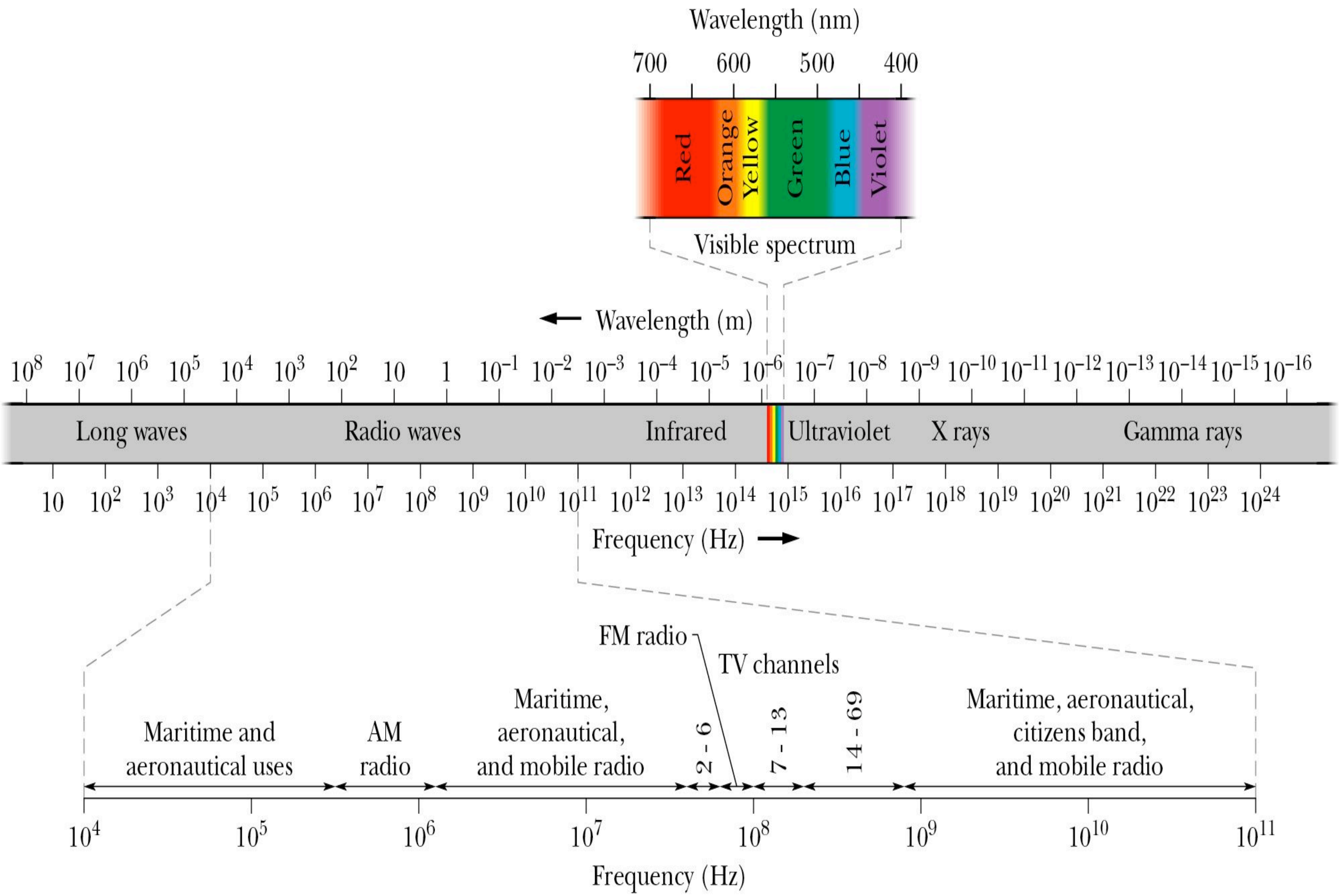
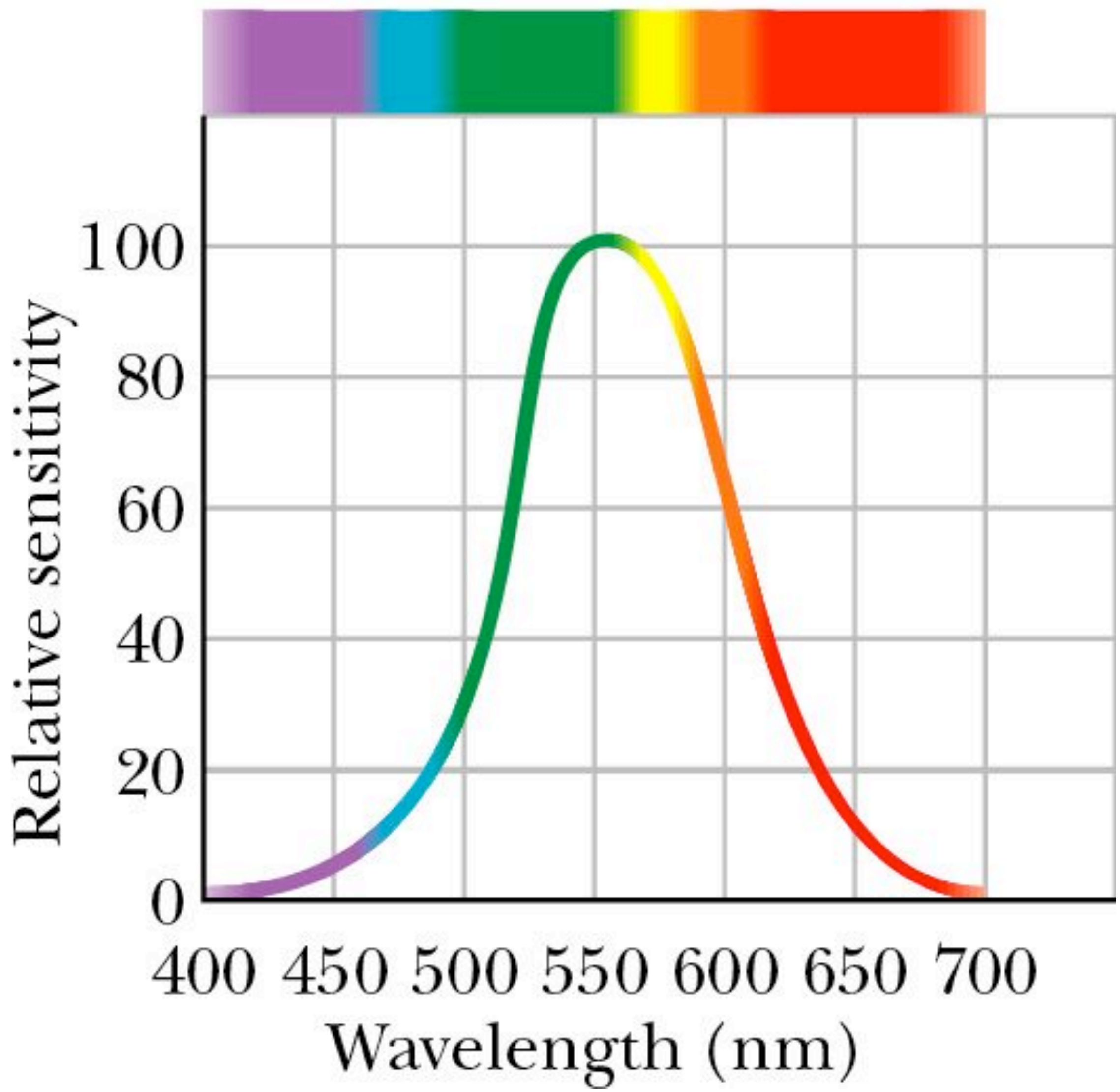
Lecture 13 Electromagnetic Waves Ch. 33

