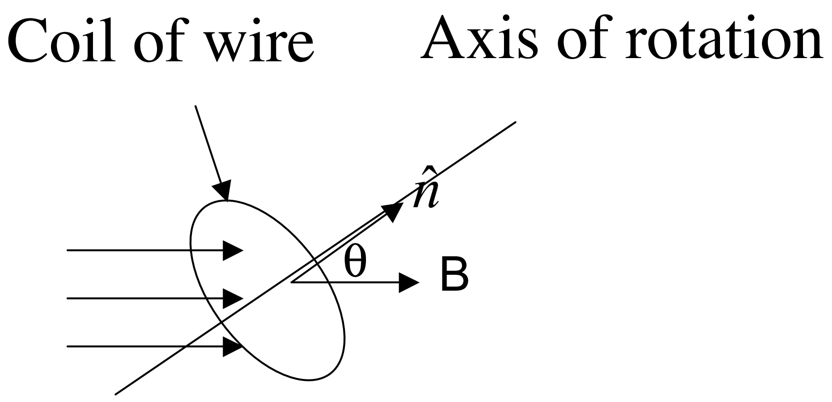
Lecture 11 Electromagnetic Oscillations and Alternating Current Chp. 31

- Warm-up problem
- Topics
  - Transformers
  - LC Circuit Qualitatively
  - Electrical and Magnetic energy oscillations
  - Alternating current
  - Pure R and L circuits
  - Series RLC circuit
  - Power and Transformers
- Demos



- $\phi_m = \vec{B} \cdot \hat{n} dA$

$$= \vec{B} \cdot d\vec{A}$$

$$= B \cos\theta dA$$

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(BA\cos\theta)}{dt} = -BA\frac{d\cos\theta}{dt} = BA\sin\theta\frac{d\theta}{dt}$$

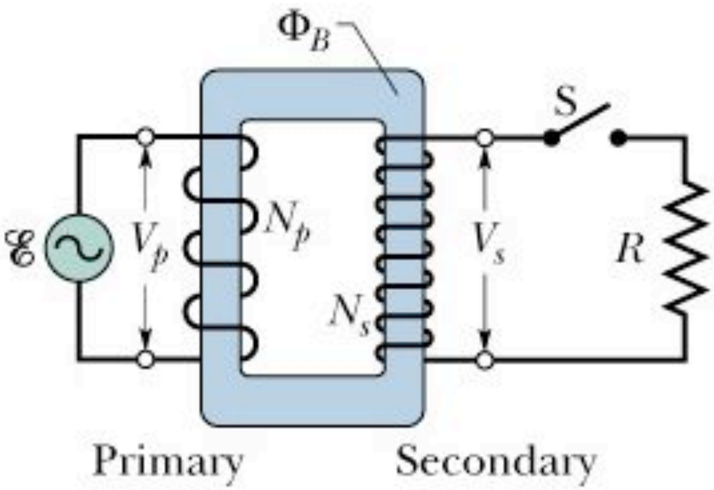
$$= BA\omega\sin\theta \quad \text{but} \quad \theta = \omega t \text{ so} \quad \frac{\mathrm{d}\theta}{\mathrm{dt}} = \omega$$

$$\varepsilon = BA\omega\sin\omega t$$

$$\varepsilon = \varepsilon_m \sin\omega t$$

ω=2πf and f= 60 Hz

Where ω is the rotational angular frequency of the generator

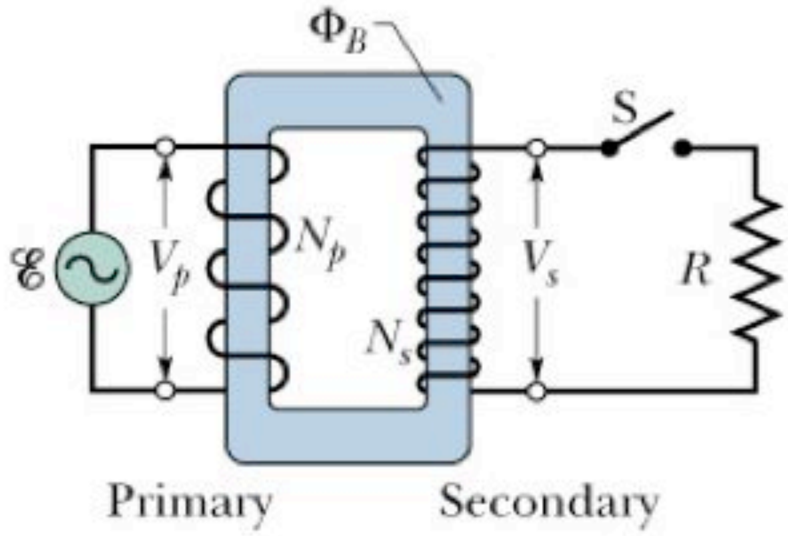


Transformers

A transformer is a device used to increase or decrease the AC voltage in a circuit

A typical device consists of two coils of wire, a primary and a secondary, wound around an iron core. The primary coil, with turns N_1, is connected to alternating voltage source . The secondary coil has N_2 turns and is connected to a "load resistance" R.

The way transformers operate is based on the principle that an alternating current in the primary coil will induce an alternating emf on the secondary coil due to their mutual inductance.



$$V_p = -N_p \frac{d\Phi_B}{dt}$$

The iron core, which extends from the primary to the secondary coils, serves to increase the magnetic field produced by the current in the primary coil and ensure that nearly all the magnetic flux through the primary coil also passes through each turn of the secondary coil. Thus, the voltage (or induced emf) across the secondary coil is

$$V_s = -N_s \frac{d\Phi_B}{dt}$$

With no flux leakage out of the iron, the flux Φ_B through each turn is the same in the primary as the secondary coils

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

second coil is greater than the input voltage in the primary coil. A transformer with $N_s > N_p$ is called a step-up transformer.

If $N_s < N_p$ the output voltage is smaller than the input voltage and the transformer is called a step-down transformer.

For an ideal transformer, power loss due to Joule heating can be ignored, so that the power supplied by the primary coil is completely transferred to the secondary coil:

$$I_p V_p = I_s V_s$$

Combining the last two equations the transformation of currents in the two coils is

$$\boxed{I_p = \frac{V_s}{V_p} I_s = \frac{N_s}{N_p} I_s}$$

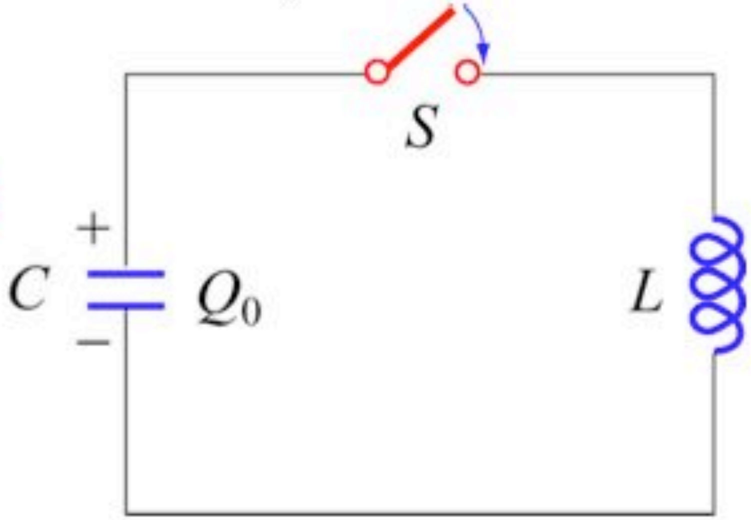
Transformers play a critical role in transmitting power. - transmit at high voltage and utilize at low voltage.

LC Oscillations, Qualitatively

Suppose the capacitor initially has charge Q_0 . When the switch is closed, the capacitor begins to discharge and the electric energy is decreased.

On the other hand, the current created from the is discharging process generates magnetic energy which then gets stored in the inductor.

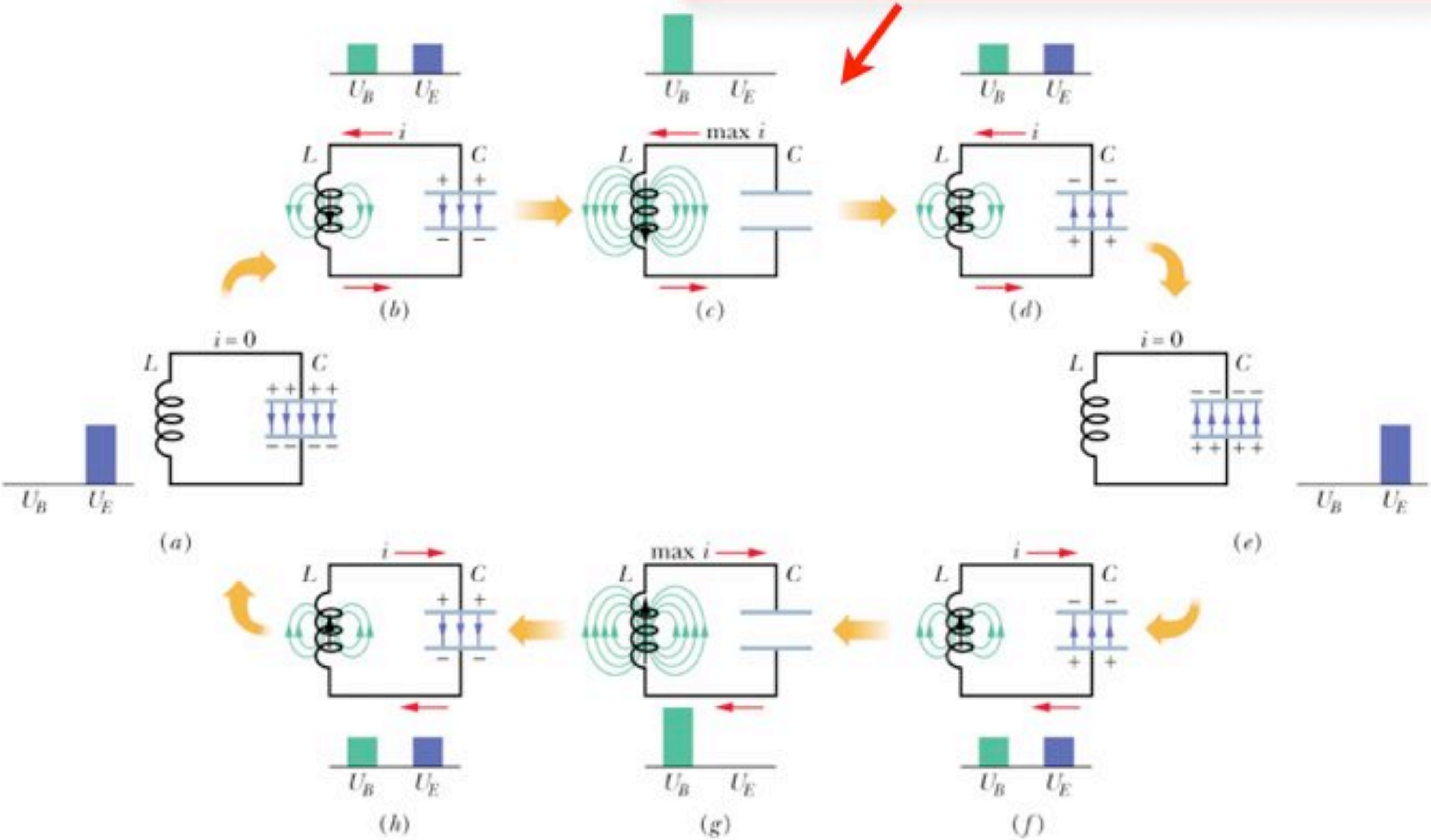
In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.



