Lecture 12 Magnetism of Matter: Maxwell's Equations Chp. 32

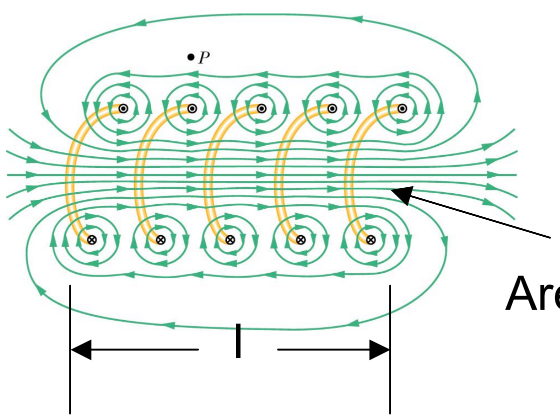
- Cartoon
- Warm-up problem
- Opening Demo
- Topics
  - Finish up Mutual inductance
  - Ferromagnetism
  - Maxwell equations
  - Displacement current

- Demos

What is the magnetic energy stored in a solenoid or coil

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \qquad dU_B = Lidi \qquad \int_0^{U_B} dU_B = \int_0^i Lidi \qquad U_B = \int_0^i Lidi = \tfrac{1}{2}Li^2$$

$U_B = \tfrac{1}{2}Li^2$ For an inductor L



Now define the energy per unit volume

$$u_B = \frac{U_B}{Al}$$

$$u_B = \frac{\tfrac{1}{2}Li^2}{Al} = \frac{L}{l}\frac{i^2}{2A} \qquad \frac{L}{l} = \mu_0 n^2 A$$

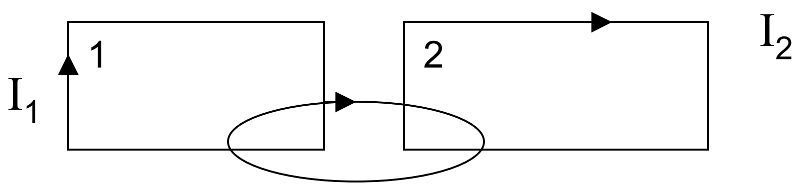
$$u_B = \frac{\tfrac{1}{2}Li^2}{Al} = \tfrac{1}{2}\mu_0 n^2 i^2 \qquad B = \mu_0 ni$$

The energy density formula is valid in general

$$u_B = \frac{B^2}{2\mu_0} \qquad u_E = \frac{E^2}{2\varepsilon_0}$$

What is Mutual Inductance? M

When two circuits are near one another and both have currents changing, they can induce emfs in each other.