- Cartoon
- Opening Demo
- Topics
  - Electromagnetic waves
  - Traveling E/M wave - Induced electric and induced magnetic amplitudes
  - Plane waves and spherical waves
  - Energy transport Poynting vector
  - Pressure produced by E/M wave
  - Polarization
  - Reflection, refraction,Snell's Law, Internal reflection
  - Prisms and chromatic dispersion
  - Polarization by reflection-Brewsters angle

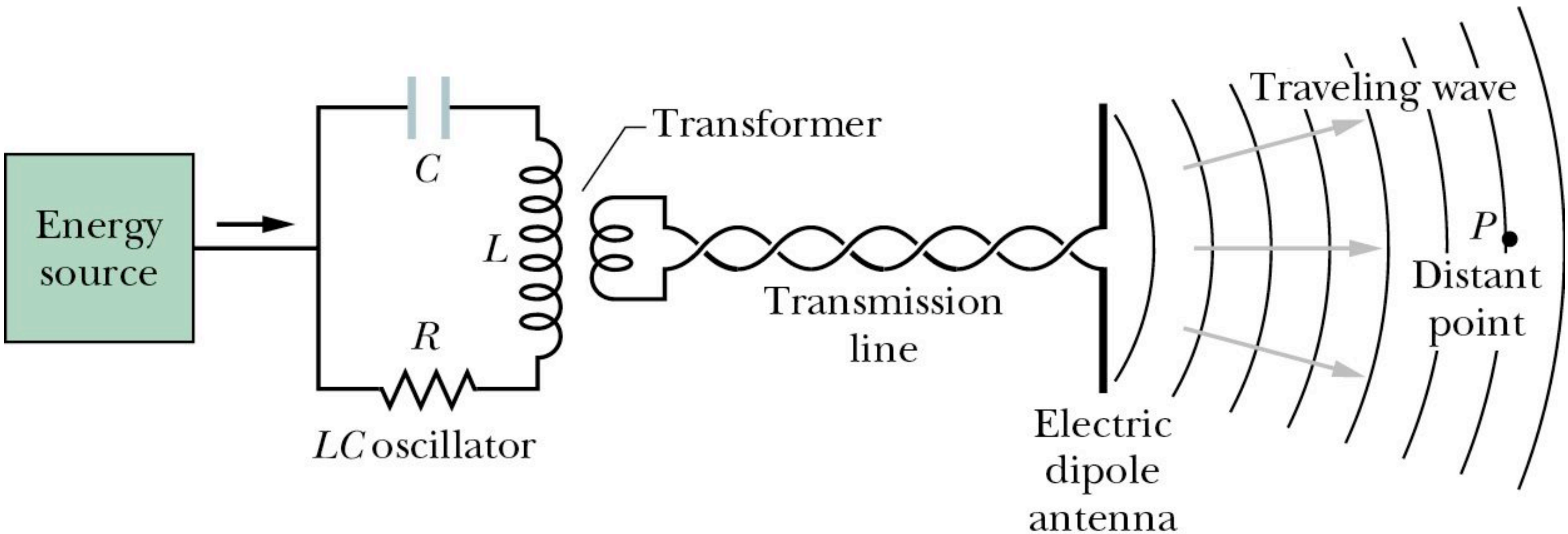
Electromagnetic Waves



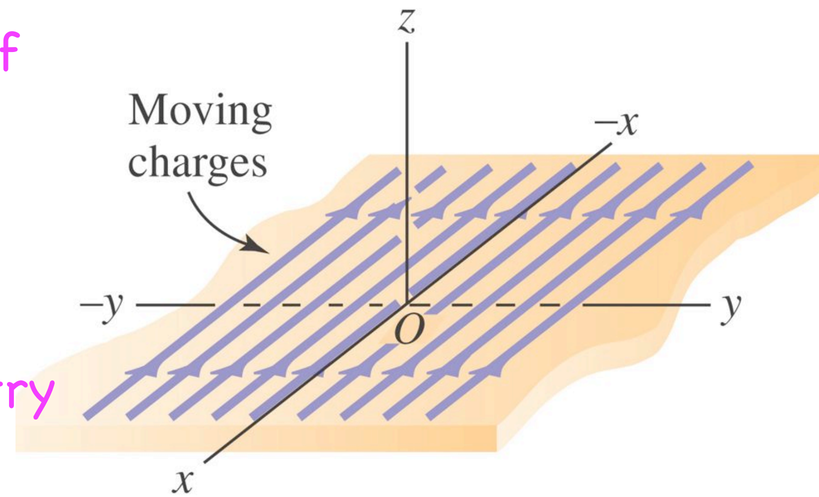
Eye Sensitivity to Color



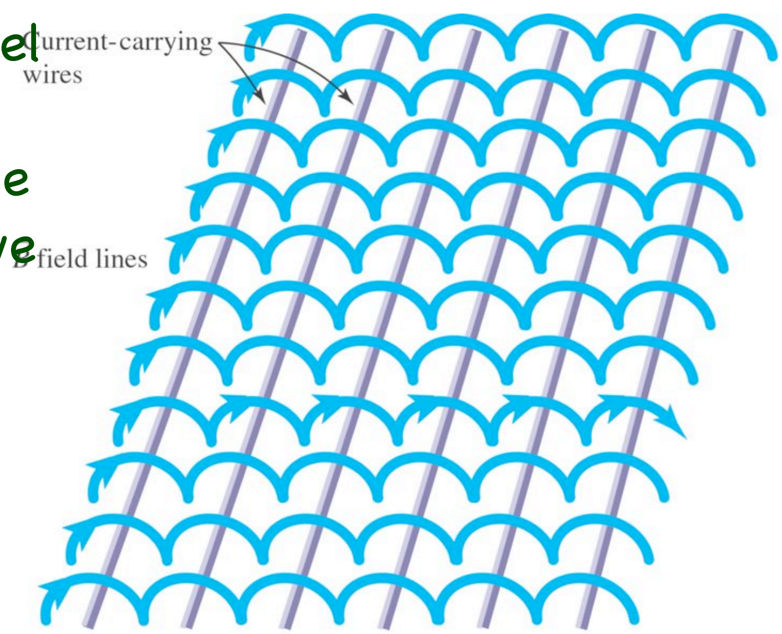
Production of Electromagnetic waves



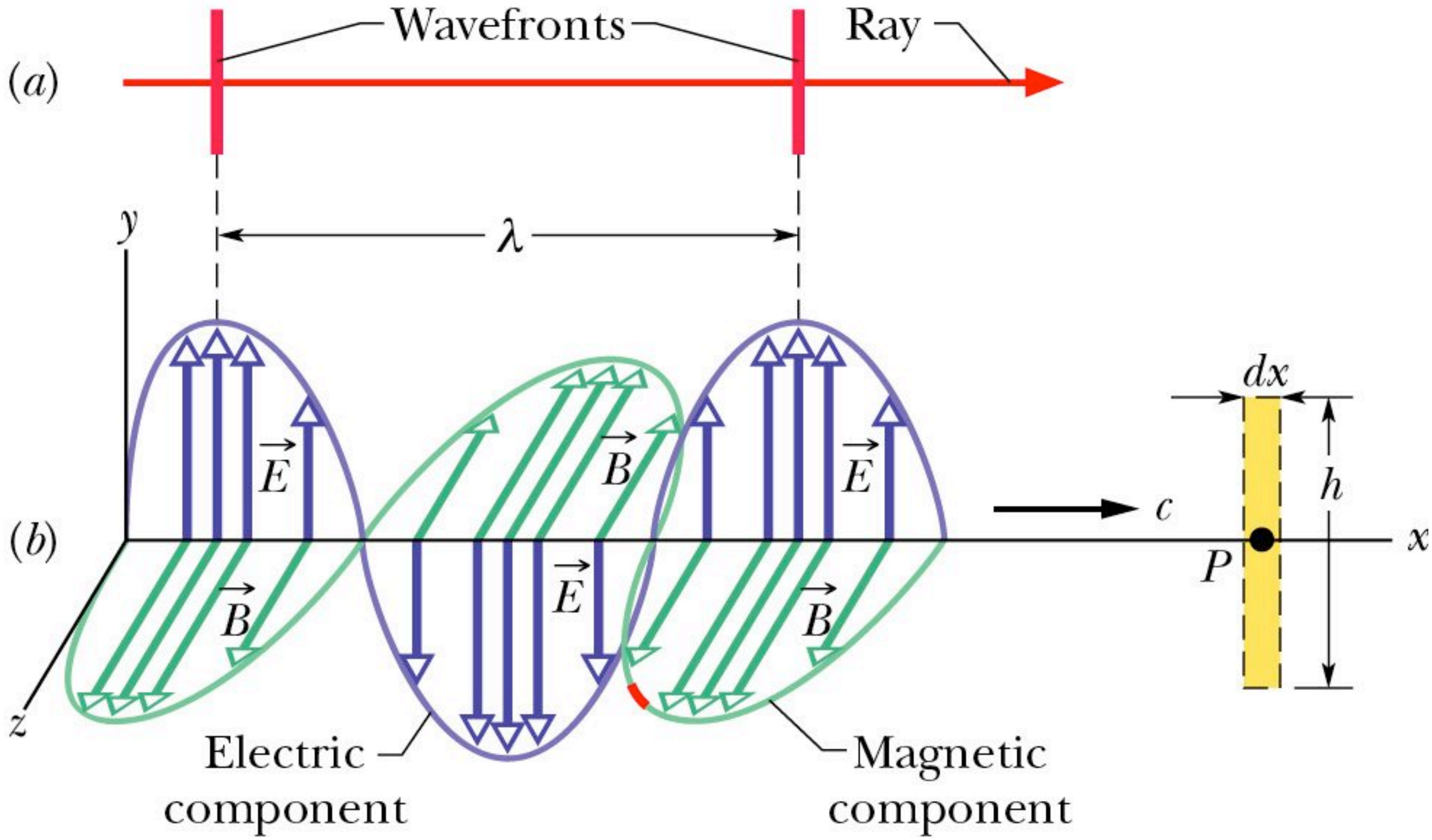
To investigate further the properties of electromagnetic waves we consider the simplest situation of a plane wave. A single wire with variable current generates propagating electric and magnetic fields with cylindrical symmetry around the wire.



If we now stack several wires parallel to each other, and make this stack wide enough (and the wires very close together), we will have a (plane) wave propagating in the z direction, with E-field oriented along x, $E = E_x$ (the current direction) and B-field along y $B=B_y$ (Transverse waves)



Electromagnetic Wave

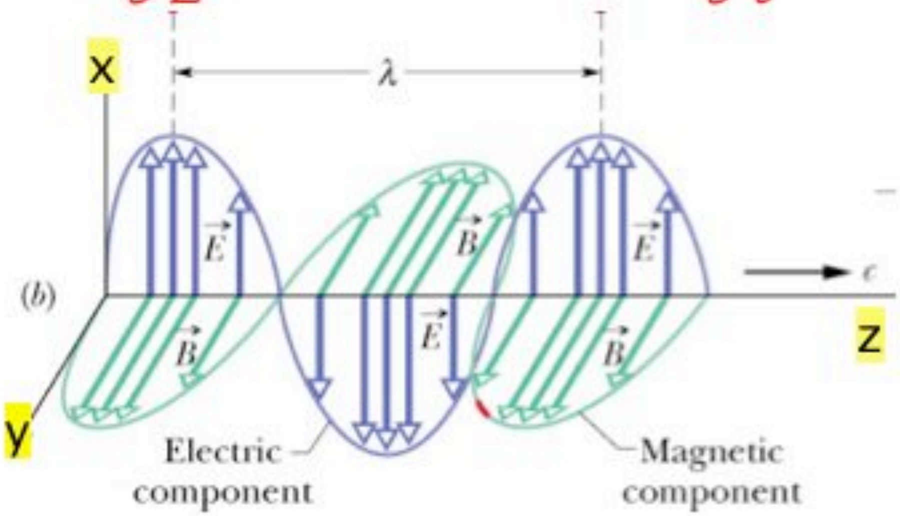