In absence of resistance the U is constant

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

inductor does not allow the current to go to zero and current continues to flow



LC Oscillations, Quantitatively

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

U is constant so time derivative is zero

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

Rewrite taking I = -dQ/dt (and dI/dt = $-d^2Q/dt^2$): The current is equal to the rate of decrease of the charge

$$Q(t) = Q_0 \cos(\omega t + \phi)$$

Solution to 2nd order differential equation

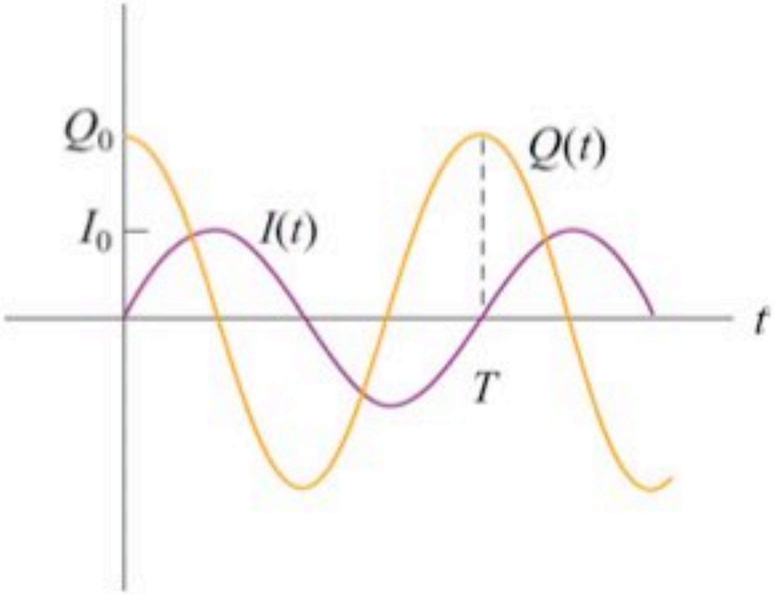
ϕ is the phase constant, We see that the charge travels to and from the plates of the capacitor, oscillating with angular frequency $\omega = 1/(\sqrt{LC})$

$$Q(t) = Q_0 \cos(\omega t + \phi)$$

Q_0 and ϕ are determined by the initial conditions: Q_0 is the initial charge and ϕ sets the phase at $t = 0$.

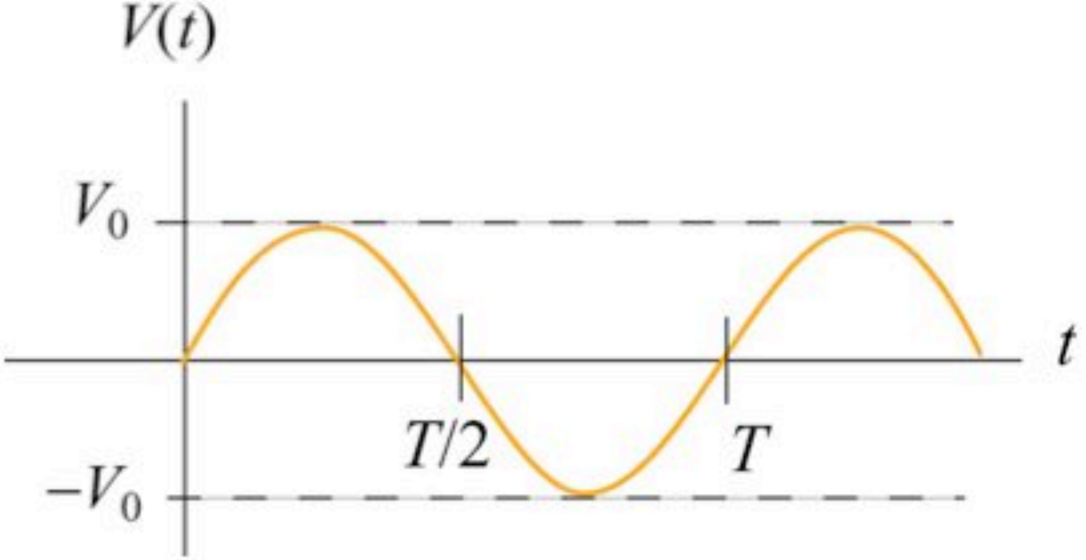
The current, dQ/dt, undergoes similar oscillations:

$$I(t) = \frac{dQ}{dt} = -\omega Q \sin(\omega t + \phi) \qquad \omega = \frac{1}{\sqrt{LC}}$$



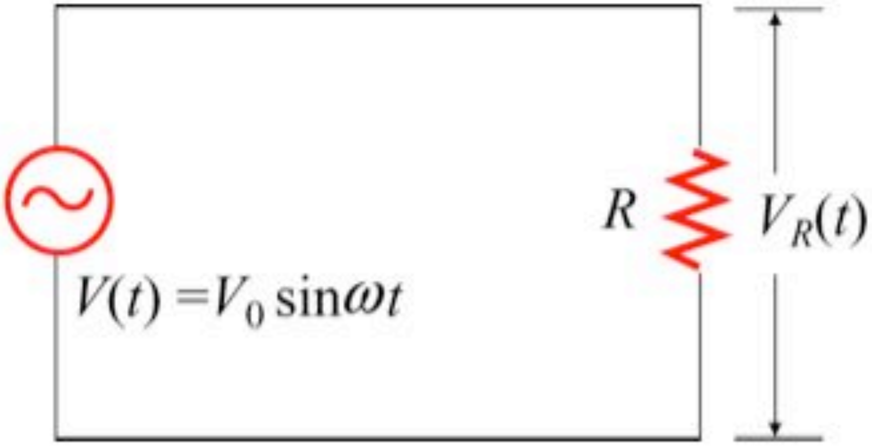
$\omega Q = I_0$

AC Circuits with R, L and C



We assume that we have a driving emf given by $V(t)=V_0 \sin(\omega t)$ and start by examining one element at a time

Purely resistive load: we can just apply Ohm's Law



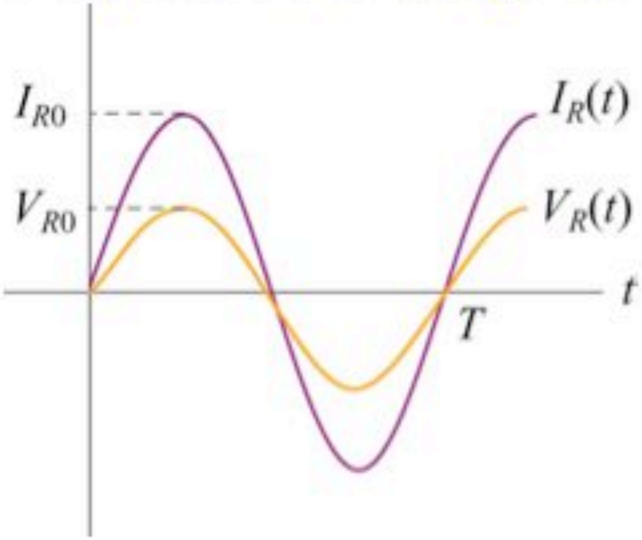
Applying Kirchhoff's loop rule yields $V(t) - V_R(t) = V(t) - I_R(t)R = 0$

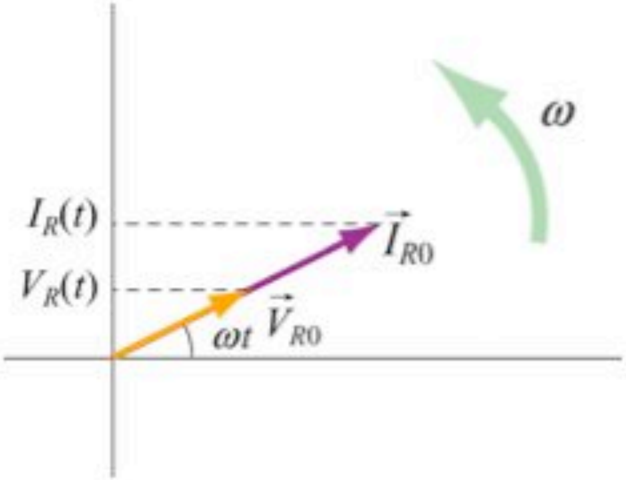
$$V_R(t) = I_R(t)\,R$$

$$I_R(t) = \frac{V_R(t)}{R} = \frac{V_0 \sin\omega t}{R}$$

and this expression tells us that current and voltage are "in phase"

