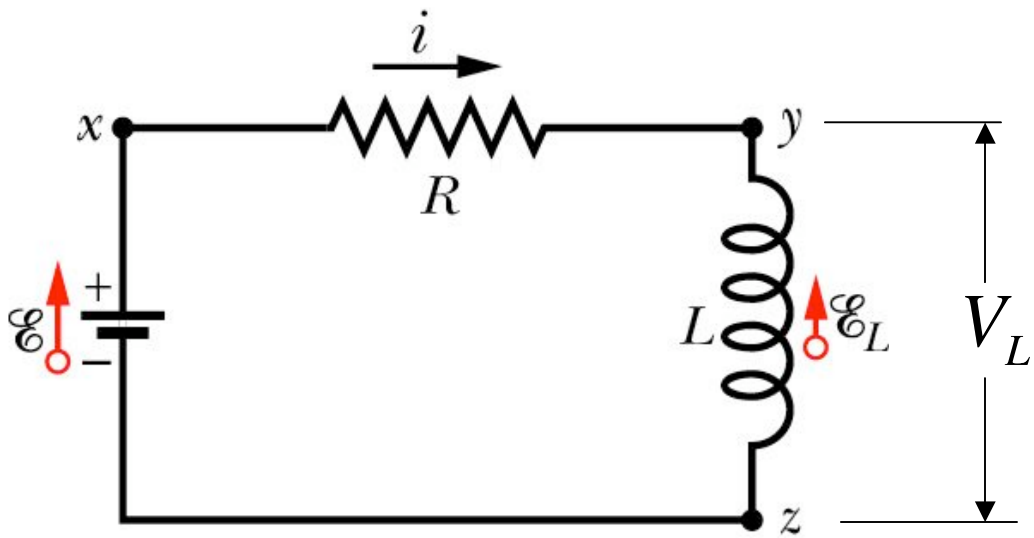
Coil resistance = 10 Ω

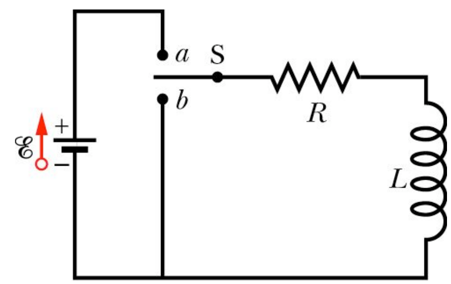
Because coils have resistance of 10 Ω, induced current has a voltage drop so that emf = IR =

$$-N\frac{d\phi_m}{dt}$$

RL Circuits

$$V_R = Ri$$





Close the switch to a.

What happens? Write down the loop rule.

Loop Rule: Sum of potentials =0

$$\varepsilon + V_R + V_L = 0$$

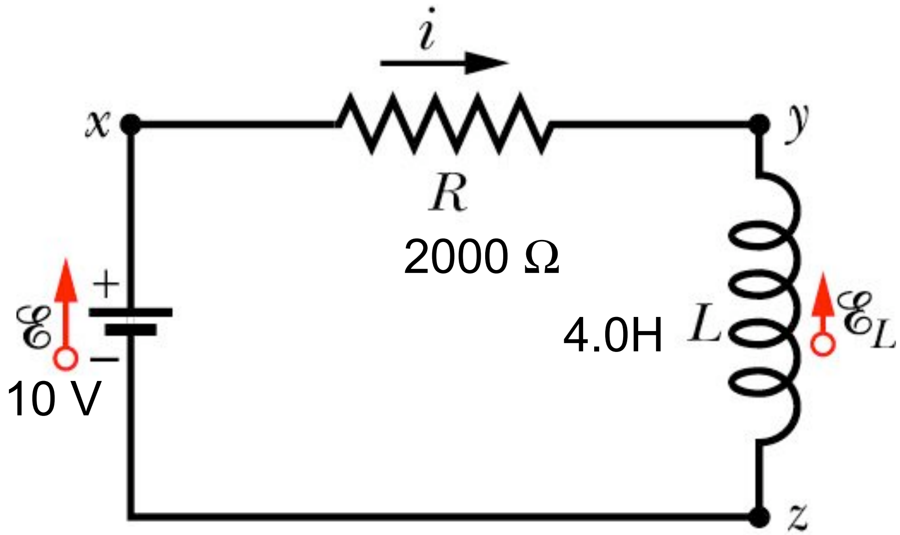
$$\varepsilon - iR - L\frac{di}{dt} = 0$$

The potential can be defined across the inductor outside the region where the magnetic flux is changing.