$$\frac{\partial^2 E_x(z,t)}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E_x(z,t)}{\partial t^2}$$

Also we get

$$\frac{\partial^2 B_y(z,t)}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 B_y(z,t)}{\partial t^2}$$

showing that both E_x and B_y obey the wave equation



We know that the coefficient of the time derivative is related to the wave's velocity of propagation $\mu_0\epsilon_0 = 1/v^2$. Inserting the known values of $\mu_0\epsilon_0$, one gets

$$v^2 = \frac{1}{\mu_0\epsilon_0} = (3.00 \times 10^8 m/s)^2$$

This is the speed of light, c!

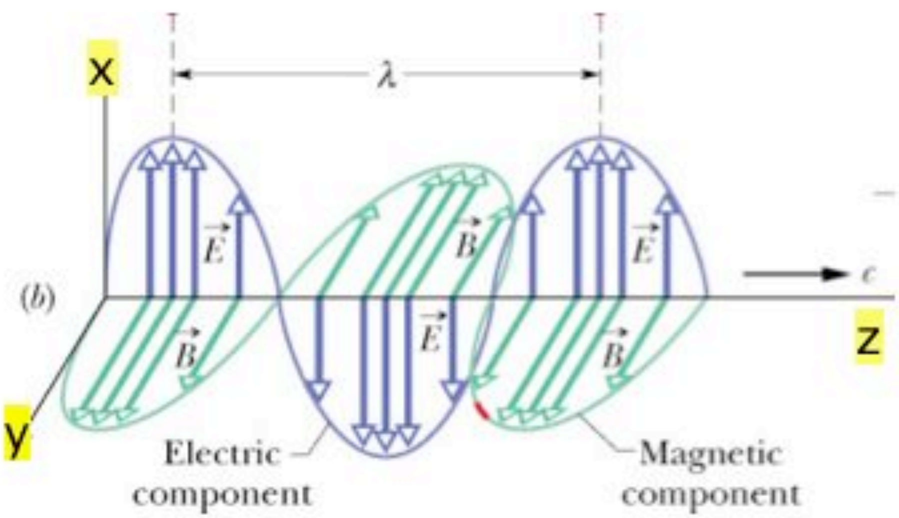
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Wave Equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\vec{E} = E_x(z,t)\hat{x} = E_0 \cos(kz - \omega t)\hat{x}$$
$$\vec{B} = B_y(z,t)\hat{y} = B_0 \cos(kz - \omega t)\hat{y}$$

where $k = \frac{2\pi}{\lambda}$ and the angular frequency $\omega = kv = 2\pi\frac{v}{\lambda} = 2\pi f$



This turns out to be the general relationship, $E = cB$

Summarize important features of electromagnetic waves

1. The wave is transverse since both E and B fields are perpendicular to the direction of propagation, which points in the direction of the cross product, $E \times B$