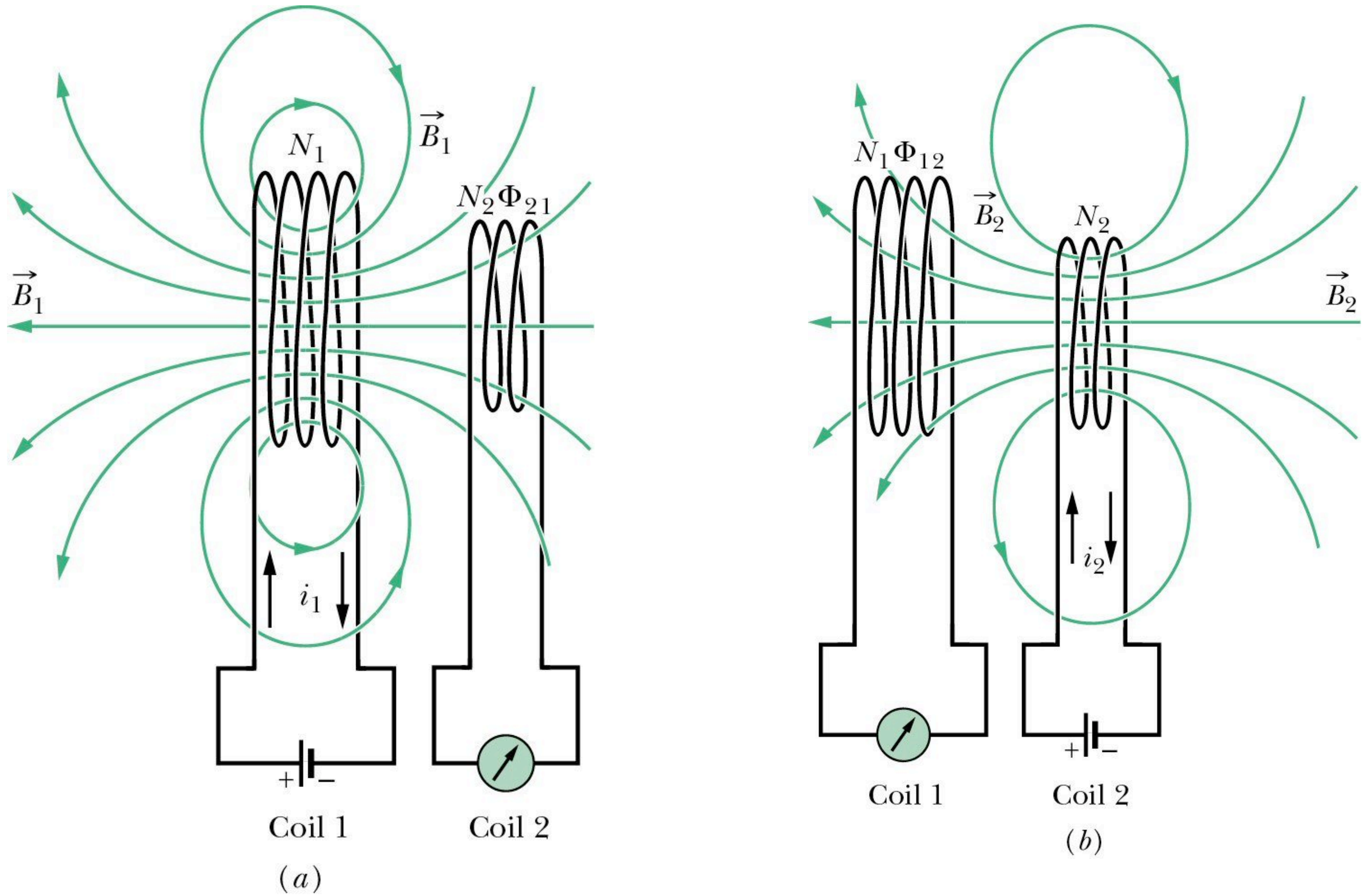
$$\phi_{m1} = L_1 I_1 + M_{21} I_2$$

$$\phi_{m2} = L_2 I_2 + M_{12} I_1$$

$$M_{12} = M_{21} = M$$

On circuit boards you have to be careful you do not put circuits near each other that have large mutual inductance.

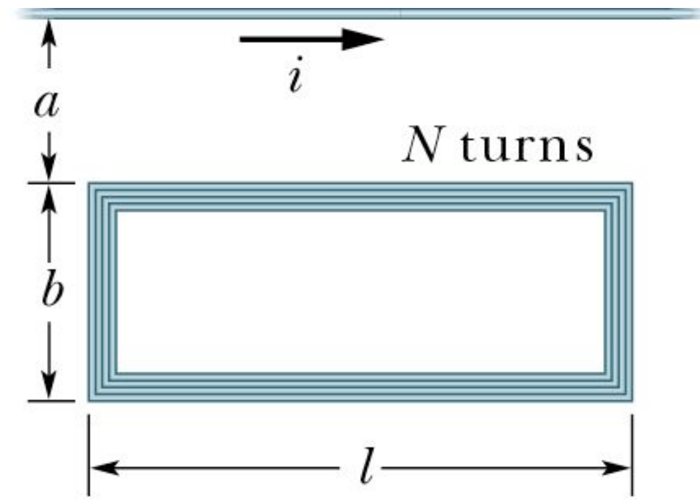
They have to be oriented carefully and even shielded.


N_1
$\vec{B}_1$
$N_2\Phi_{21}$
$\vec{B}_1$
i_1
+ −
Coil 1
Coil 2
(a)
$N_1\Phi_{12}$
$\vec{B}_2$
N_2
$\vec{B}_2$
i_2
+ −
Coil 1
Coil 2
(b)

75. A rectangular loop of N closely packed turns is positioned near a long, straight wire as shown in the figure.

(a) What is the mutual inductance M for the loop-wire combination?

(b) Evaluate M for N = 100, a = 1.0 cm, b = 8.0 cm, and l = 30 cm.



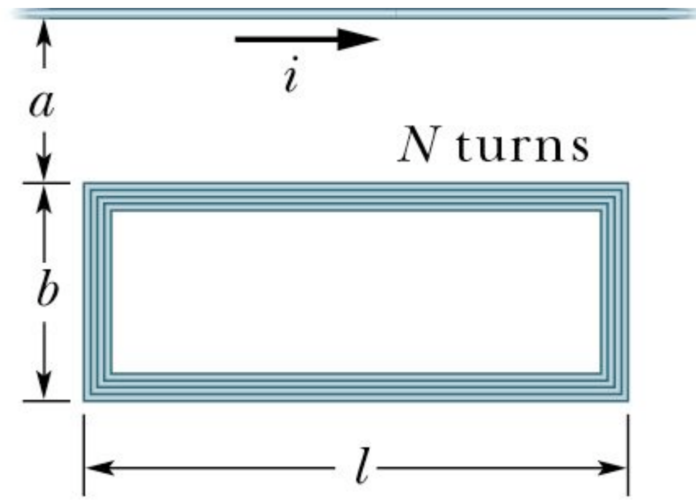
(a) The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{wire} l dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} \ln r \Big|_a^{a+b} = \frac{\mu_0 i l}{2\pi} \ln(1 + \frac{b}{a})$$

$$M = \frac{N\Phi}{i}$$

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1 + \frac{b}{a}\right)$$

(b) Evaluate M for N = 100, a = 1.0 cm, b = 8.0 cm, and l = 30 cm.



$$M = \frac{N\mu_0 l}{2\pi}\ln\left(1+\frac{b}{a}\right)$$

(b) From the formula for M obtained,

$$M = \frac{100\left(4\pi\times10^{-7}\ H/m\right)(0.30m)}{2\pi}\ln\left(1+\frac{8.0}{1.0}\right)$$

$$= 1.3\times10^{-5}\,H$$

Ferromagnetism

Iron, cobalt, nickel, and rare earth alloys exhibit ferromagnetism.
The so called exchange coupling causes electron magnetic moments of one atom to align with electrons of other atoms. This alignment produces magnetism. Whole groups of atoms align and form domains.