Purely resistive load


Time dependence of Voltage and Current


Phasor representation

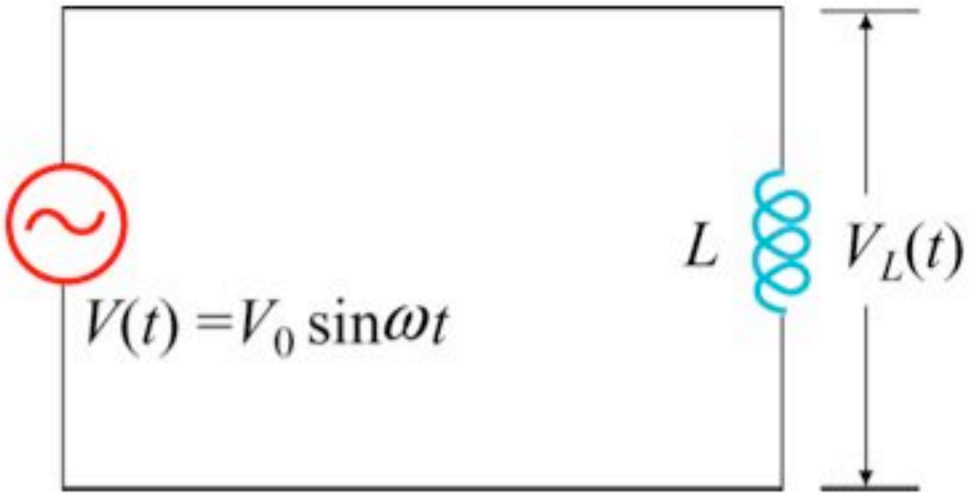
A phasor is a rotating vector having the following properties:

(i) length: the length corresponds to the amplitude.

(ii) angular speed: the vector rotates counterclockwise with an angular speed ω.

(iii) projection: the projection of the vector along the vertical axis corresponds to the value of the alternating quantity at time t.

Purely inductive load:



Here the loop rule is
$V_0 \sin(\omega t) - L\, dI/dt = 0 \Rightarrow$ $dI/dt = V_0/L \sin(\omega t)$

and the current is obtained from integration, giving
$I = -V_0/\omega L \cos(\omega t) = V_0/\omega L \sin(\omega t - \pi/2)$

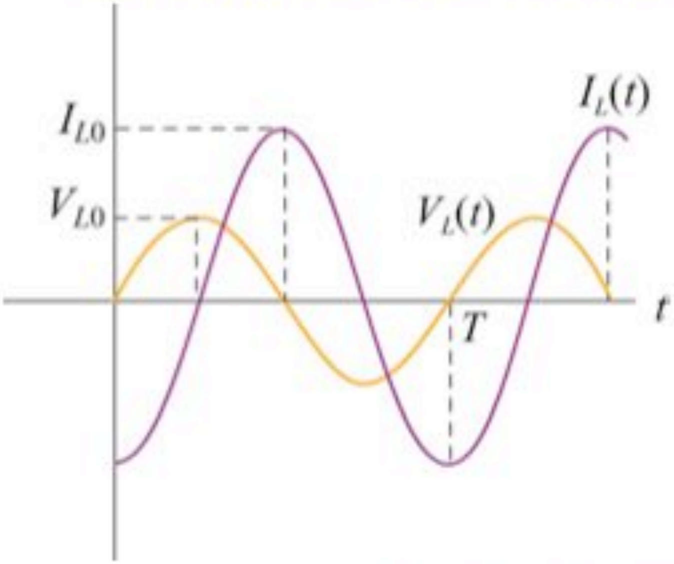
we have used $-\cos \omega t = \sin(\omega t - \pi/2)$

Amplitude of current through the inductor is

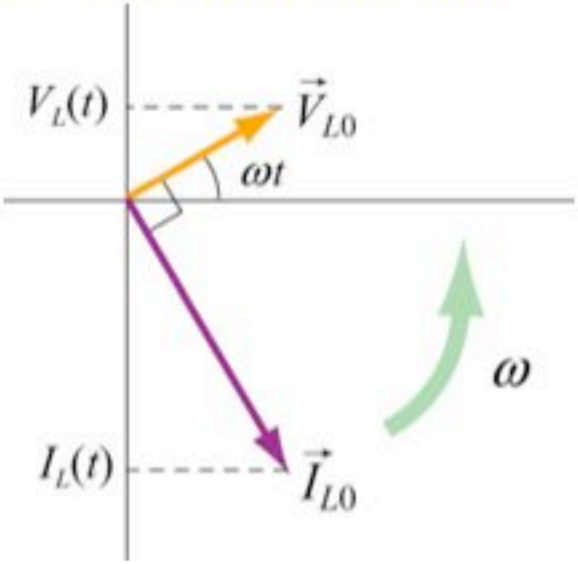
$$I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$

X_L is the inductive reactance and has the units of ohms (Ω). Note dependence on ω.

Time dependence of Voltage and Current



Phasor representation

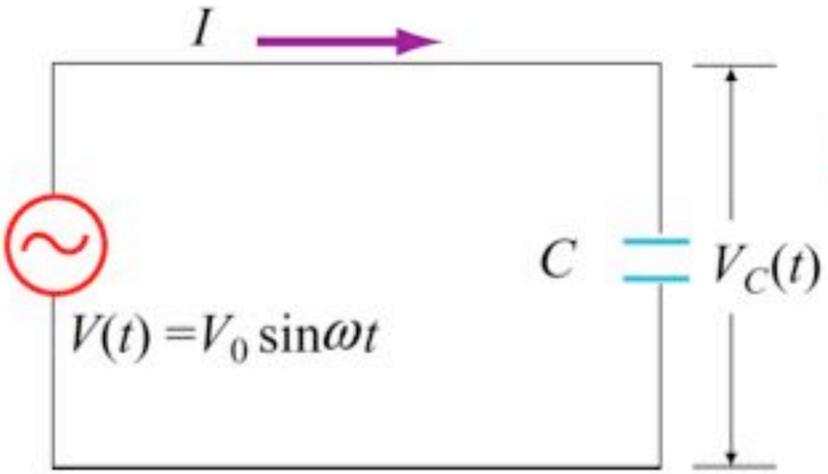


In the purely inductive circuit then

the current "lags" the voltage by a quarter period

The quantity relating current to voltage is represented by the inductive reactance $X_L = \omega L$.

$$I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$ Ohm's Law, I = V/R $I \Rightarrow 0$ as $\omega \Rightarrow \infty$



Purely capacitive circuit

The loop rule gives
$V_0 \sin(\omega t) - Q/C = 0 \Rightarrow$ $Q = CV_0 \sin(\omega t)$, therefore, for the current $I = \frac{dQ}{dt} = \omega C V_0 \cos(\omega t) = \omega C V_0 \sin(\omega t + \frac{\pi}{2})$

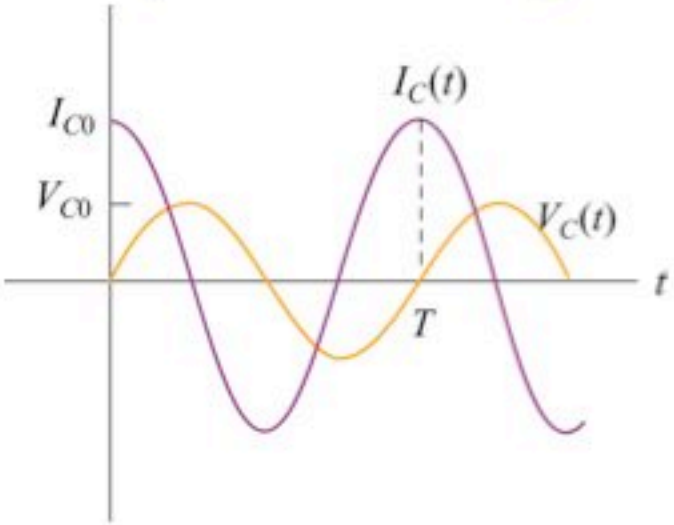
The quantity ωC is the proportionality factor relating current to voltage. Instead of $I = V_0/R$, we have $I = V_0/X_C$

We see that the equivalent of the resistance is the quantity $X_C = 1/\omega C$, called capacitive reactance

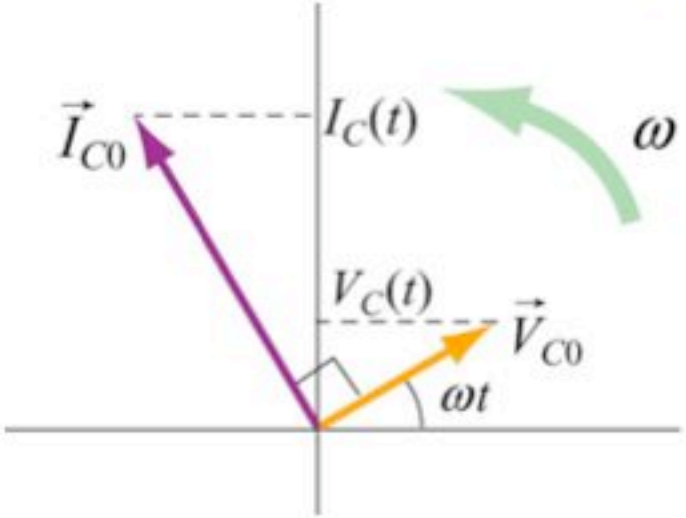
$I \Rightarrow 0$ as $\omega \Rightarrow 0$. No DC current flows through a capacitor

Purely capacitive circuit

Time dependence of Voltage and Current



Phasor representation



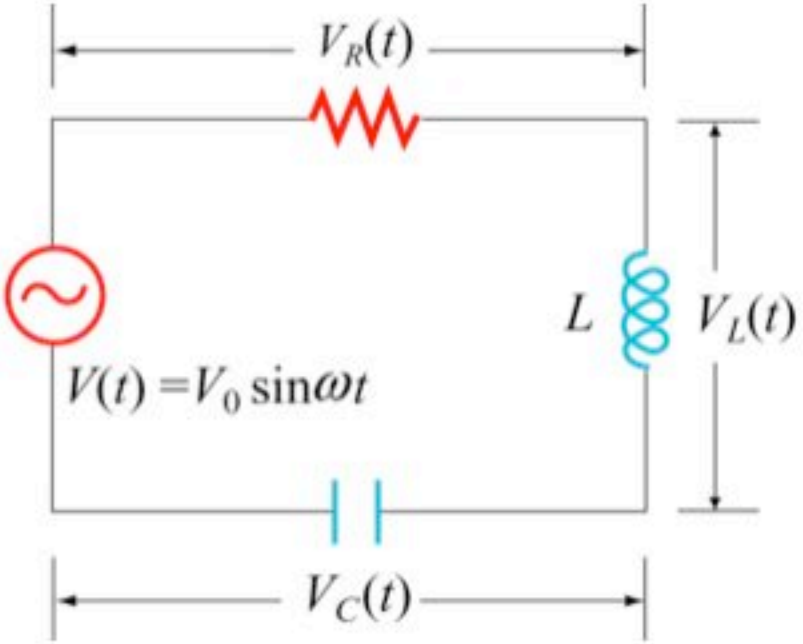
The current "leads" the voltage by π/2 in a capacitive circuit

The quantity relating current to voltage is represented by the capacitive reactance $X_L = 1/(\omega C)$, Ohm's Law, $I = V/R$

(a) How does the capacitive reactance change if the driving frequency is doubled? halved?