2. The E and B fields are perpendicular to each other. Therefore, their dot product vanishes, $\vec{E} \cdot \vec{B} = 0$

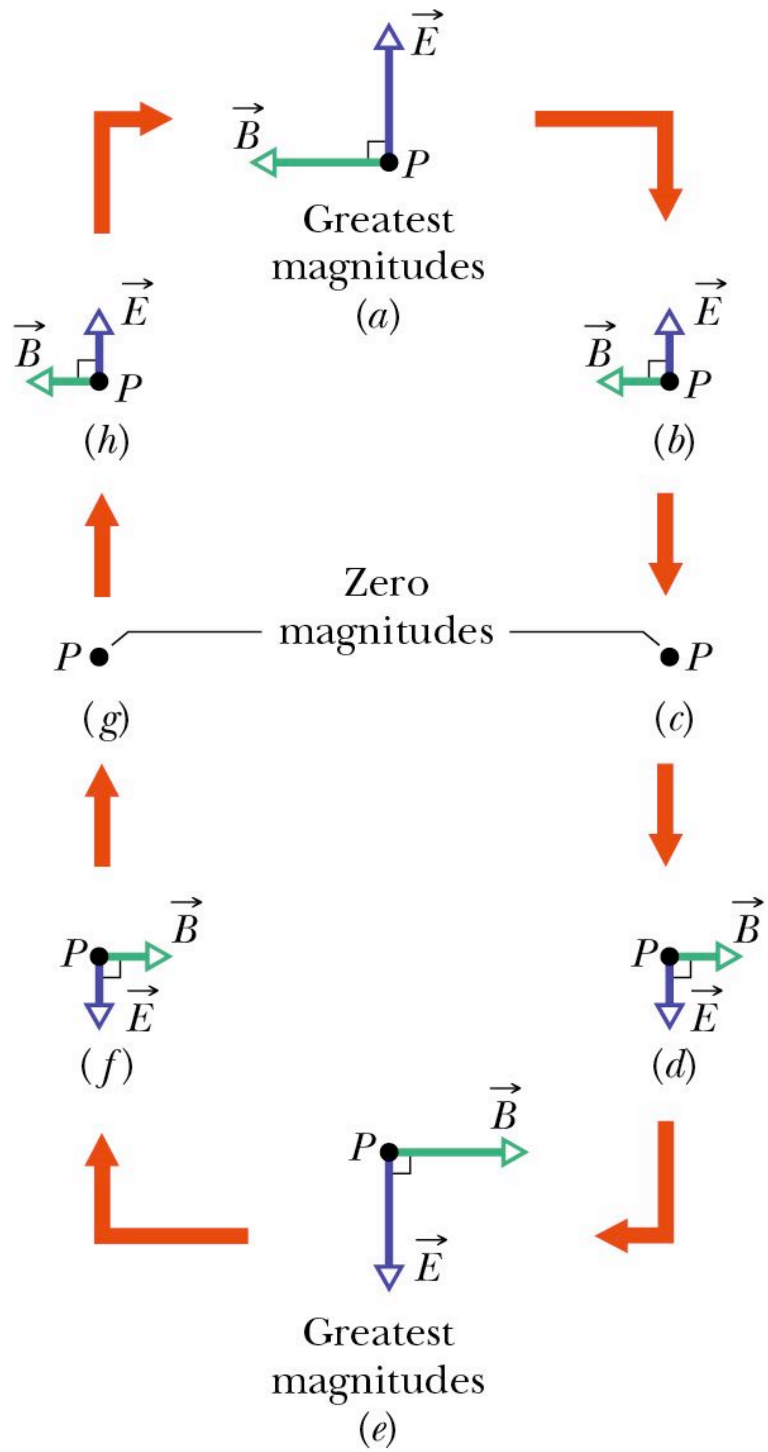
3. The ratio of the magnitudes and the amplitudes of the fields

$$\frac{E}{B} = \frac{E_0}{B_0} = \frac{\omega}{k} = c$$

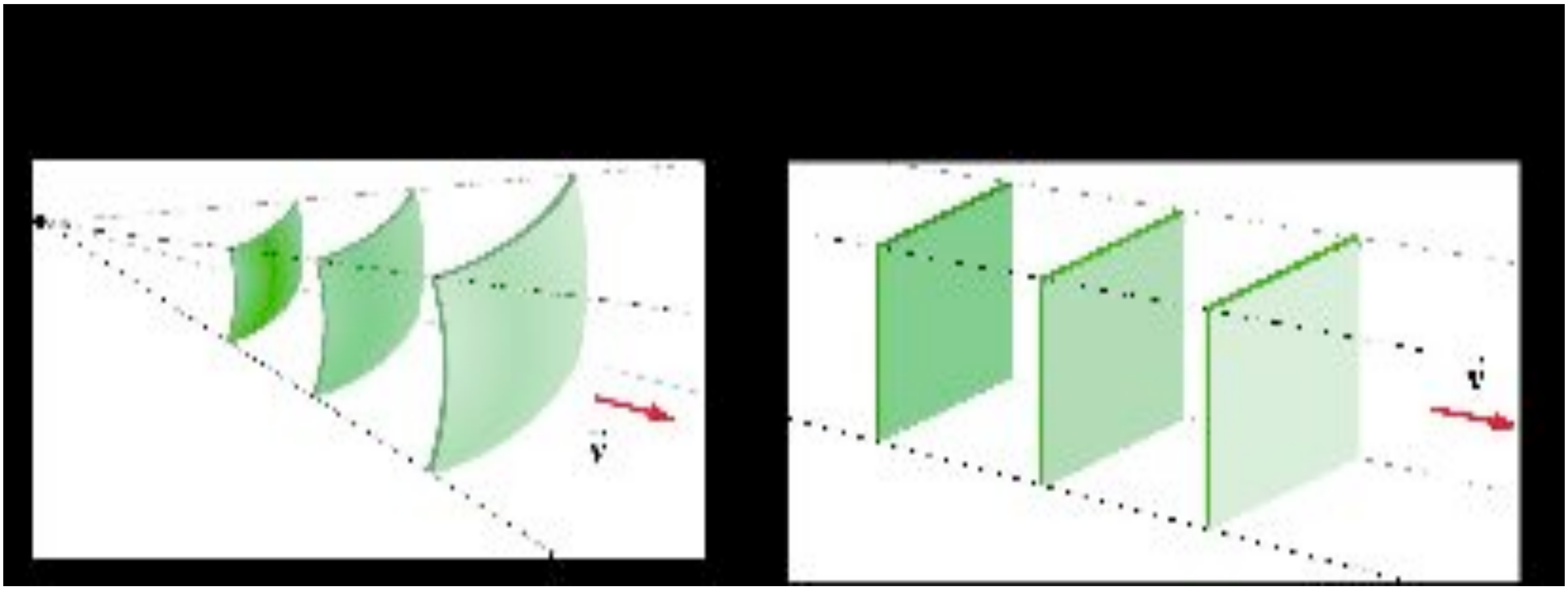
4. The speed of propagation in vacuum is equal to the speed of light, $c = 1/\sqrt{\mu_0 \epsilon_0}$.