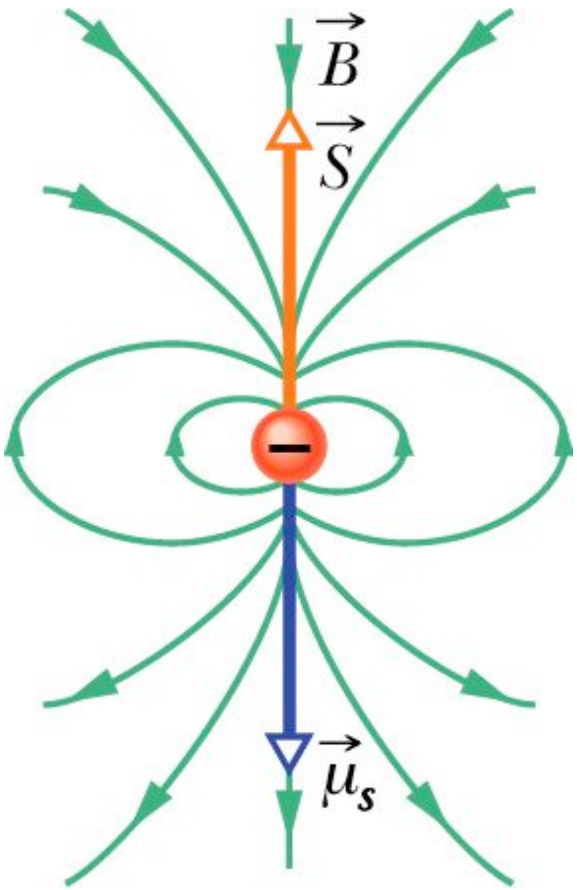
(See Figure 32-12 on page 756)

A material becomes a magnet when the domains line up adding all the magnetic moments.You can actually hear the domains shifting by bringing up an magnet and hear the induced currents in the coil. Barkhausen Effect

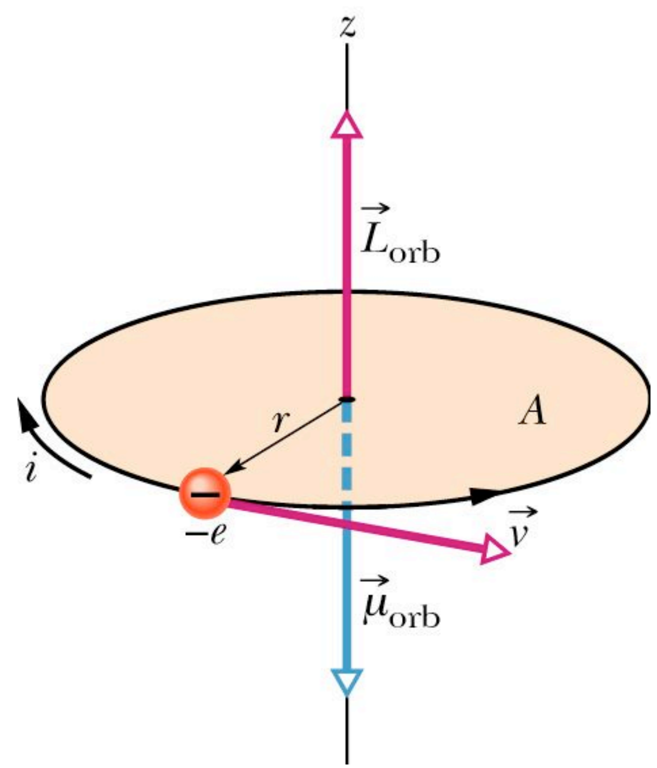
Two other types of magnetic behavior are paramagnetism or diamagnetism.

What is the atomic origin of magnetism?

Electron spinning on its axis



Electron orbiting around the nucleus



Spin Magnetic Dipole Moment of the Electron

$$\vec{\mu} = -\frac{e}{m}\vec{S}$$

$$e = 1.6 \times 10^{-9} Coulombs$$

$$m = 9.11 \times 10^{-31} kg$$

S is the angular momentum due to the electron’s spin. It has units kg.m^2/s. μ has units of A.m^2 - current times area
Recall for a current loop, the magnetic dipole moment = current times area of loop

In the quantum field theory of the electron, S can not be measured. Only it’s component along the z axis can be measured. In quantum physics, there are only two values of the z component of the electron spin.

Therefore, only the z component of μ can be measured.Its two possible values are:

$$\mu_z = \pm \frac{eh}{4\pi m}$$

Corresponding to the two values of the electron spin quantum number +1/2 and -1/2

The above quantity is called the Bohr magneton and is equal to:

$$\mu_B = \mu_z = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} A.m^2$$

The magnetic moment of the electron is the prime origin of ferromagnetism in materials.

22. The dipole moment associated with an atom of iron in an iron bar is 2.1x10^{-23} J/T. Assume that all the atoms in the bar, which is 5.0 cm long and has a cross-sectional area of 1.0 cm^2, have their dipole moments aligned.

(a) What is the dipole moment of the bar?