(b) Are there any times when the capacitor is supplying power to the AC source?

The Driven RLC Series Circuit



This is similar to the RLC oscillating circuit we saw before, with the addition of a driving alternating emf. We will still have a sinusoidal behavior, but the presence of the "restoring emf" will prevent the oscillations from dying away.

Apply the loop rule to get

$$V(t) - V_R(t) - V_L(t) - V_C(t) = V(t) - IR - L\frac{dI}{dt} - \frac{Q}{C} = 0$$

and we get the following differential equation

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = V_0 \sin\omega t$$

Driven RLC Series Circuit

The Driven RLC Series Circuit

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = V_0 \sin\omega t$$

Taking the capacitor as initially uncharged, and I = + dQ/dt is proportional to the increase of charge in the capacitor,

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \sin\omega t$$

A solution is

$$Q(t) = -Q_0 \cos(\omega t - \phi)$$ where the amplitude

$$Q_0 = \frac{V_0/L}{\sqrt{(R\omega/L)^2 + (\omega^2 - 1/LC)^2}} = \frac{V_0}{\omega\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$= \frac{V_0}{\omega\sqrt{R^2 + (X_L - X_C)^2}}$$

and the phase

$$\tan\phi = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \frac{X_L - X_C}{R}$$

The Driven RLC Series Circuit

The corresponding current is

$$I(t) = +\frac{dQ}{dt} = I_0 \sin(\omega t - \phi)$$

with an amplitude

$$I_0 = Q_0 \omega = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

ϕ, the phase angle gives the relative phase between driving voltage and current

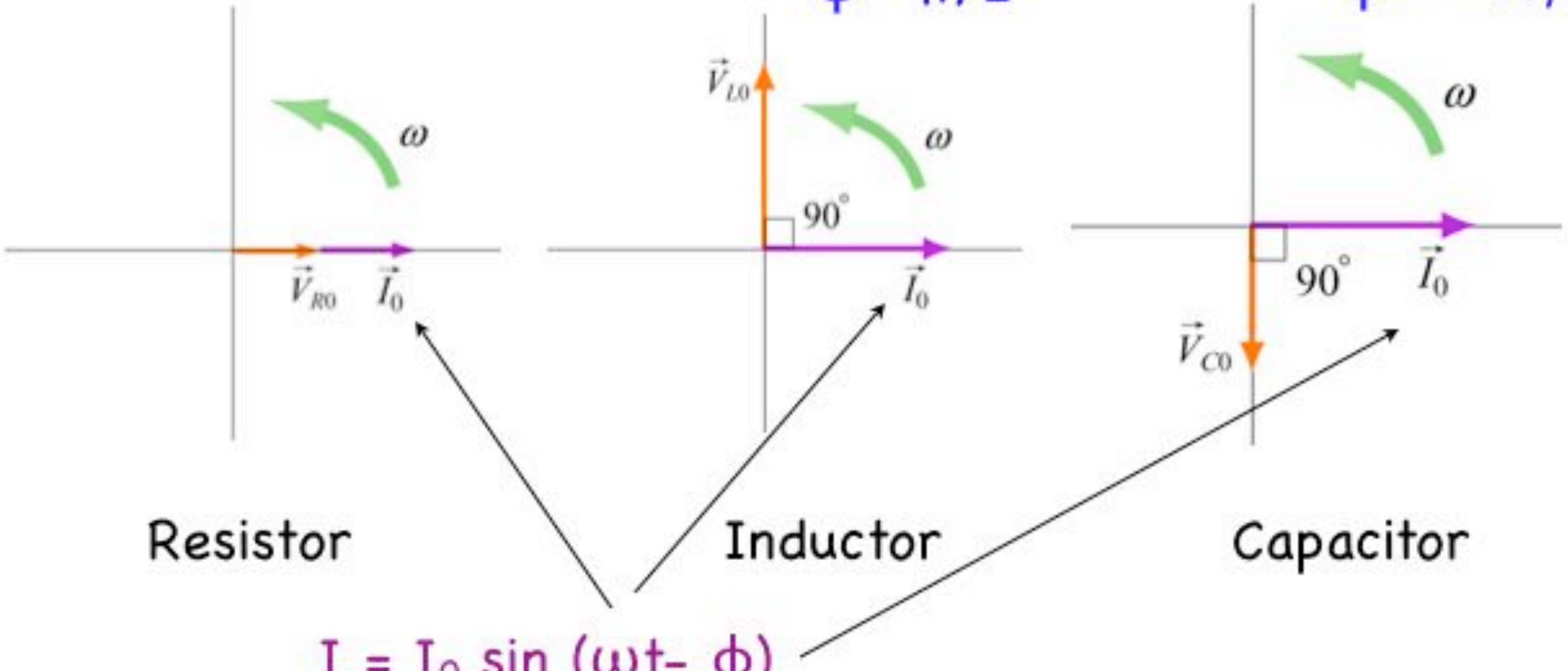
$$\tan\phi = \frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \frac{X_L - X_C}{R}$$

Notice that the current has the same amplitude and phase at all points in the series RLC circuit.

On the other hand, the instantaneous voltage across each of the three circuit elements R, L and C has a different amplitude and phase relationship with the current, as can be seen from the phasor diagrams

