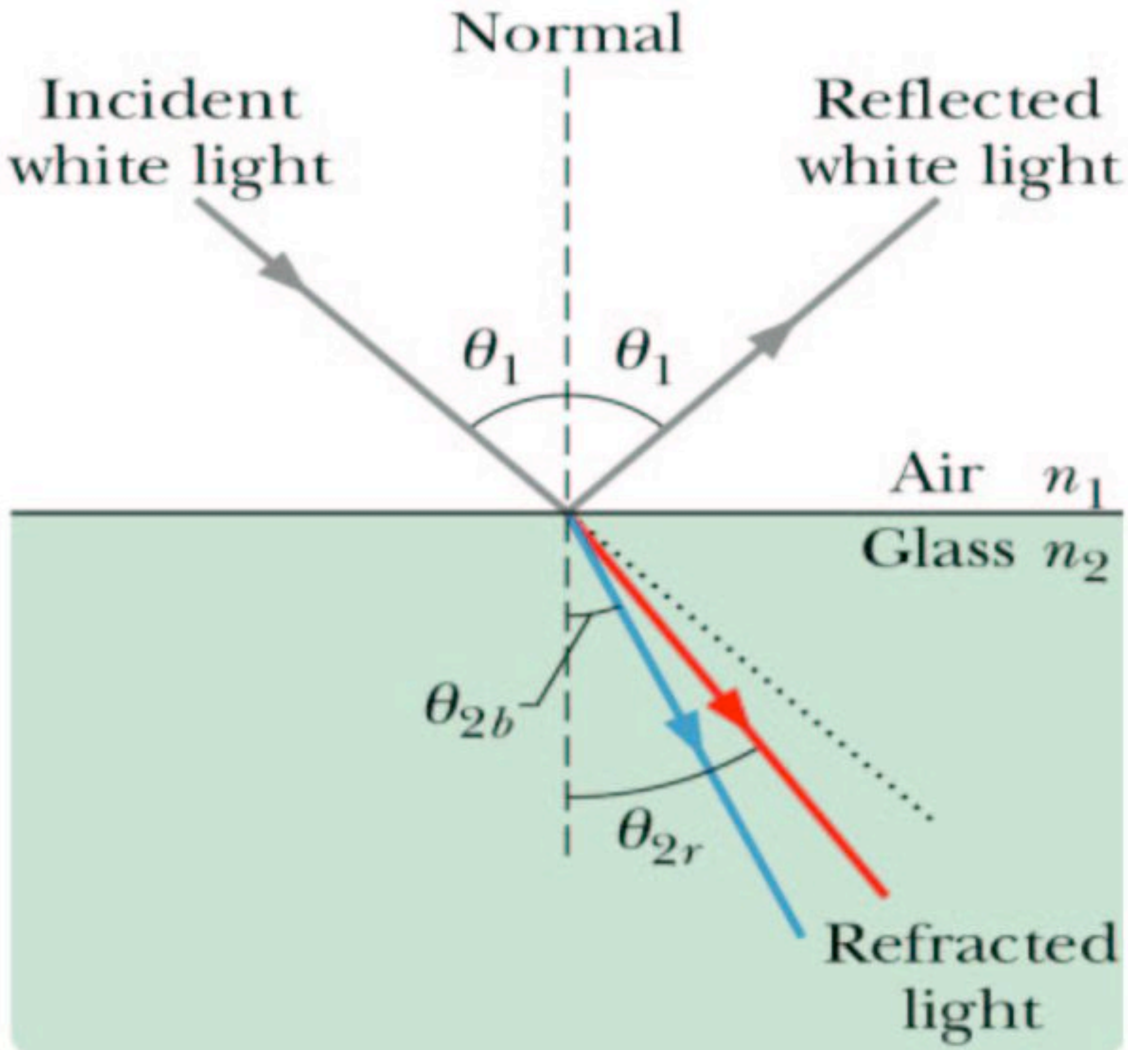
Lecture 14 Images Chapter 34

- Opening Demo
- Topics
  - Reflection
  - Refraction
  - Spherical mirror/Plane mirror comparison in forming image
  - Spherical refracting surfaces
  - Thin lenses
  - Optical Instruments

    Magnifying glass, Microscope, Refracting telescope
- Warm-up problem

Preliminary topics before mirrors and lenses

- Law of Reflection
- Dispersion
- Snell's Law
- Brewsters Angle



(a)