The number of iron atoms in the iron bar is

$$N = \frac{(7.9\,g/cm^3)(5.0cm)(1.0cm^2)}{(55.847\,g/mol)/(6.022\times 10^{23}\,mol)} = 4.3\times 10^{23}.$$

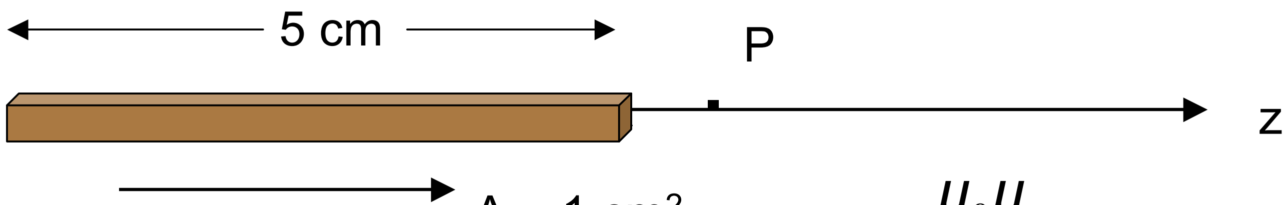
Thus, the dipole moment of the bar is

$$\mu = (2.1\times 10^{-23}\,J/T)(4.3\times 10^{23}) = 9.03A\cdot m^2.$$

(b) What torque must be exerted to hold this magnet perpendicular to an external field of 1.5 T? (The density of iron is 7.9 g/cm³)

$$(b)\quad \tau = \mu B \sin 90^o = \left(9.03 A \cdot m^2\right)\left(1.5T\right) = 13.5 N \cdot m$$

(C) Use the dipole formula to find the magnitude and direction of the magnetic field 1cm from the end of the bar magnet on its central axis at P.



$$B = \frac{\mu_0 \mu}{2\pi z^3}$$

$$B_T = \int dB = \frac{\mu_0}{2\pi} \int \frac{d\mu}{z^3}$$

$$d\mu = \frac{\mu}{AL} A dz = \frac{\mu}{L} dz$$

$$B_T = \frac{\mu_0 \mu}{2\pi L} \int_{.06}^{.01} \frac{dz}{z^3} = \left(\frac{\mu_0 \mu}{2\pi L}\right)\left(-\frac{2}{z^2}\Big|_{0.06}^{0.01}\right) = \left(\frac{\mu_0 \mu}{2\pi L}\right)\left(-\frac{2}{(0.01)^2}\right)$$

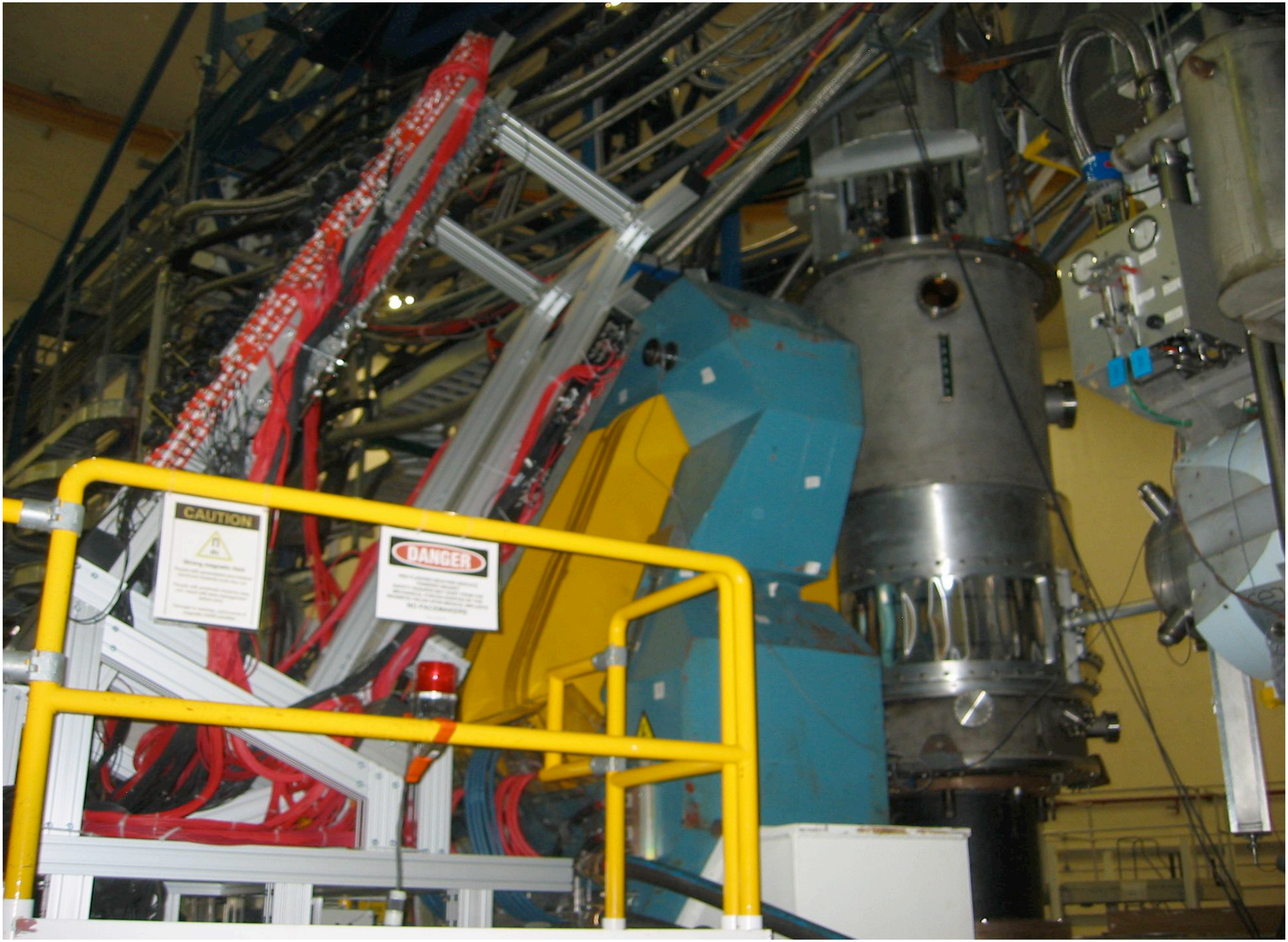
$$B = \frac{4\pi \times 10^{-7} \frac{N}{A^2} 8.9 A.m^2}{\pi \times 0.05m(.01m)^2}$$