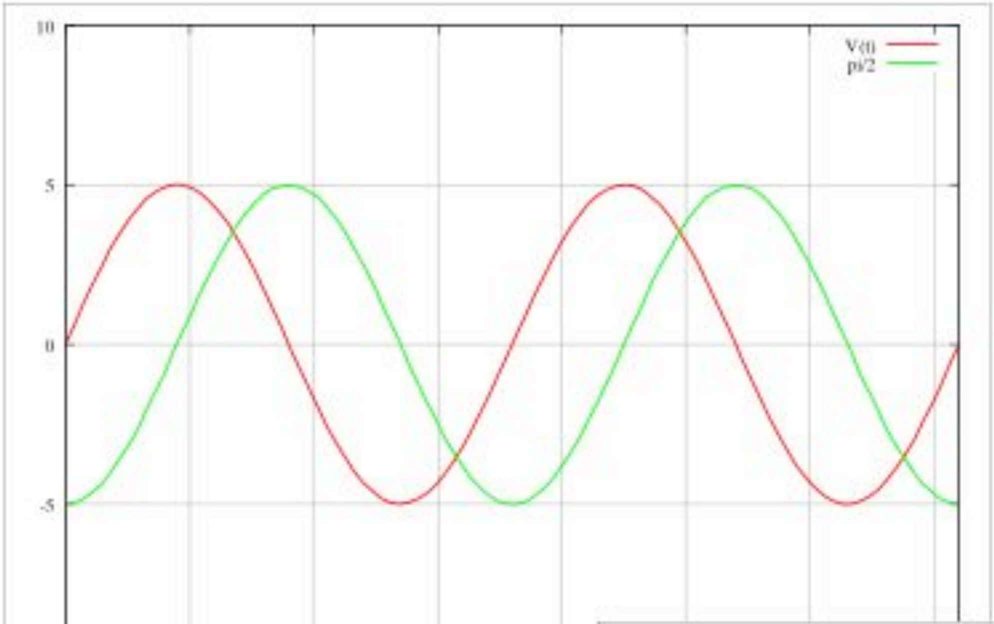
Phasor diagrams

$\phi = \pi/2$ $\quad$ $\phi = -\pi/2$



Resistor $\quad$ Inductor $\quad$ Capacitor

$I = I_0 \sin(\omega t - \phi)$

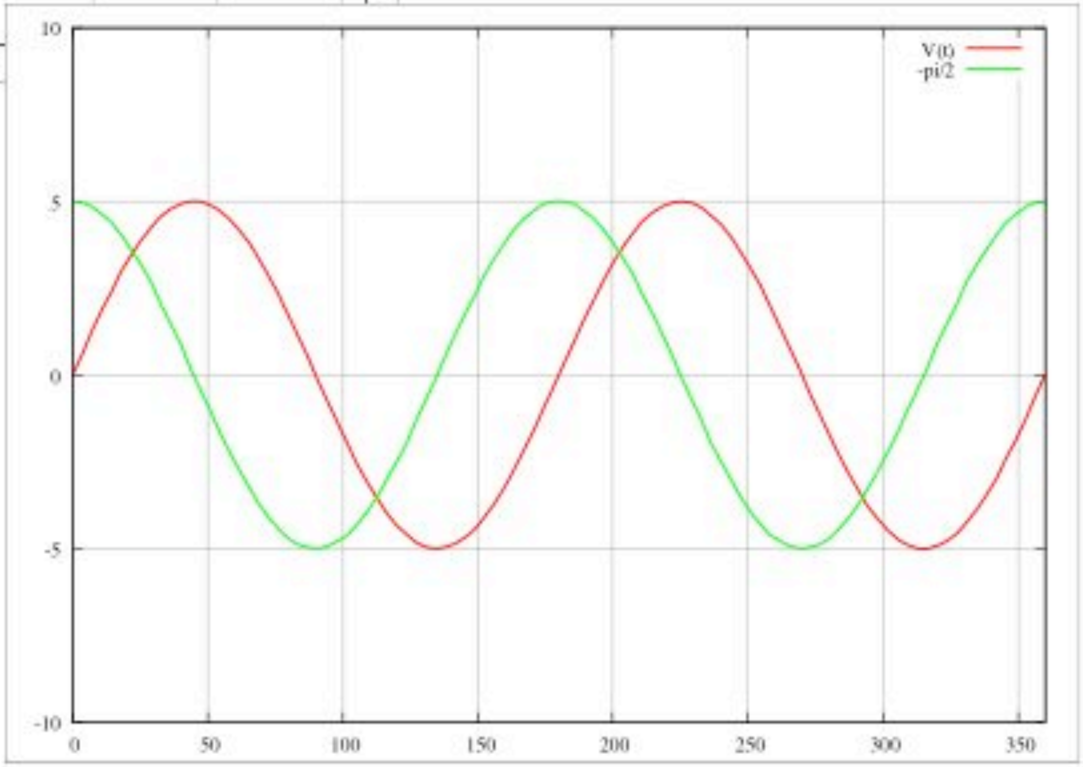


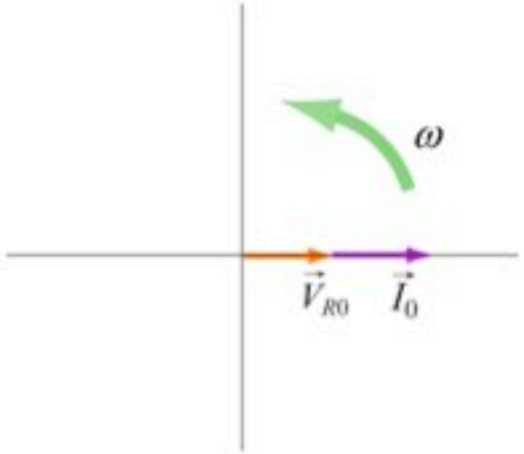
$\phi = \pi/2$

Purely inductive

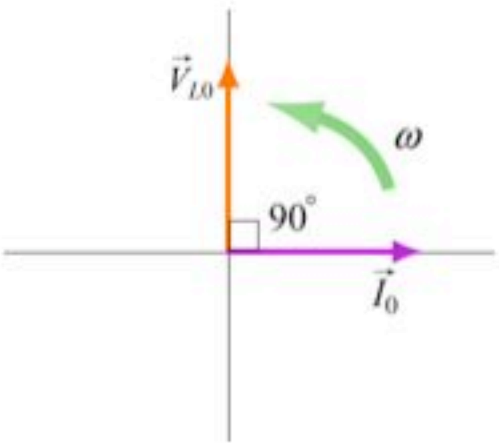
Purely capacitive

$\phi = -\pi/2$

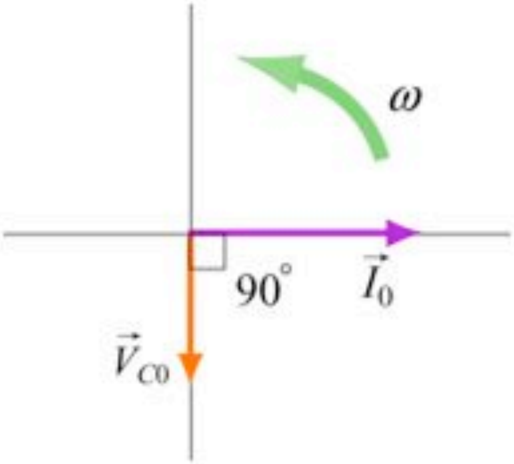




Resistor



Inductor



Capacitor

$$
\begin{aligned}
V_R(t) &= I_0 R \sin\omega t = V_{R0}\sin\omega t \\
V_L(t) &= I_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right) = V_{L0}\cos\omega t \\
V_C(t) &= I_0 X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -V_{C0}\cos\omega t
\end{aligned}
$$

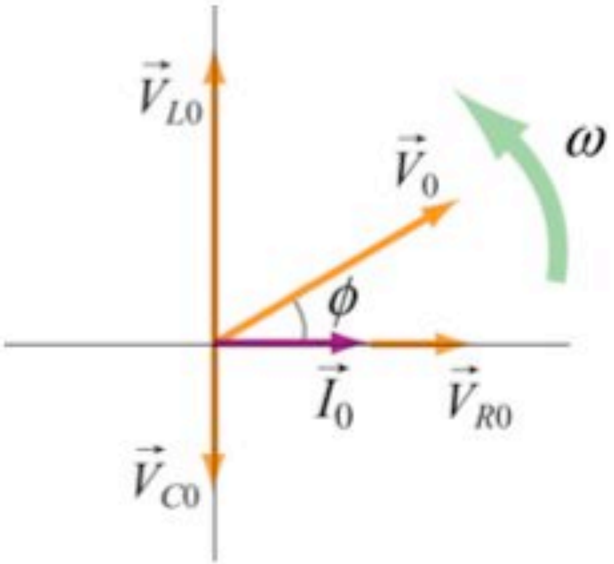
The sum of all three voltages is equal to the instantaneous voltage supplied by the AC source

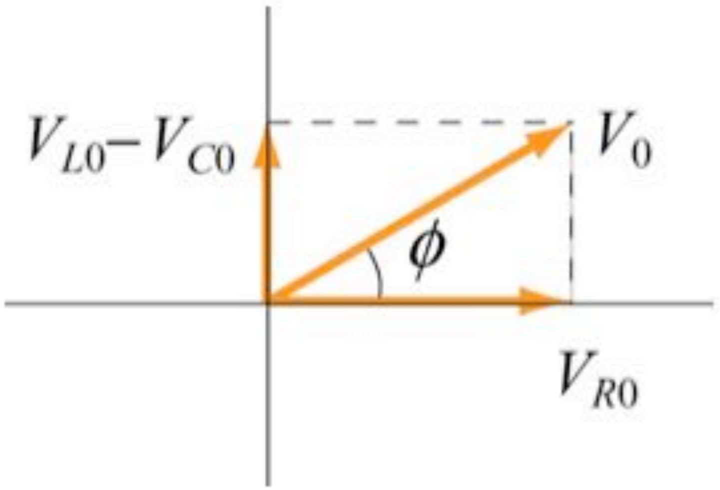
$$V(t) = V_R(t) + V_L(t) + V_C(t)$$

Phasor notation

$$\tilde{V}_O = \widetilde{V_{RO}} + \widetilde{V_{LO}} + \widetilde{V_{CO}}$$

AC voltage V_0 is not equal to the sum of the maximum voltage amplitudes across the three circuit elements





$$\begin{aligned} V_O &= |\tilde{V}_O| = |\widetilde{V_{RO}} + \widetilde{V_{LO}} + \widetilde{V_{CO}}| = \sqrt{V_{RO}^2 + (V_{LO} - V_{CO})^2} \\ &= \sqrt{(I_O R_O)^2 + (I_O X_L - I_O X_C)^2} \\ &= I_O \sqrt{R^2 + (X_L - X_C)^2} \end{aligned}$$

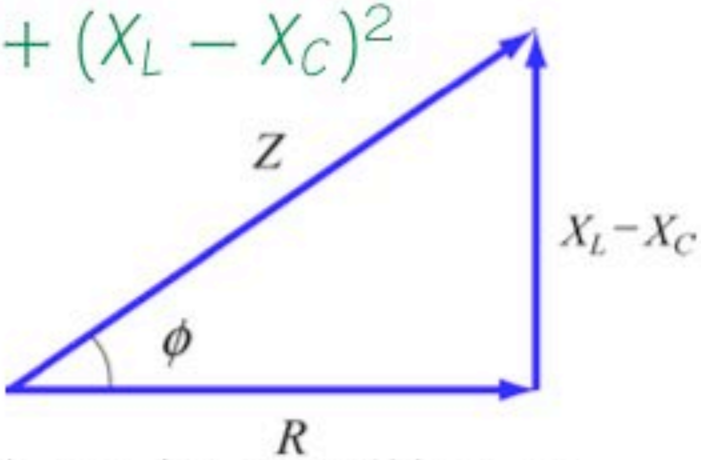
Just what we obtained a few slides back.

Impedance

We have already seen that the inductive reactance and capacitance reactance $X_C = 1/(\omega C)$ and $X_L = \omega L$ play the role of an effective resistance in the purely capacitive and inductive circuits, respectively. In the series RLC circuit, the effective resistance is the impedance, defined as

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{R^2 + (X_L - X_C)^2}$$

Relationship between Z, X_L and X_C

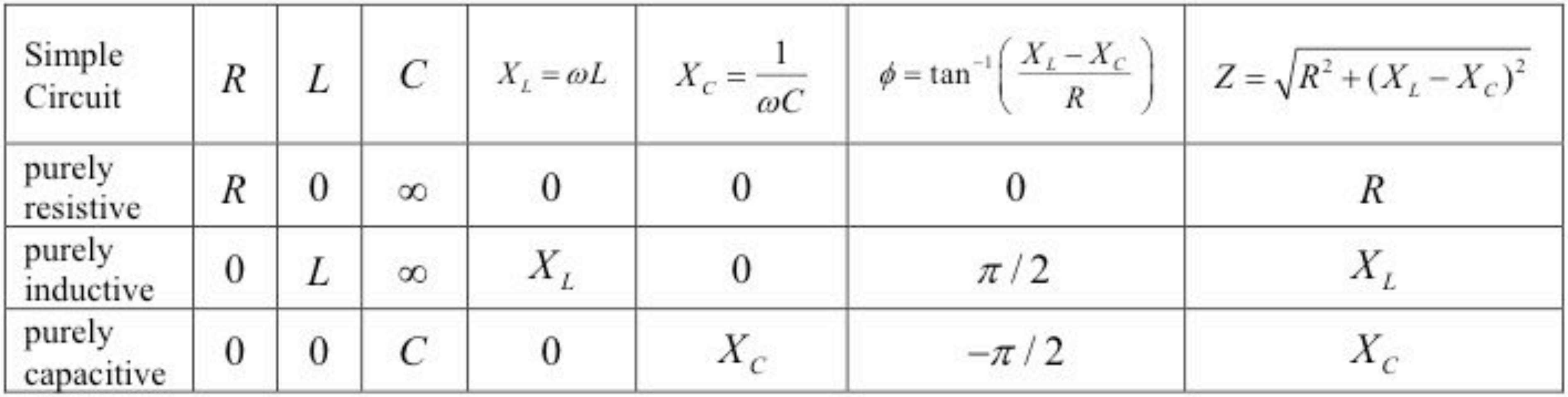


Z has the units of Ohms and the current can be rewritten as

$$I(t) = \frac{V_0}{Z} \sin(\omega t - \phi)$$

Z, like X_L and X_C depends on the angular frequency, ω

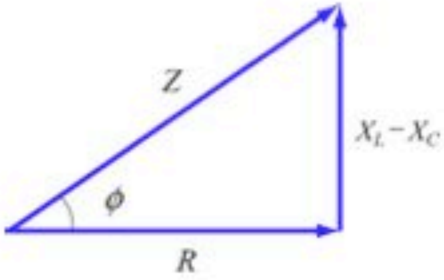
Simple-circuit limits of the series RLC circuit