5. Electromagnetic waves obey the superposition principle.

How the fields vary at a Point P in space as the wave goes by

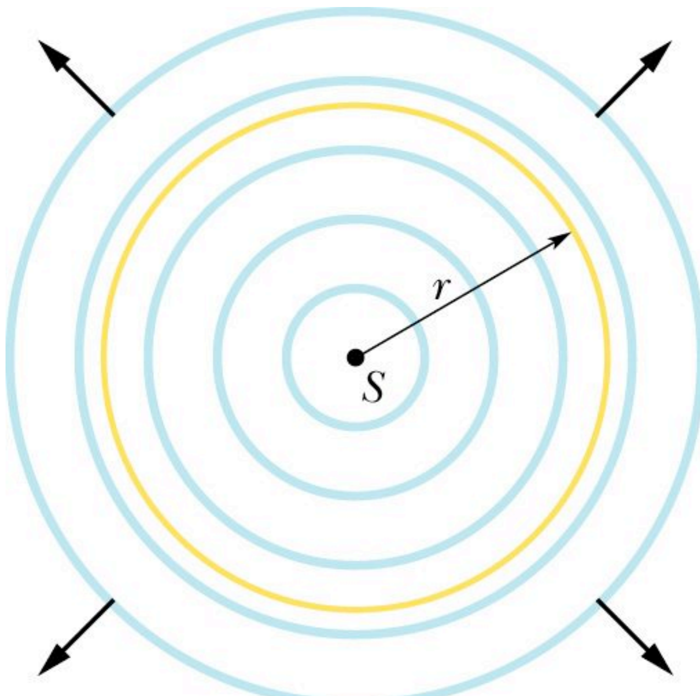


Spherical waves Plane waves



Spherical Waves

A point source of light generates a spherical wave. Light is emitted isotropically and the intensity of it falls off as $1/r^2$



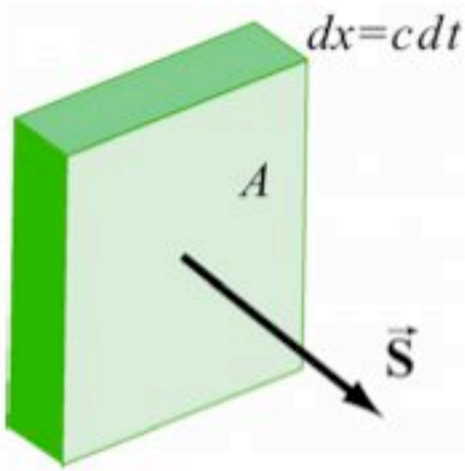
Let P be the power of the source in joules per sec. Then the intensity of light at a distance r is

$$I = \frac{P}{4\pi r^2}$$

What do we mean by Intensity of light?

Poynting Vector

We saw previously that the electric and magnetic fields store energy. Thus energy can be carried by the electromagnetic wave that consists of both fields.



A plane e/m wave passing through a small volume element of Area A and thickness dx

$$dU = uAdx = (u_E + u_B)Adx = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)Adx$$

where u_E and u_B are $\frac{1}{2}\epsilon_0 E^2$ and $\frac{B^2}{2\mu_0}$

Since the electromagnetic wave propagates with the speed of light c, the amount of time it takes for the wave to move through the volume element is $dt = dx/c$. Thus, the rate of change of energy per unit area, denoted with the symbol S, as

$$S = \frac{dU}{Adt} = \frac{c}{2}\left(\epsilon_0 E^2 + \frac{B^2}{2\mu_0}\right)$$

The SI unit of S is W/m^2

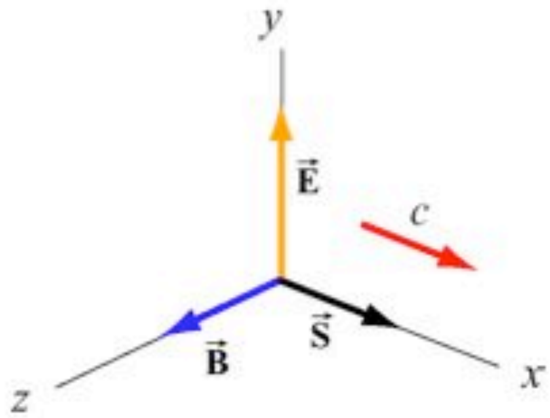
Recall that $E = cB$ and $c = 1/\sqrt{\mu_0\epsilon_0}$ we can write

$$S = \frac{c}{2}\left(\epsilon_0 E^2 + \frac{B^2}{2\mu_0}\right) = c\epsilon_0 E^2 = \frac{EB}{\mu_0}$$

More generally, the rate of the energy flow per unit area may be described by the Poynting vector S (after the British physicist John Poynting), which is defined as

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

S points in the direction of propagation



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

S points in the direction of propagation

The energy density and the Poynting vector vary with time. We then introduce the intensity of the wave, I, defined as the time average of S, is given by

$$I = \langle S \rangle = \frac{E_0 B_0}{\mu_0}\left\langle \cos^2(kx - \omega t)\right\rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$$

We can relate intensity to the energy density

$$\left\langle \cos^2(kx - \omega t)\right\rangle = \frac{1}{2}$$

17. The maximum electric field at a distance of 10 m from an isotropic point light source is 2.0 V/m. Calculate

(a) the maximum value of the magnetic field and

(b) the average intensity of the light there?

(c) What is the power of the source?

(a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\frac{V}{m}}{2.998\times 10^8\,\frac{m}{s}} = 6.7\times 10^{-9}\,T$$

(b) The average intensity is

$$I_{avg} = \frac{E^2{}_m}{2\mu_0 c} = \frac{\left(2.0\,\frac{V}{m}\right)^2}{2\left(4\pi\times 10^{-7}\,T\cdot\frac{V}{m}\right)\left(2.998\times 10^8\,\frac{m}{s}\right)} = 5.3\times 10^{-3}\,\frac{W}{m^2}$$

(c) The power of the source is

$$P = 4\pi r^2 I_{avg} = 4\pi(10m)^2\left(5.3\times 10^{-3}\,\frac{W}{m^2}\right) = 6.7W$$

- So far, we have implicitly assumed that the waves are propagating in a vacuum. When going through a medium, the speed of propagation will be modified, according to :