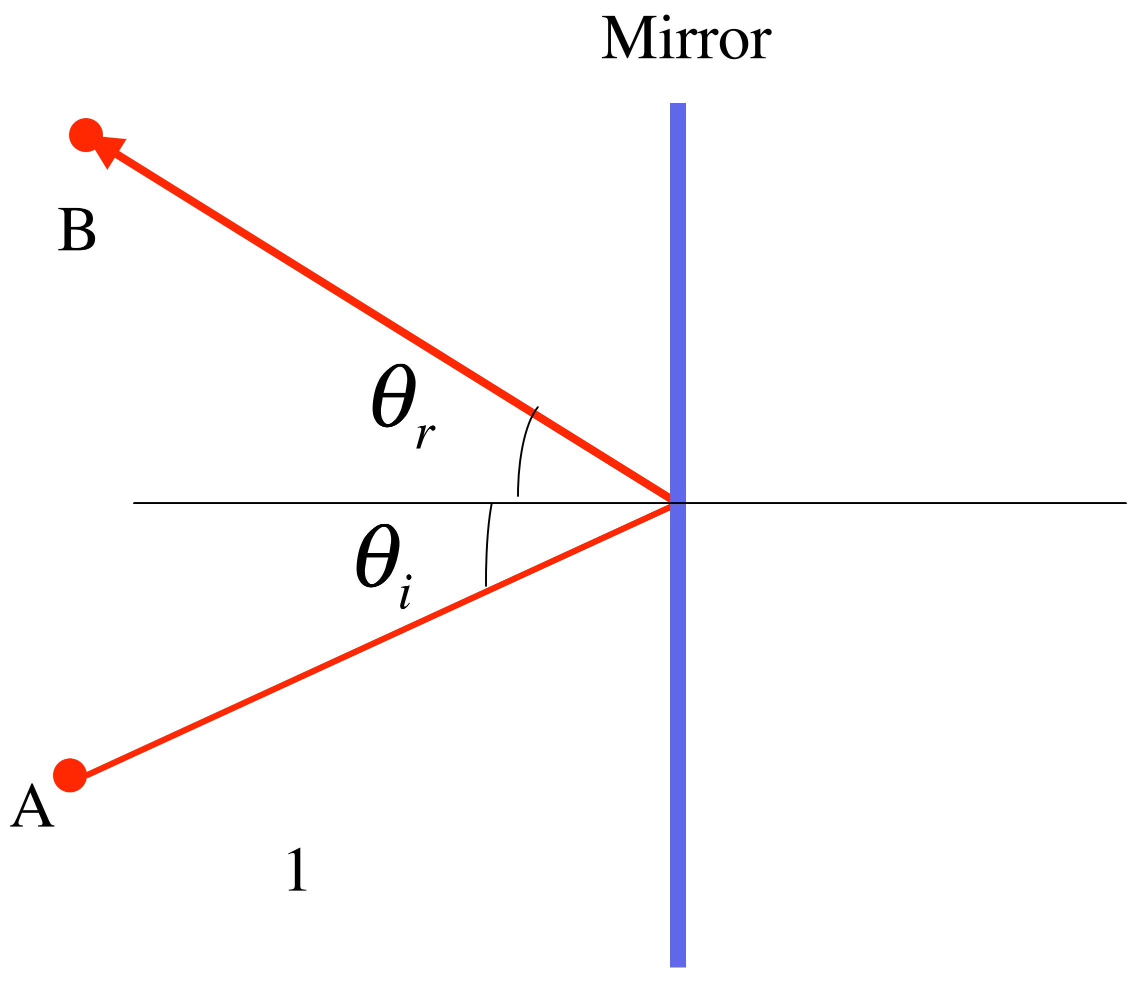
- Law of Reflection
- Dispersion
- Snell’s Law
- Brewsters Angle

Geometrical Optics: Study of reflection and refraction of light from surfaces

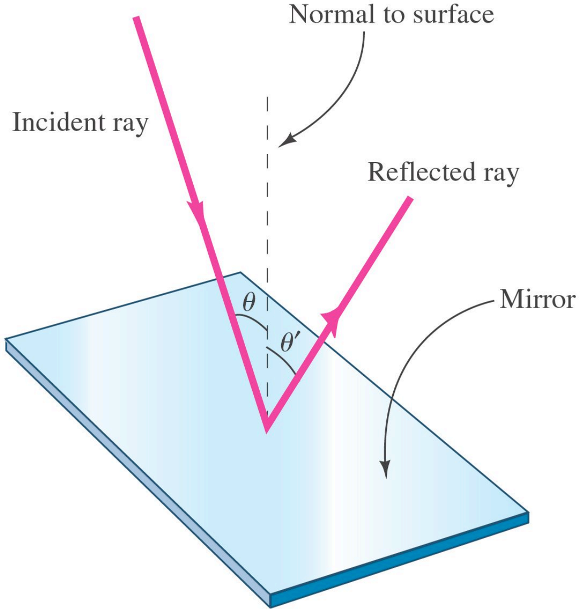
The ray approximation states that light travels in straight lines until it is reflected or refracted and then travels in straight lines again. The wavelength of light must be small compared to the size of the objects or else diffractive effects occur.