Simple Circuit	R	L	C	$X_L = \omega L$	$X_C = \dfrac{1}{\omega C}$	$\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
purely resistive	R	0	∞	0	0	0	R
purely inductive	0	L	∞	X_L	0	$\pi/2$	X_L
purely capacitive	0	0	C	0	X_C	$-\pi/2$	X_C

Resonance

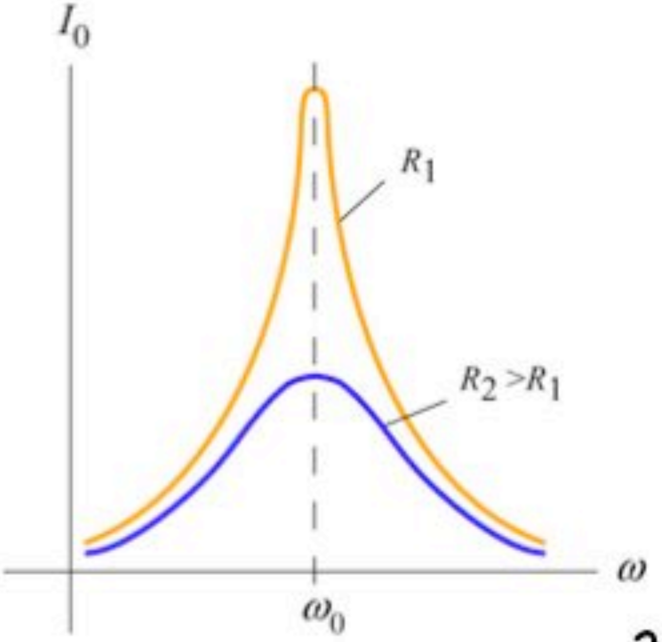
$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{R^2 + (X_L - X_C)^2}$$



Indicates that the amplitude of the current $I_0 = V_0/Z$ reaches a maximum when Z is at a minimum. This occurs when $X_L = X_C$ or $\omega L = 1/\omega C$, leading to

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

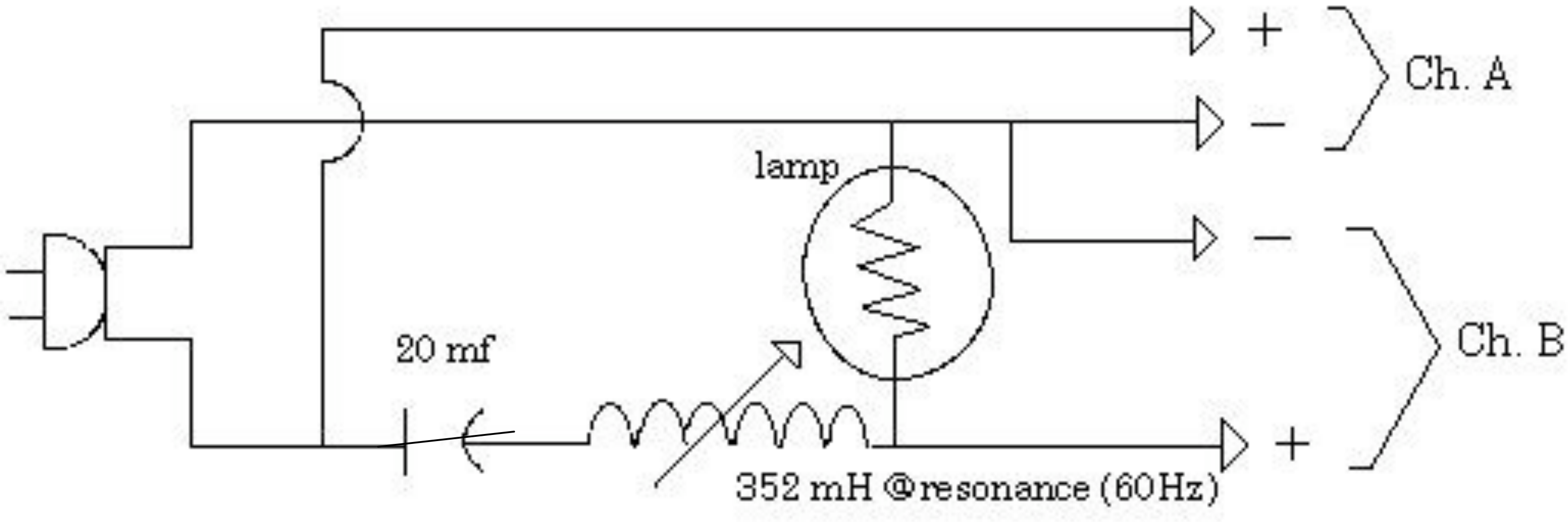
The phenomenon at which I_0 reaches a maximum is called a resonance, and the frequency ω_0 is called the resonant frequency.



At resonance $Z = R$ and $I_0 = V_0/R$ and $\phi = 0$. Acts like a purely resistive circuit.

Demonstration

Series LCR Circuit, Resonance



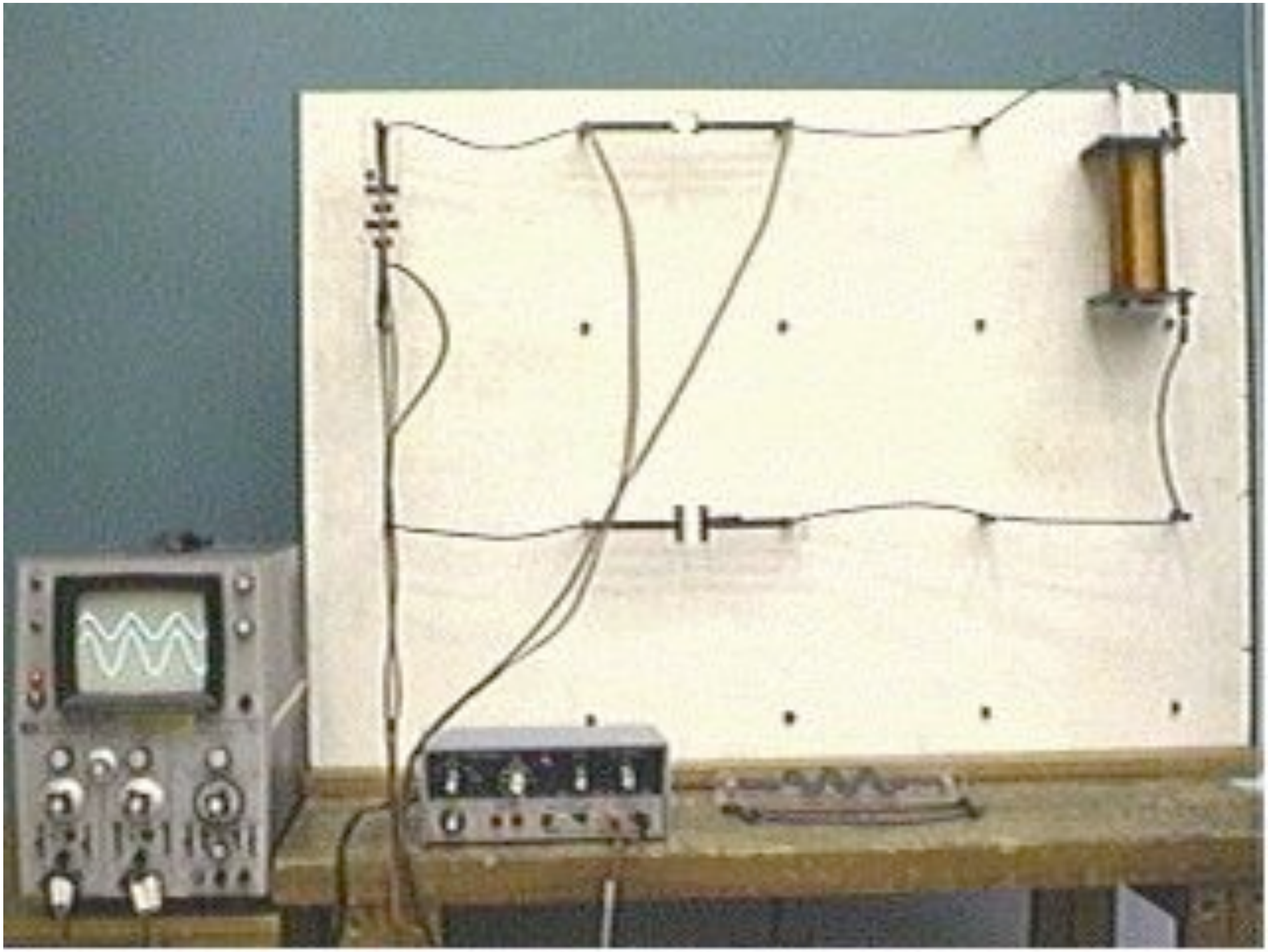
1.0 uF

4.25 mH

$$f = \frac{1}{6.28\sqrt{LC}} = \frac{1}{6.28\sqrt{4.25 \times 10^{-3} H \times 10^{-6} F}}$$

$$f = 2445 Hz$$

Series LCR circuit



Scope above is measuring driving voltage (top trace) and voltage across the resistor (bottom trace)

Power in AC Circuits

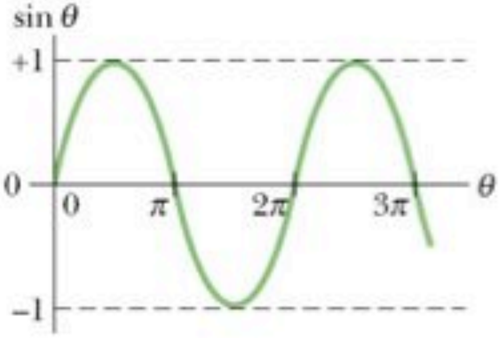
Resistive elements absorb net electric energy in ac circuits. Capacitors store electric field energy and feed it back into the circuit when they discharge and inductors store magnetic energy while current flows but they too return energy to the circuit when the current goes to zero.