$$v = \sqrt{\frac{1}{\mu\epsilon}} \equiv \frac{c}{n}$$

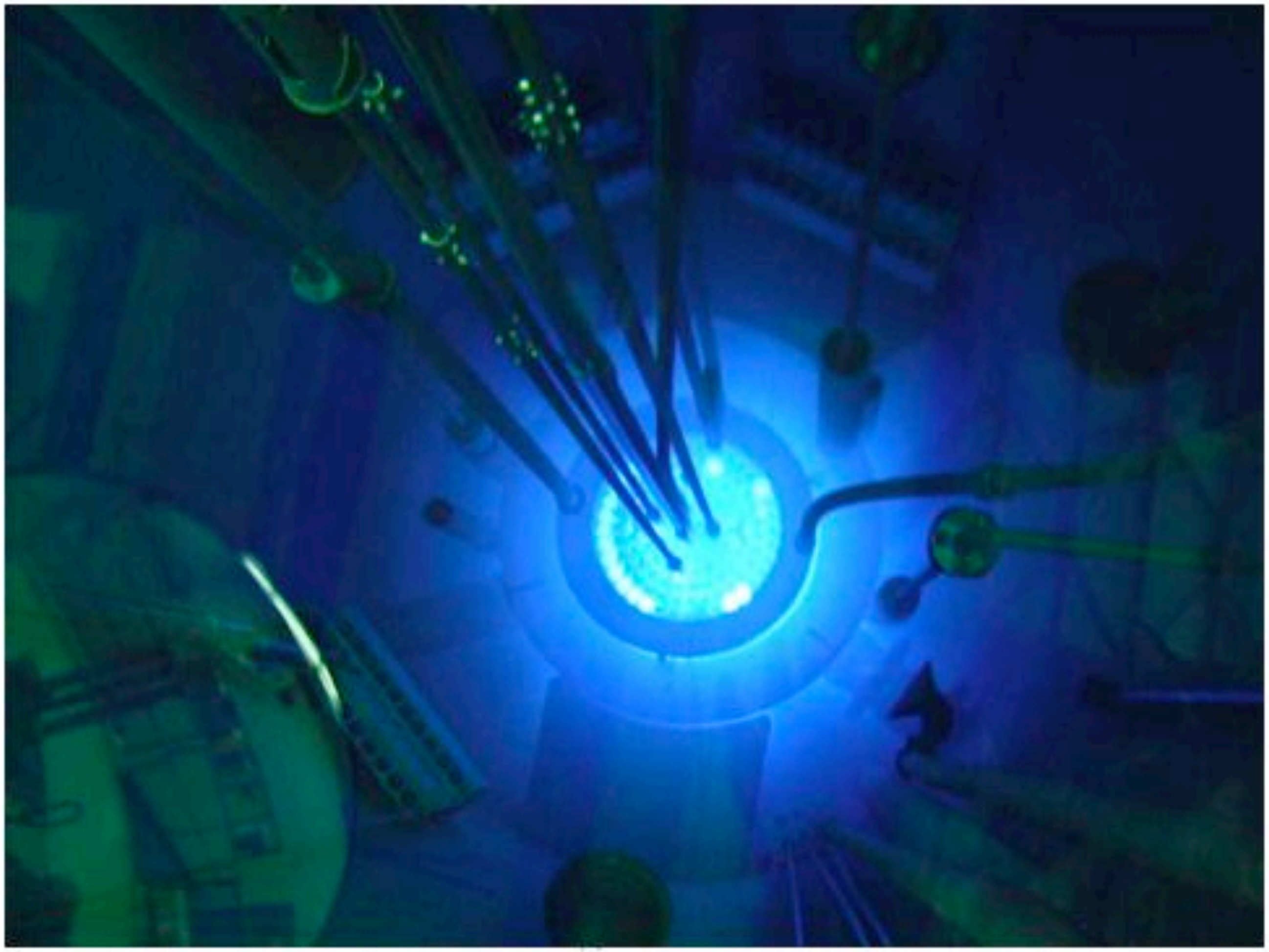
- This defines n, the index of refraction.

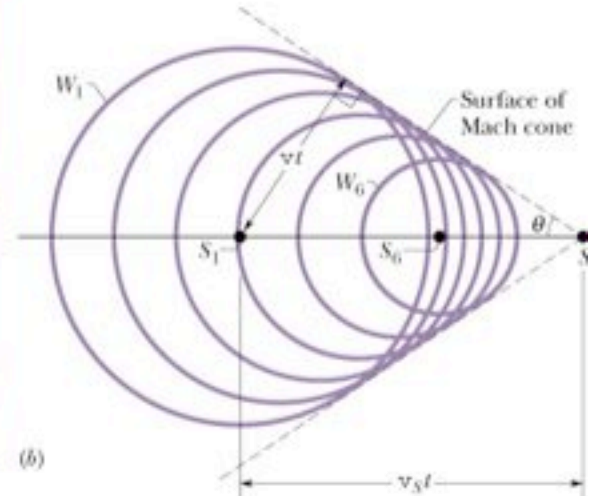
Index of Refraction

Material	Index
Vacuum	1.00000
Air at STP	1.00029
Ice	1.31
Water at 20 C	1.33
Acetone	1.36
Ethyl alcohol	1.36
Sugar solution(30%)	1.38
Fluorite	1.433
Fused quartz	1.46
Glycerine	1.473
Sugar solution (80%)	1.49
Typical crown glass	1.52
Crown glasses	1.52-1.62
Spectacle crown, C-1	1.523
Sodium chloride	1.54
Polystyrene	1.55-1.59
Carbon disulfide	1.63
Flint glasses	1.57-1.75
Heavy flint glass	1.65
Extra dense flint, EDF-3	1.7200
Methylene iodide	1.74
Sapphire	1.77
Rare earth flint	1.7-1.84
Lanthanum flint	1.82-1.98
Arsenic trisulfide glass	2.04
Diamond	2.417

Cerenkov radiation

Particles moving faster than the speed of light in the medium emit light at an angle related to the index of refraction





Particle identification: particles with the same momentum but different masses have different velocities. Those who v exceeds c/n, emit Cerenkov radiation

Another property of light

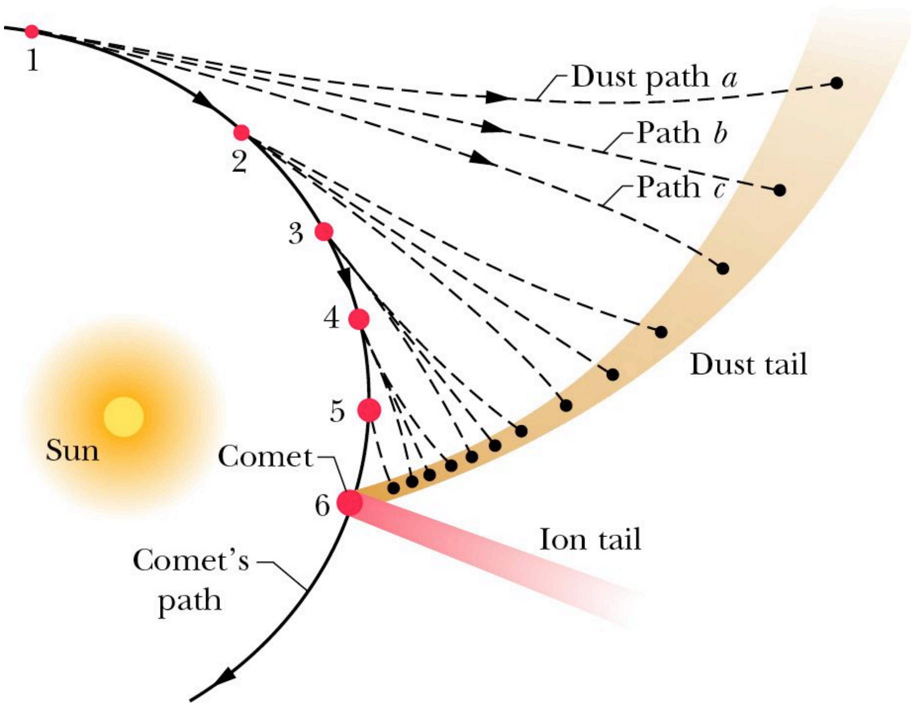
Radiation pressure: Light carries momentum

$$P_r = \frac{I}{c}$$

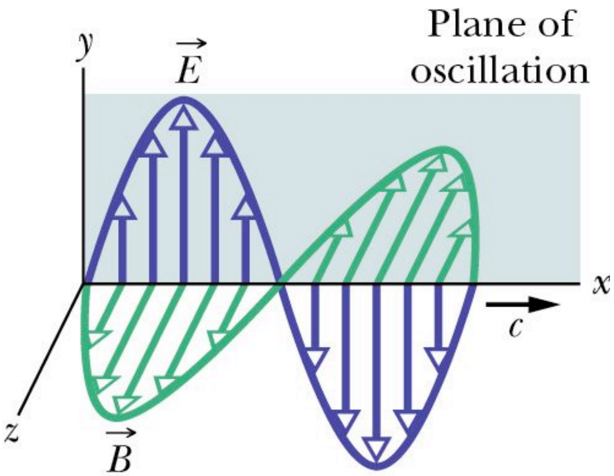
This is the force per unit area felt by an object that absorbs light. (Black piece of paper))

$$P_r = \frac{2I}{c}$$

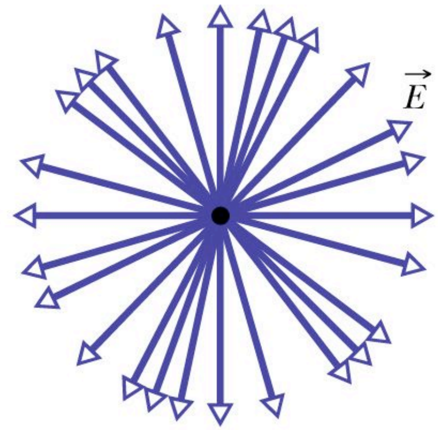
This is the force per unit area felt by an object that reflects light backwards. (Aluminum foil)