$$B = \frac{4 \times 10^{-7} 8.9 N / A.m}{5 \times 10^{-6}}$$

$$B = 0.71\ T$$

BigBite is a 50 ton electromagnet with a 25 cm by 100 cm gap

B = 1 Tesla

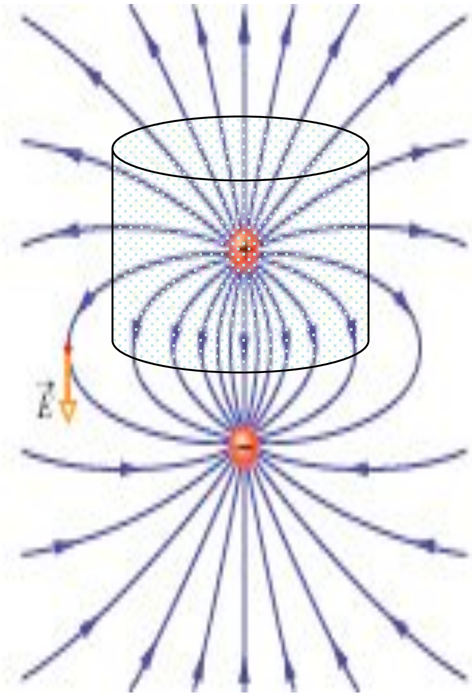

CAUTION
DANGER

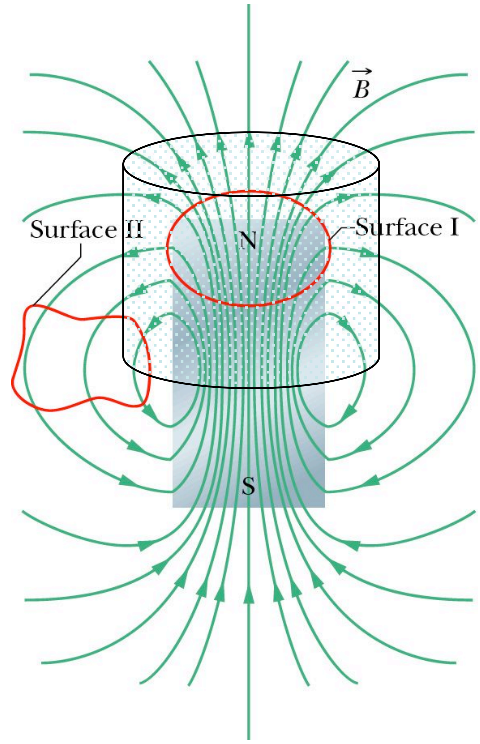
Maxwells Equations:
In 1873 he wrote down 4 equations which govern all classical electromagnetic phenomena.

You already know two of them.

1. $\Phi_E = \oint \vec{E}.dA = q_{enc} / \varepsilon_0$

2. $\Phi_B = \oint \vec{B}.dA = 0$





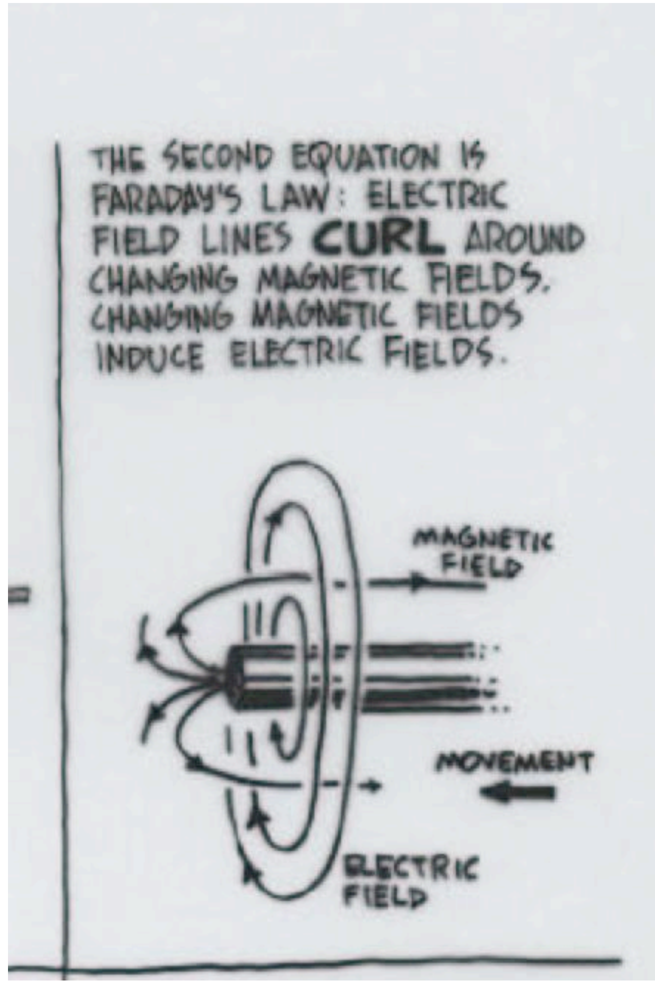
A magnetic field changing with time can produce an electric field: Faraday's law

$$3. \quad \oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

Line integral of the electric field around the wire equals the change of Magnetic flux through the area Bounded by the loop