How to calculate the power transferred from an ac source to any kind of ac circuit?

We must know the ac current and voltage at the input to the circuit as well as the relative phase of the current and voltage.

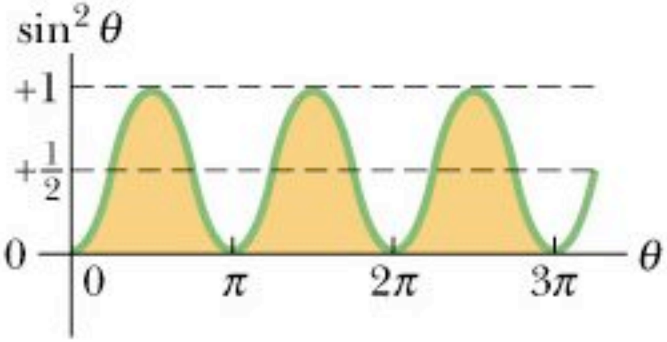
Instantaneous rate of energy dissipation is

$$P = I^2R = [I_0 \sin(\omega t - \Phi)]^2 R = I_0^2 R \sin^2(\omega t - \Phi)$$



(a)

P



(b)

The average rate of energy dissipation is the average of equation above. Note that over one complete cycle average value of sin θ is zero. But the average of $\sin^2\theta$ is 1/2.

Peak value

Effective Power

$$P = I_{rms}^2 R$$

$$I_{rms} = \frac{I}{\sqrt{2}}$$

rms=root-mean-square

The quantity I/(√2) is called the root-mean-square or rms, value of the current.

$$P_{avg} = I_{rms}^2 R$$

We can also define rms values for the voltage and emf

$$V_{rms} = \frac{V}{\sqrt{2}} \quad \xi_{rms} = \frac{\xi}{\sqrt{2}}$$

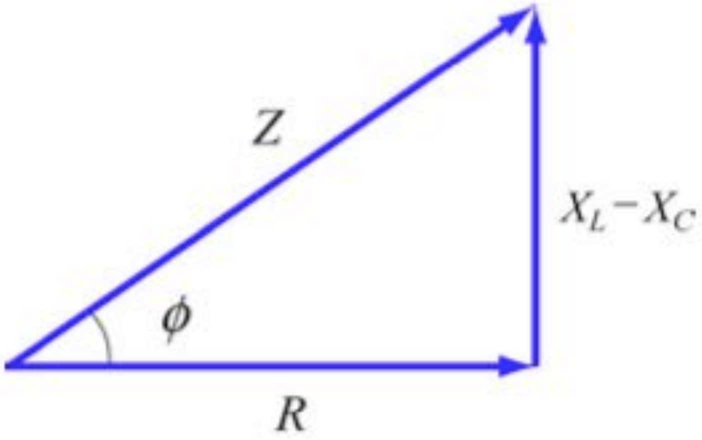
Ammeters and voltmeters are calibrated to read in $I_{rms,}$ V_{rms} etc. If your house ac line reads 120 Volts then the peak values is (120) (√2) = 170 V

More on Power in AC Circuits:

From $\tan(\phi) = (\omega L - 1/\omega C)/R$ and $\cos^2\phi = (\tan^2\phi + 1)^{-1}$ one could show that $\cos\phi = R/Z$, so that

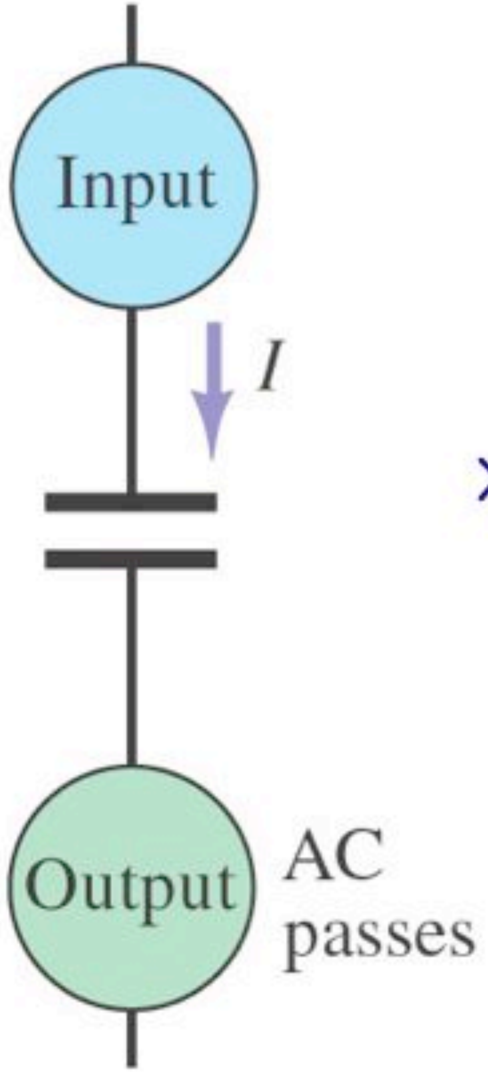
$$\langle P\rangle = I_{rms}^2 R = I_{rms}^2 Z\cos\Phi \qquad (33\text{–}46)$$

and $\cos\Phi = \frac{R}{Z}$ is called the power factor



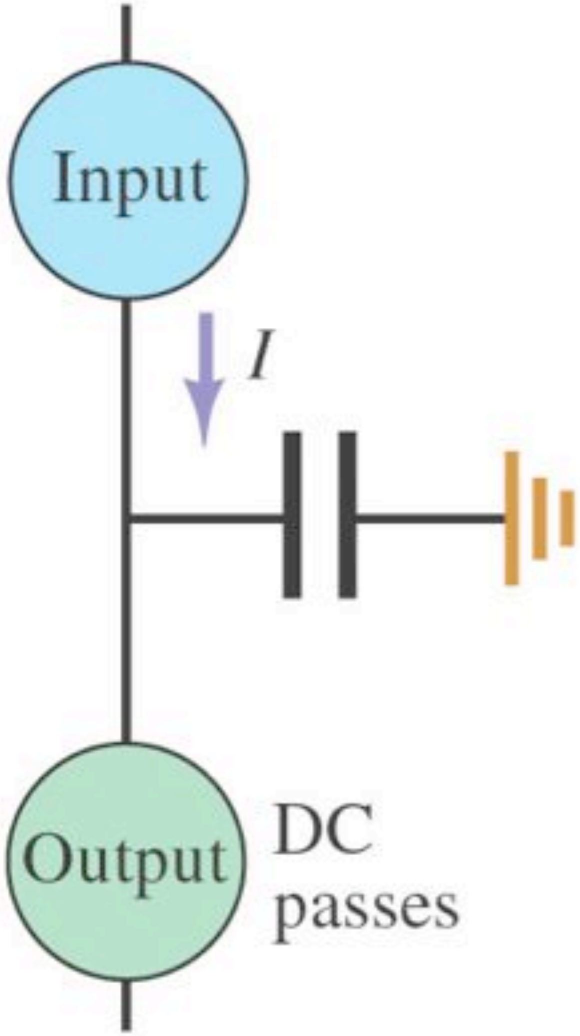
Filters:

Either capacitors or inductors can be used to make either AC or DC filters:

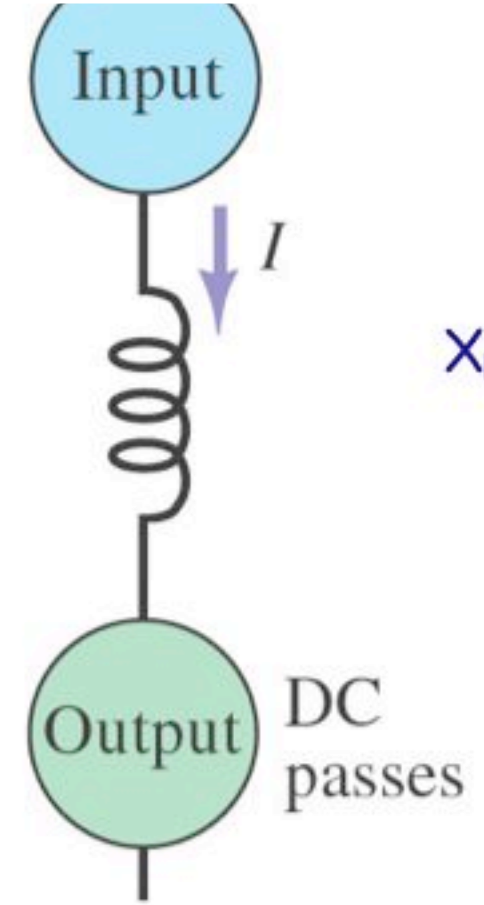


(a)