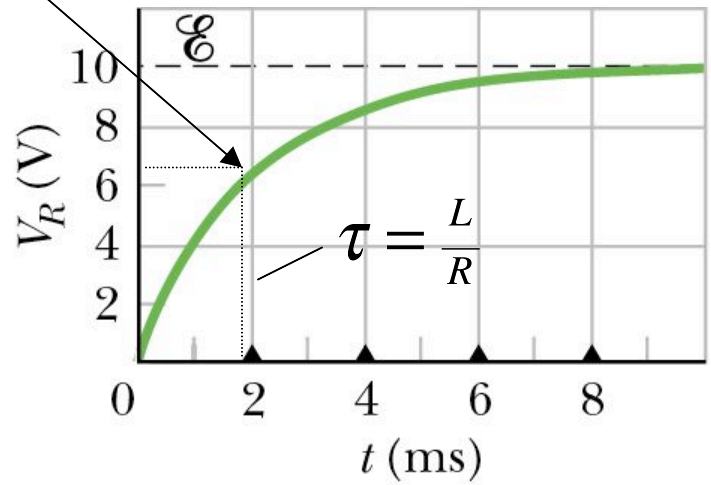
Solve this equation for the current i.

$$i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$V_R = Ri$$



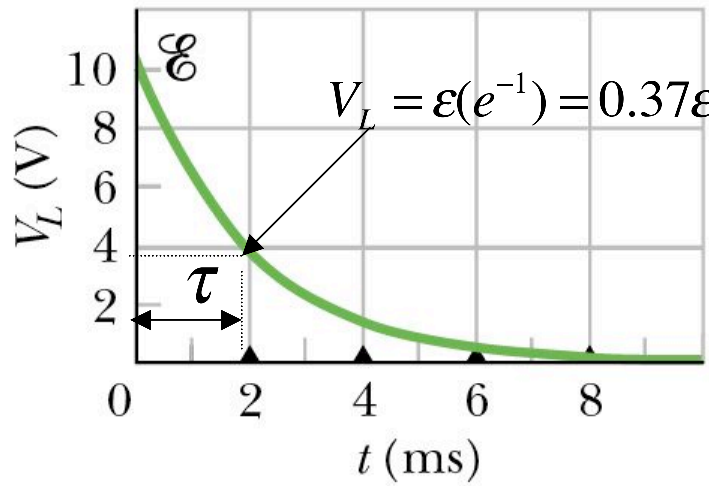
$V_R = 0.63\varepsilon$ $\quad V_R = \varepsilon\left(1 - e^{-\frac{Rt}{L}}\right)$



(a)

R i

$$V_L = \varepsilon e^{-\frac{Rt}{L}}$$



(b)

$L\frac{di}{dt}$

Note τ = L/R = 4/2000 = 0.002 s,

$$i = \frac{\varepsilon}{R}(1 - e^{-1}) = 0.63\frac{\varepsilon}{R}$$

and

$$V_R = \varepsilon(1 - e^{-1}) = 0.63\varepsilon$$

How is the magnetic energy stored in a solenoid or coil in our circuit?

$$\varepsilon - iR - L\frac{di}{dt} = 0$$

Start with Loop rule or Kirchhoff's Law I

$$\varepsilon = iR + L\frac{di}{dt}$$

Solve it for ε

$$\varepsilon i = i^2 R + Li\frac{di}{dt}$$

Multiply by i

Rate at which energy is delivered to circuit from the battery

Rate at which energy is lost in resistor

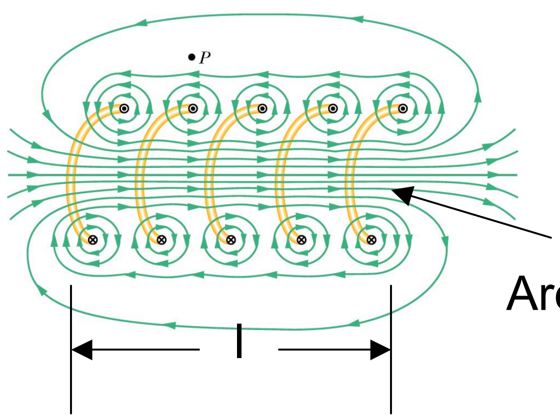
Rate at which energy is stored in the magnetic field of the coil

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

What is the magnetic energy stored in a solenoid or coil

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \qquad dU_B = Lidi \qquad \int_0^{U_B} dU_B = \int_0^i Lidi \qquad U_B = \int_0^i Lidi = \tfrac{1}{2}Li^2$$

$U_B = \tfrac{1}{2}Li^2$ For an inductor L



Now define the energy per unit volume

$$u_B = \frac{U_B}{Al}$$

$$u_B = \frac{\tfrac{1}{2}Li^2}{Al} = \frac{L}{l}\frac{i^2}{2A} \qquad \frac{L}{l} = \mu_0 n^2 A$$

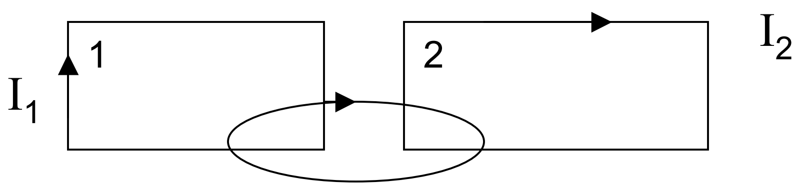
$$u_B = \frac{\tfrac{1}{2}Li^2}{Al} = \tfrac{1}{2}\mu_0 n^2 i^2 \qquad B = \mu_0 ni$$

The energy density formula is valid in general

$$u_B = \frac{B^2}{2\mu_0} \qquad u_E = \frac{E^2}{2\varepsilon_0}$$

What is Mutual Inductance? M

When two circuits are near one another and both have currents changing, they can induce emfs in each other.