Electric lines curl around changing magnetic field lines

Example



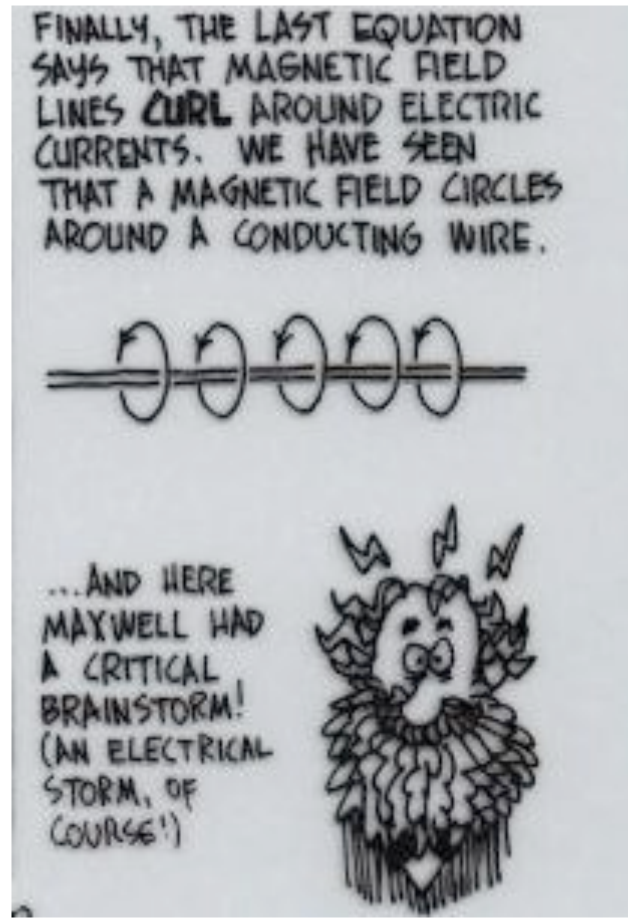
New Question: Can a changing electric field with time produce an magnetic field?

Yes it can and it is called

Maxwell’s law of induction

.