$X_C = 1/\omega C$

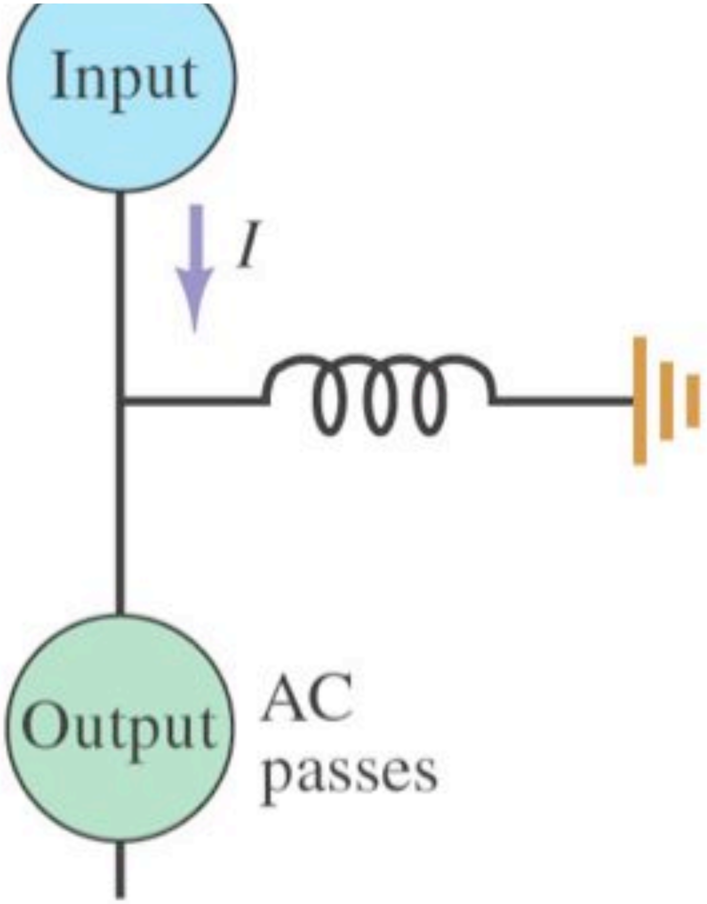


(b)

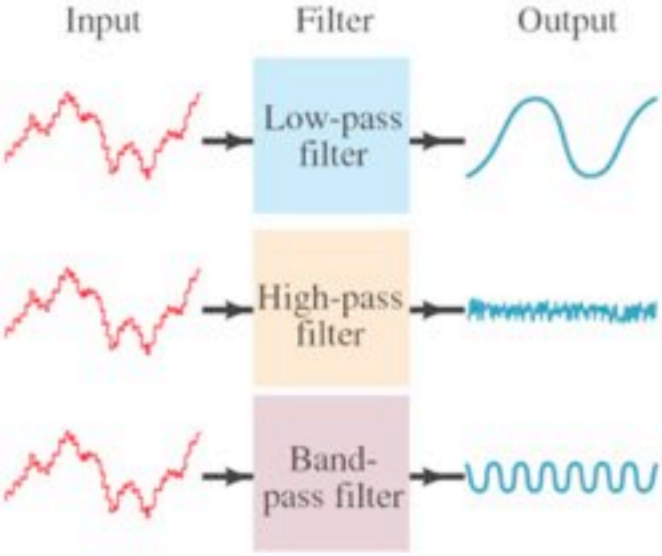


(c)

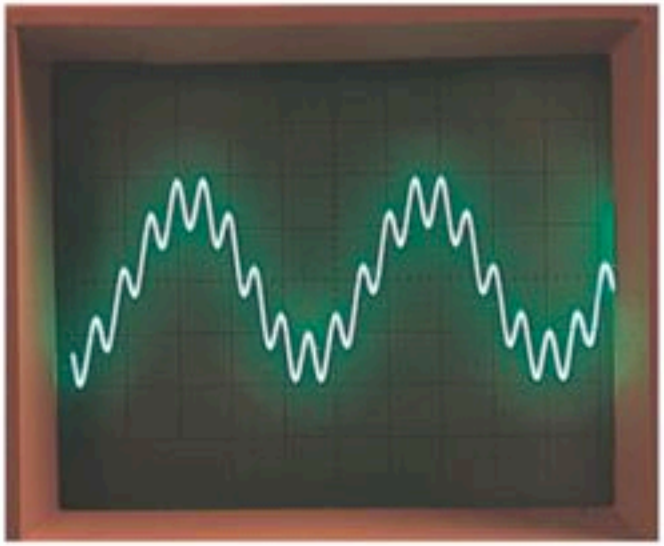
$X_L = \omega L$



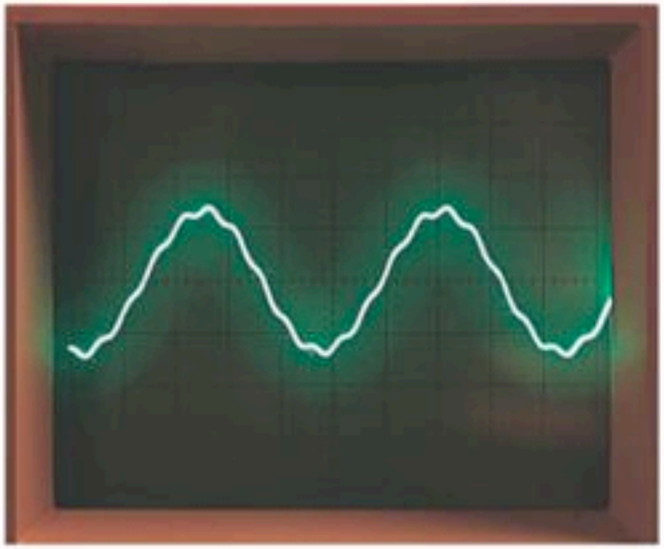
(d)

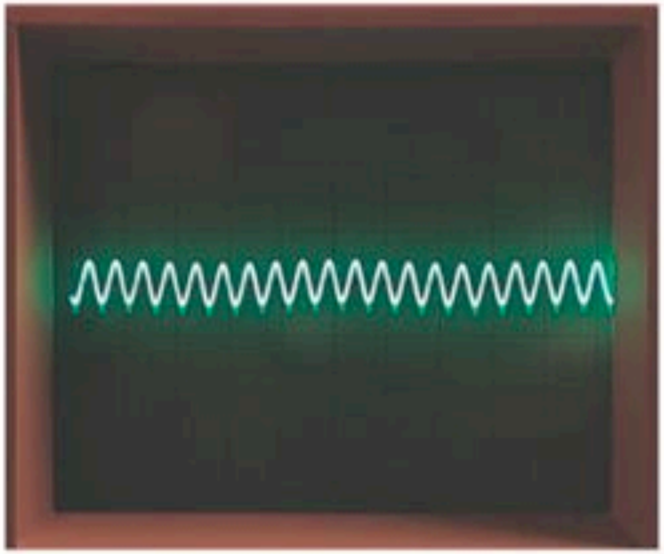

Input
Filter
Output
Low-pass filter
High-pass filter
Band-pass filter

(a)

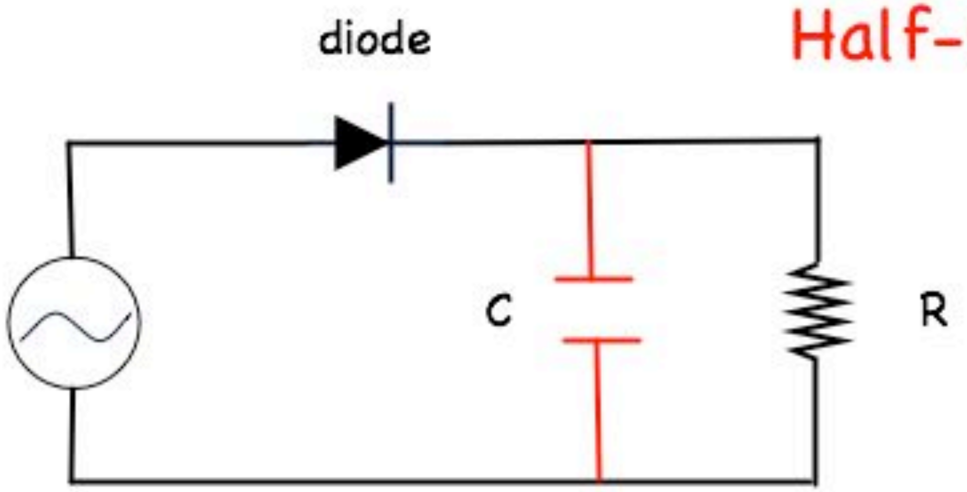


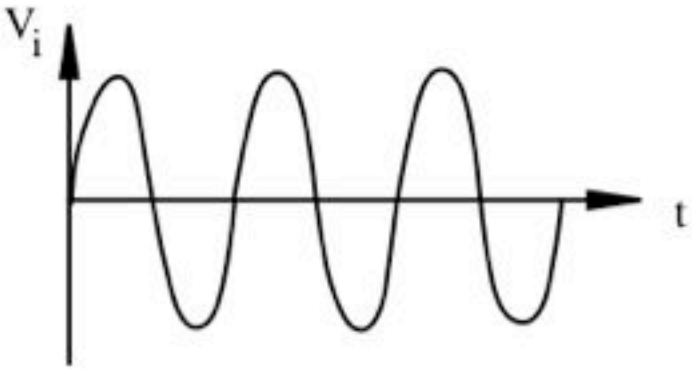
(b)

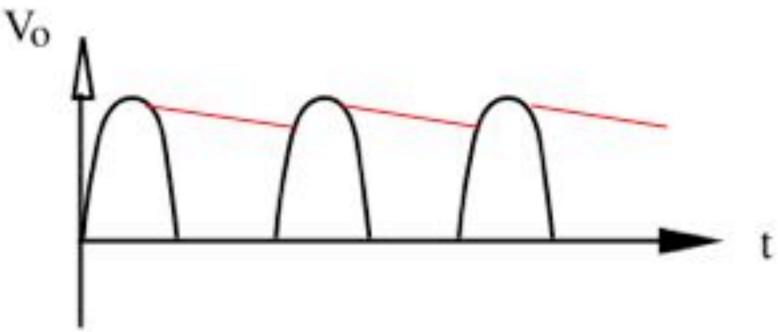




Half-wave rectifier







Problems from book