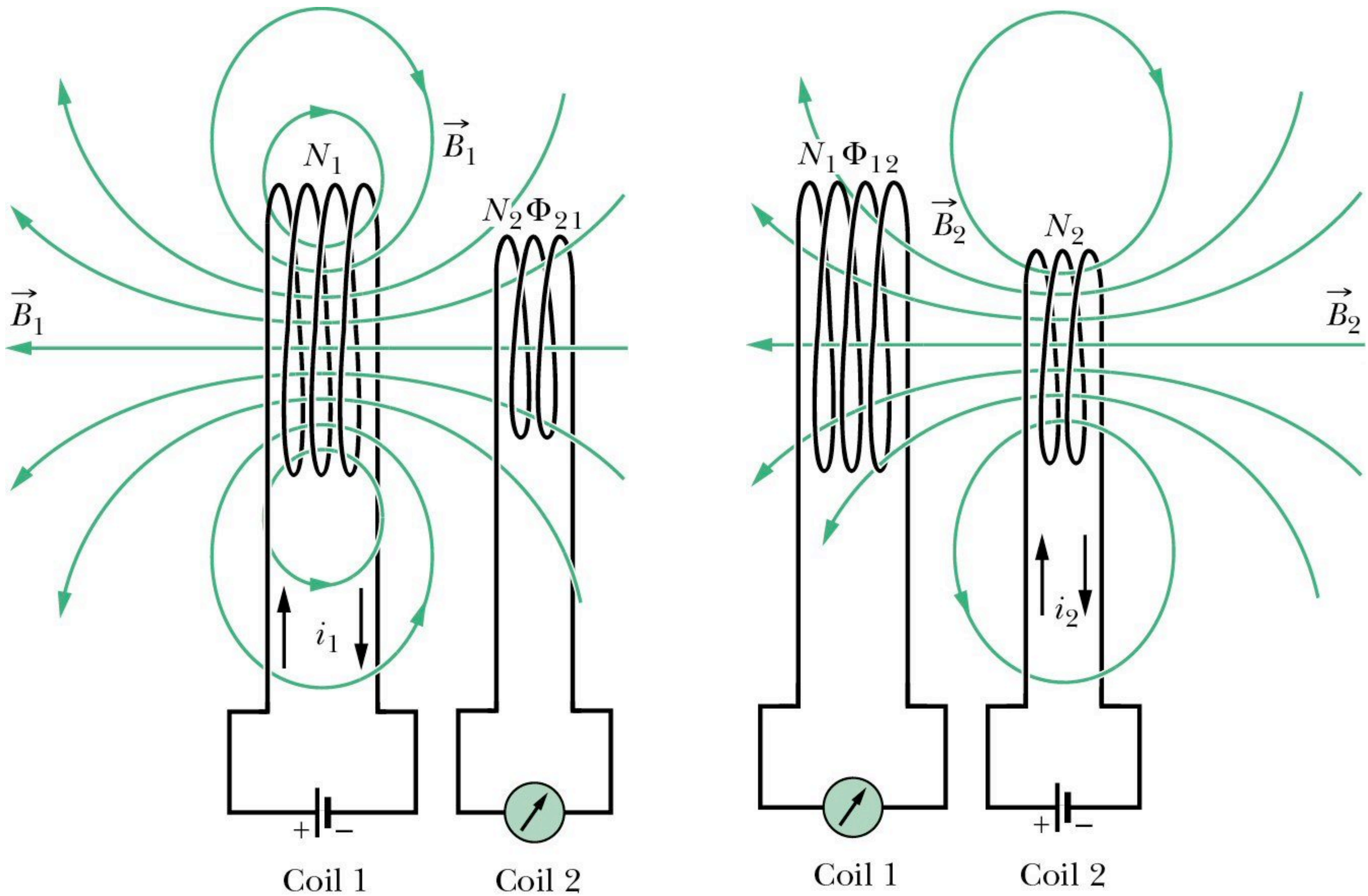
$$\phi_{m1} = L_1 I_1 + M_{21} I_2$$

$$\phi_{m2} = L_2 I_2 + M_{12} I_1$$

$$M_{12} = M_{21} = M$$

On circuit boards you have to be careful you do not put circuits near each other that have large mutual inductance.

They have to be oriented carefully and even shielded.



N_1
$\vec{B}_1$
$N_2\Phi_{21}$
$\vec{B}_1$
i_1
+
−
Coil 1
Coil 2
$N_1\Phi_{12}$
$\vec{B}_2$
N_2
$\vec{B}_2$
i_2
+
−
Coil 1
Coil 2

(a)

(b)