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



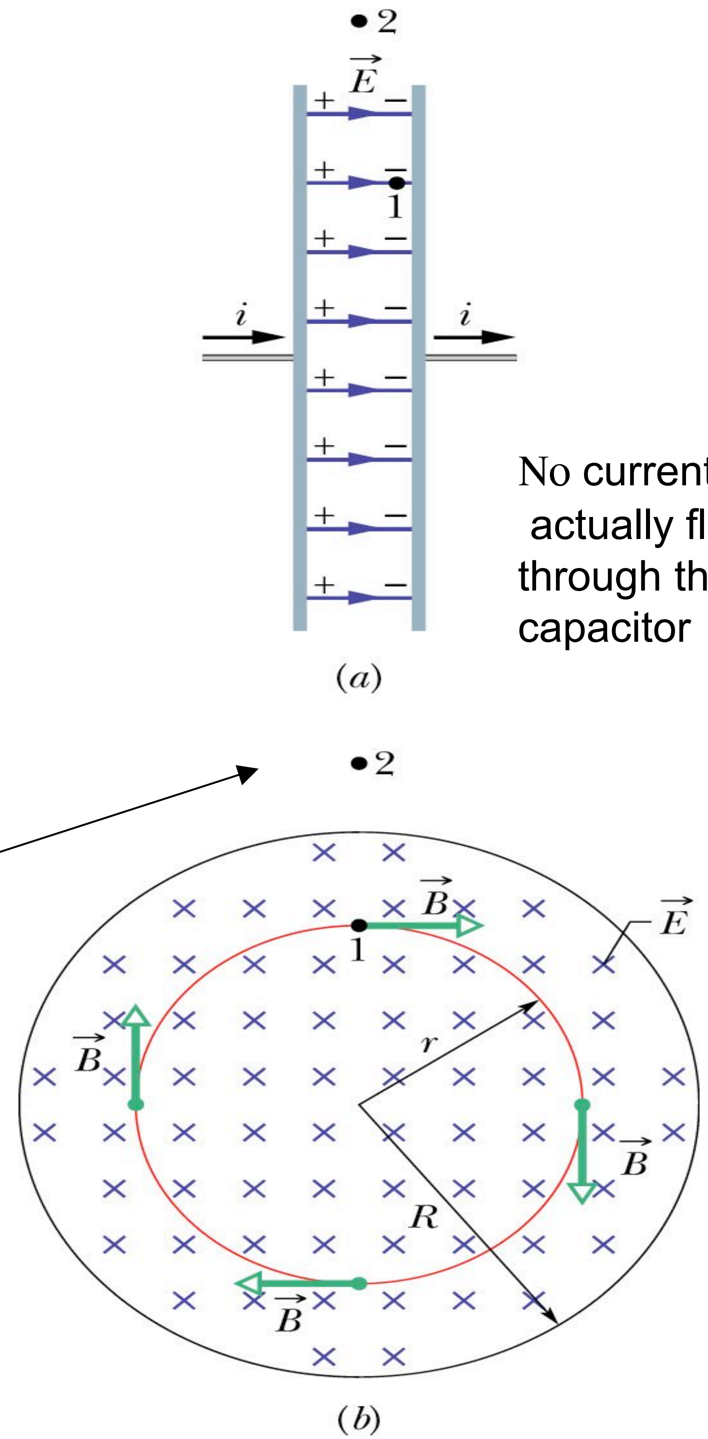
Maxwell's law of induction

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Consider the charging of our circular plate capacitor

B field also induced at point 2.

When capacitor stops charging B field disappears.



No current ever actually flows through the capacitor

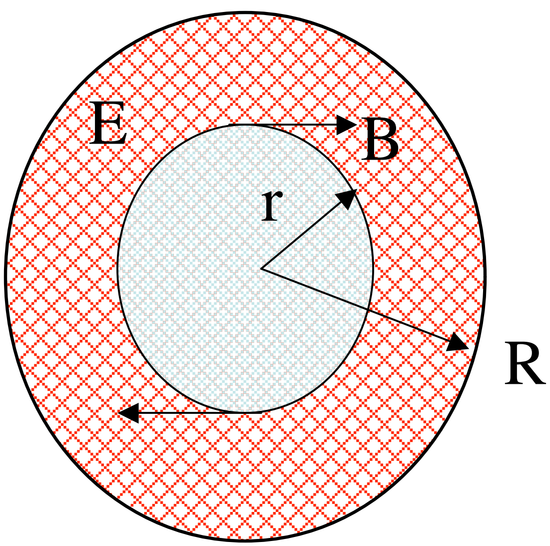
Find the expression for the induced magnetic field B that circulates around the electric field lines of a charging circular parallel plate capacitor

r < R

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Flux within the loop of radius r

0



$$\oint \vec{B}.d\vec{s} = (B)(2\pi r) \text{ since B parallel ds}$$

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{d(AE)}{dt} = \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

r< R

r > R

$$(B)(2\pi r) = \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}$$

Ampere-Maxwell's Law

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B}.d\vec{s} = \mu_0 i$$

This term has units of current

Maxwell combined the above two equations to form one equation

4. $$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

How do we interpret this equation?

What is the displacement current?

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

$$\varepsilon_0 \frac{d\Phi_E}{dt} = i_d$$

This is called the displacement current i_d

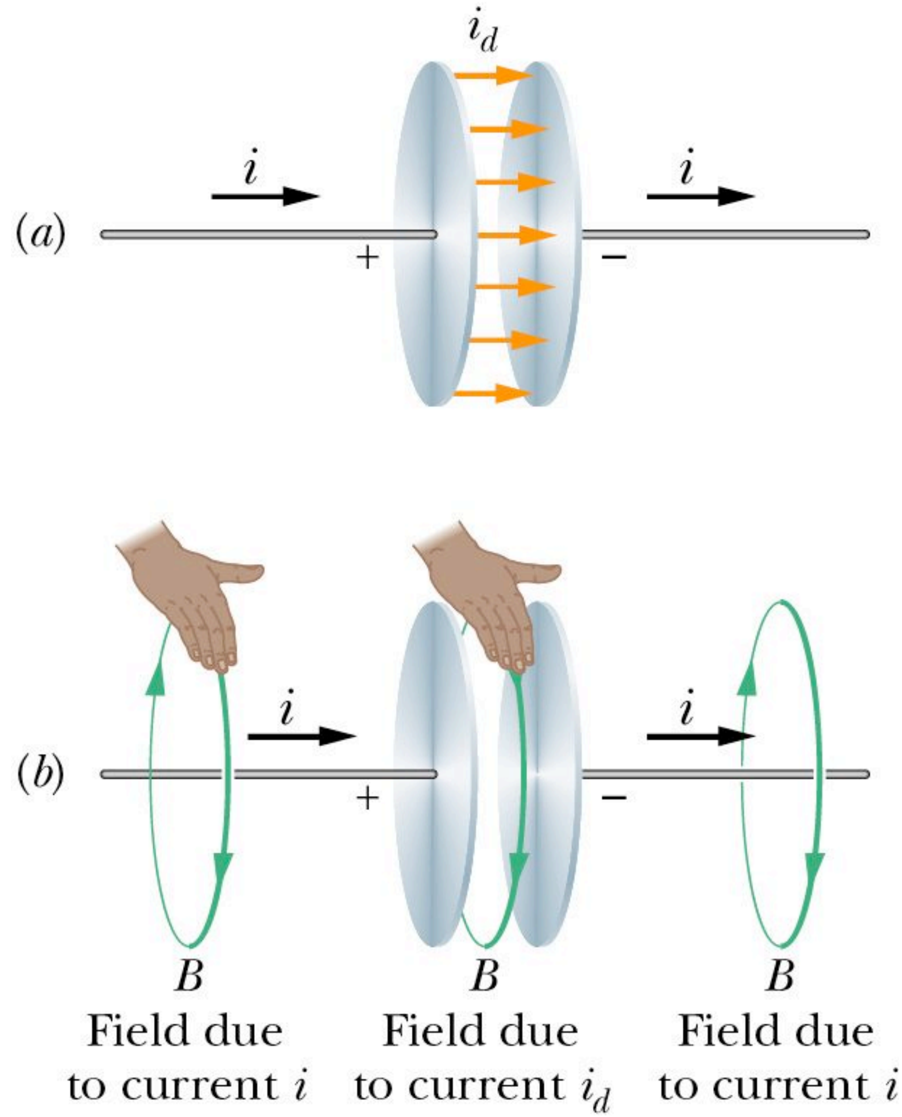
$$\oint \vec{B}.d\vec{s} = \mu_0 i_d + \mu_0 i_{enc}$$