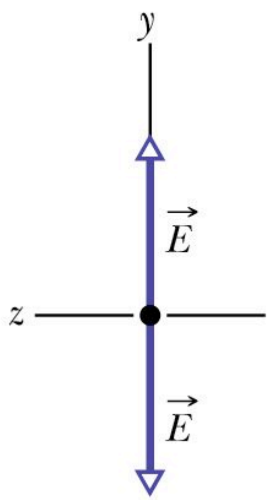
Polarization of light



(a)

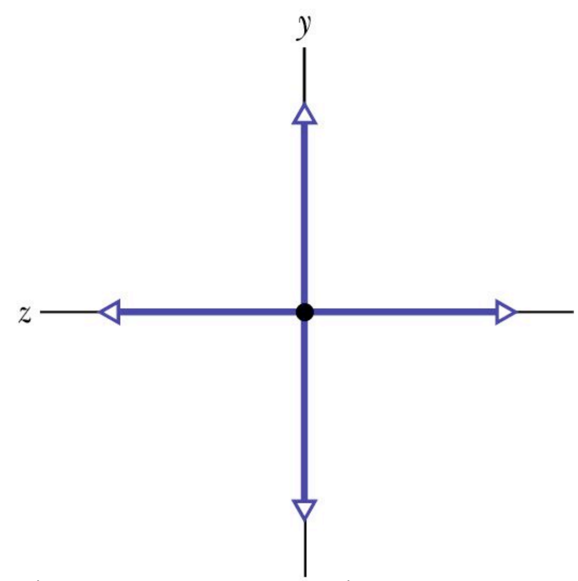


(a)



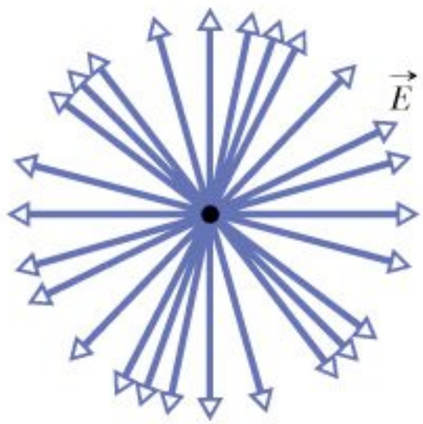
(b)

Pass though a polarizing sheet
aligned to pass only the y-component



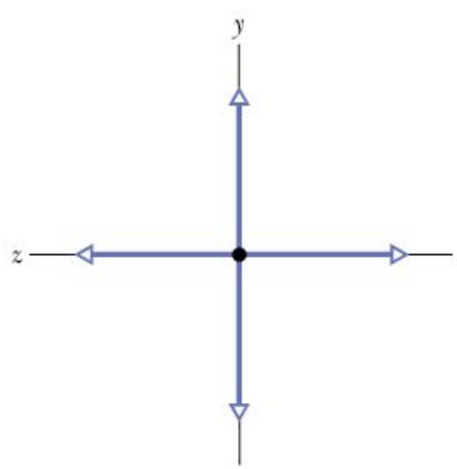
(b)

Resolved into its y and z-components
The sum of the y-components and
z components are equal



(a)

Intensity I_0



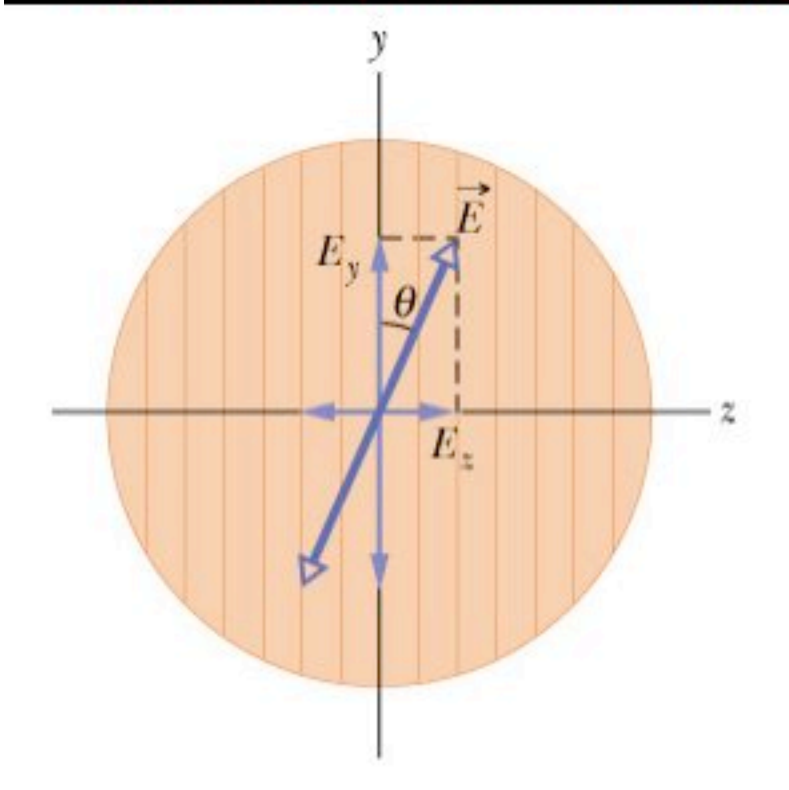
(b)

Pass though a polarizing sheet aligned to pass only the y-component
Malus's Law

$$I = \frac{I_0}{2}$$

One Half Rule
Half the intensity out

Suppose light incident on polarizing sheet is already polarized



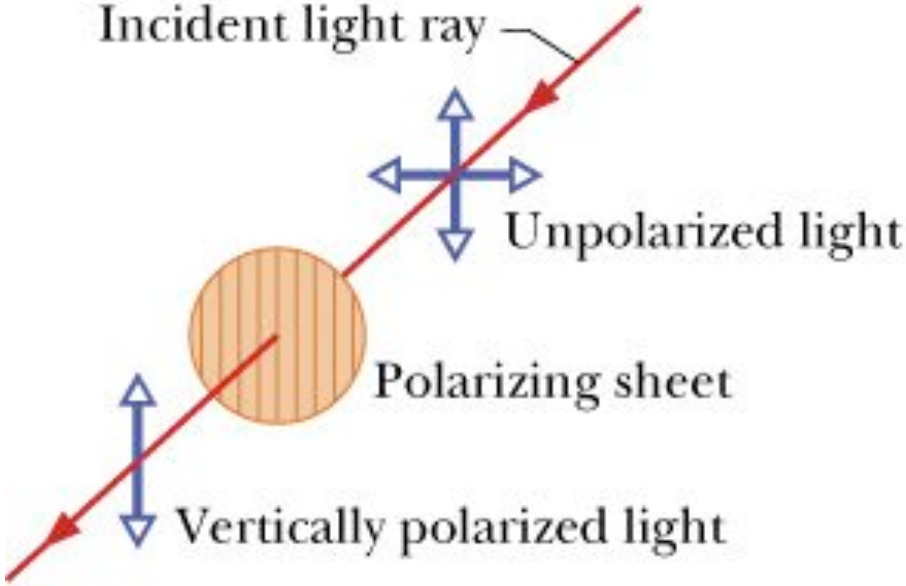
Resolve E along y-axis and z-axis, y component is passed

$$E_y = E\cos\theta$$

$$I = \frac{E^2}{c\mu_0}$$

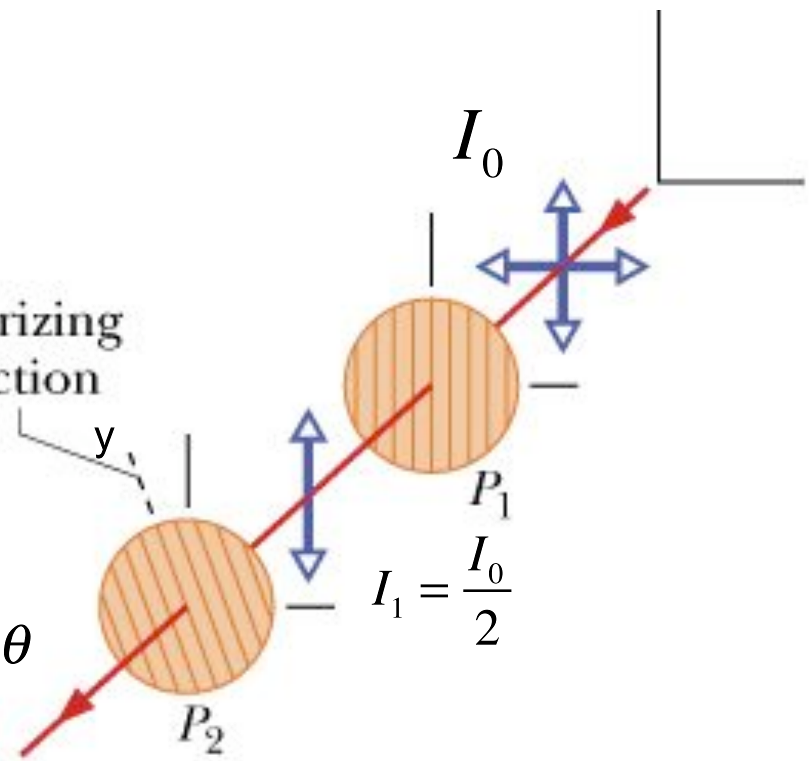
We can write $I/I_0 = \cos^2\theta$ or $I = I_0\cos^2\theta$

cosine-squared rule rule: can only be used when the incident light is polarized



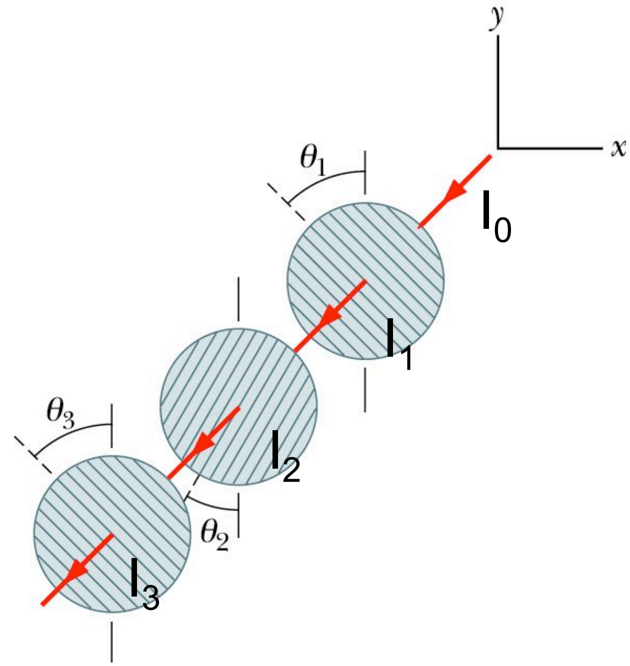
$$I \propto E_y^{\ 2}$$

$$E_y = E\cos\theta$$



$$I_2 = I_1 \cos^2\theta$$

35. In the figure, initially unpolarized light is sent through three polarizing sheets whose polarizing directions make angles of θ_1 = 40°, θ_2 = 20°, and θ_3 = 40° with the direction of the y axis. What percentage of the light's initial intensity is transmitted by the system? (Hint: Be careful with the angles.)



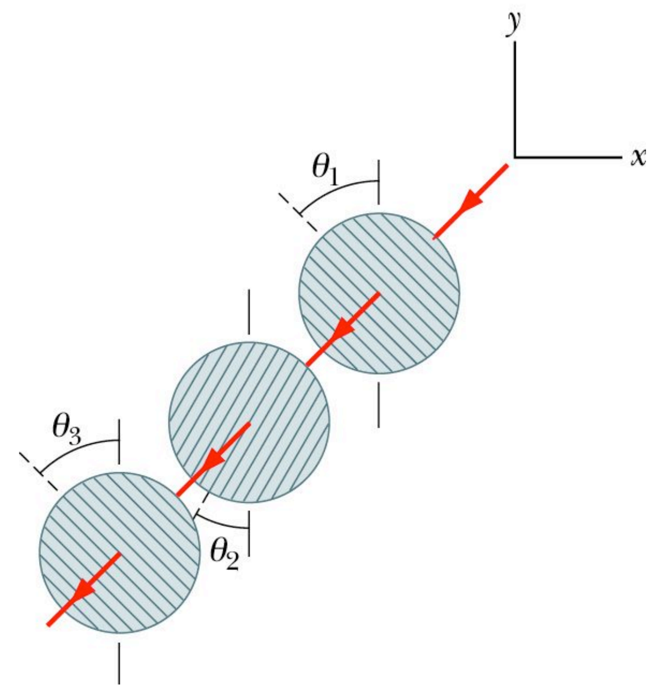
Let I_o be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity of is $I_1 = (1/2)I_0$, and the direction of polarization of the transmitted light is θ_1 = 40° counterclockwise from the y axis in the diagram. The polarizing direction of the second sheet is θ_2 = 20° clockwise from the y axis, so the angle between the direction of polarization that is incident on that sheet and the the polarizing direction of the sheet is 40° + 20° = 60°. The transmitted intensity is

$$I_2 = I_1 \cos^2 60^o = \frac{1}{2} I_0 \cos^2 60^o \quad ,$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis.

35. In the figure, initially unpolarized light is sent through three polarizing sheets whose polarizing directions make angles of θ_1 = 40°, θ_2 = 20°, and θ_3 = 40° with the direction of the y axis. What percentage of the light's initial intensity is transmitted by the system? (Hint: Be careful with the angles.)

The polarizing direction of the third sheet is θ_3 = 40° counterclockwise from the y axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is



$$I_3 = I_2 \cos^2 60^o$$

$$I_2 = I_1 \cos^2 60^o$$

$$I_1 = \frac{1}{2} I_0$$

$$\frac{I_3}{I_0} = \frac{1}{2} \cos^4 60 = 3.125 \times 10^{-2}$$

Thus, 3.1% of the light's initial intensity is transmitted.

Polarization can be produced by:

- Scattering
- Reflection

Light scattered from sky is partially polarized
Light scattered from your car hood is polarized in the plane of the hood.

See Brewsters Law

Light is also refracted when changing mediums