The term is really is a transfer of electric and magnetic energy from one plate to the other while the plates are being charged or discharged. When charging stops, this term goes to zero. Note it is time dependent.

Show that the displacement current in the gap of the two capacitor plates is equal to the real current outside the gap

$$\varepsilon_0 \frac{d\Phi_E}{dt} = i_d$$

$$\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$



Can I detect the magnetic field associated with displacement current?

Calculation of i_d

First find the real current i

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q/A}{\varepsilon_0}$$

For the field inside a parallel plate capacitor

$$q = \varepsilon_0 AE$$

Solving for q

$$i = \frac{dq}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

This is the real current i charging the capacitor.

Next find the displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$i_d = \varepsilon_0 \frac{d(AE)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

displacement current = real current. No charge actually moves across the gap.

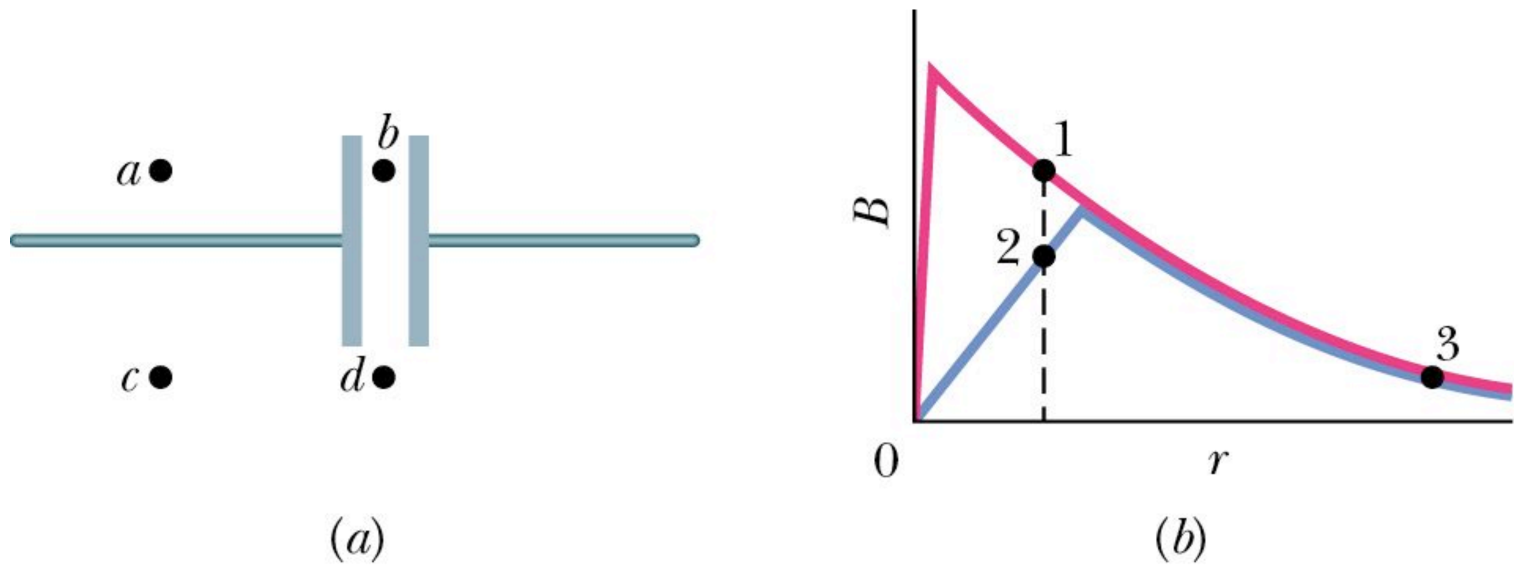
Calculate Magnetic field due to displacement current

Current is uniformly spread over the circular plates of the capacitor. Imagine it to be just a large wire of diameter R. Then use the formula for the magnetic field inside a wire.

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r$$ Inside the capacitor

$$B = \frac{\mu_0 i_d}{2\pi r}$$ Outside the capacitor

Question 11: A circular capacitor of radius R is being charged through a wire of radius R_0. Which of the points a, b, c, and d correspond to points 1, 2, and 3 on the graph



Where is the radius R_0 and R on the graph?

Summary of Maxwell Equations
Integral form

1. $\Phi_E = \oint \vec{E}.dA = q_{enc} / \varepsilon_0$

2. $\Phi_B = \oint \vec{B}.dA = 0$

3. $\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$

4. $\oint \vec{B}.d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$