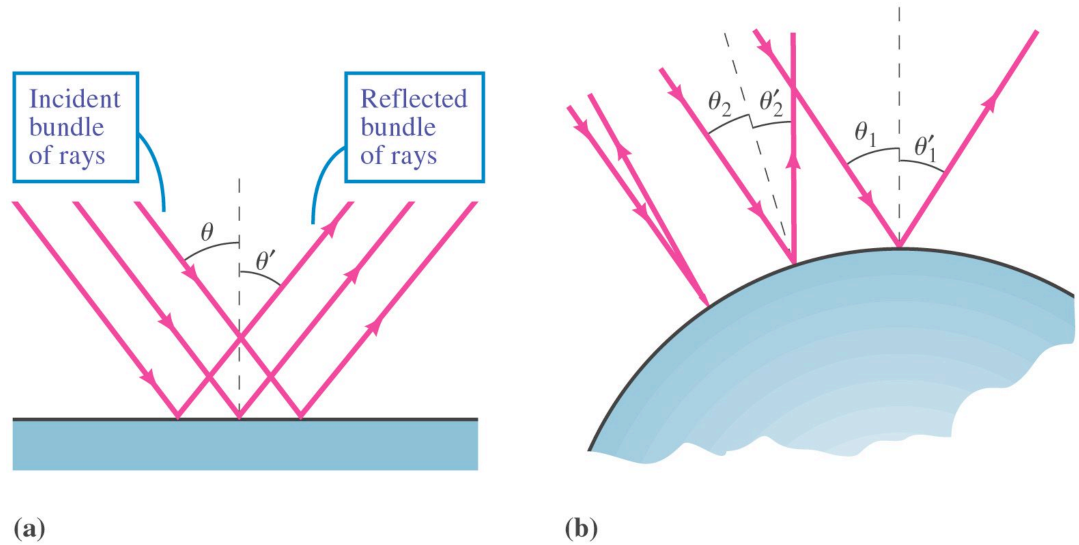
Law of Reflection

$$\theta_I = \theta_R$$



Drawing Normals

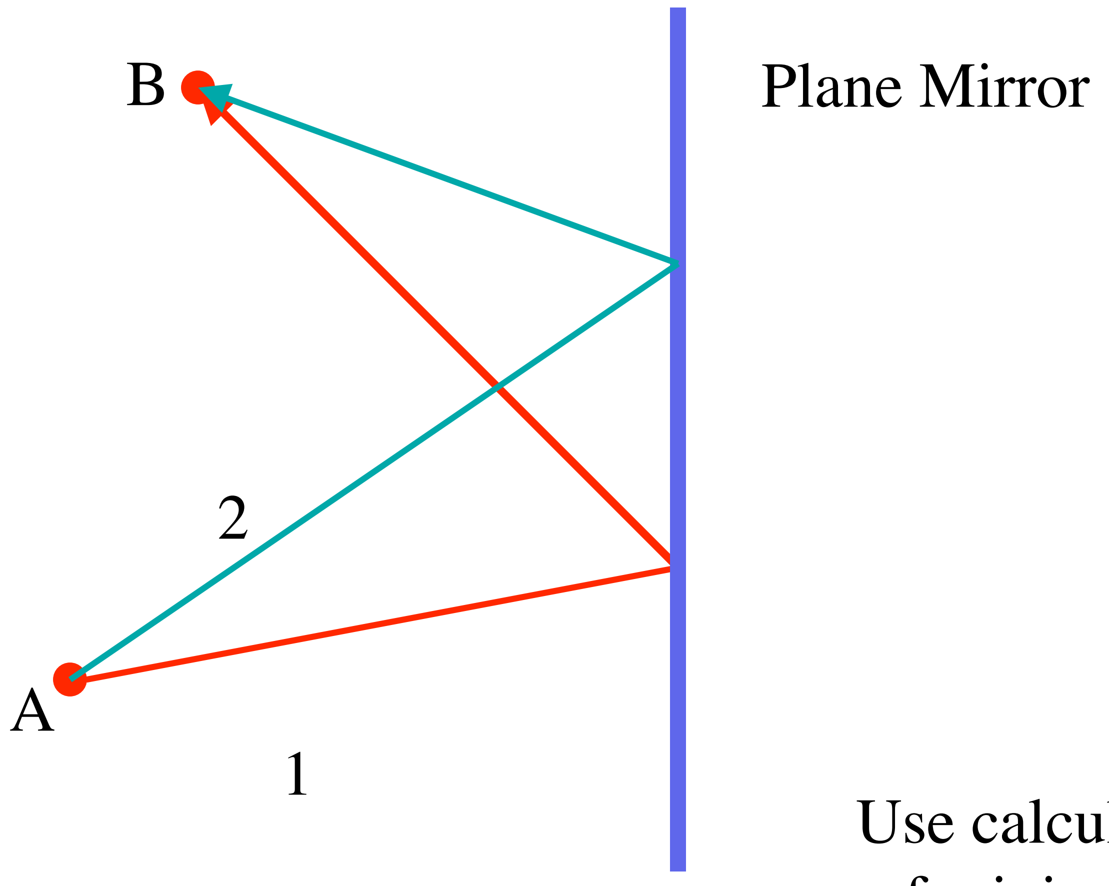




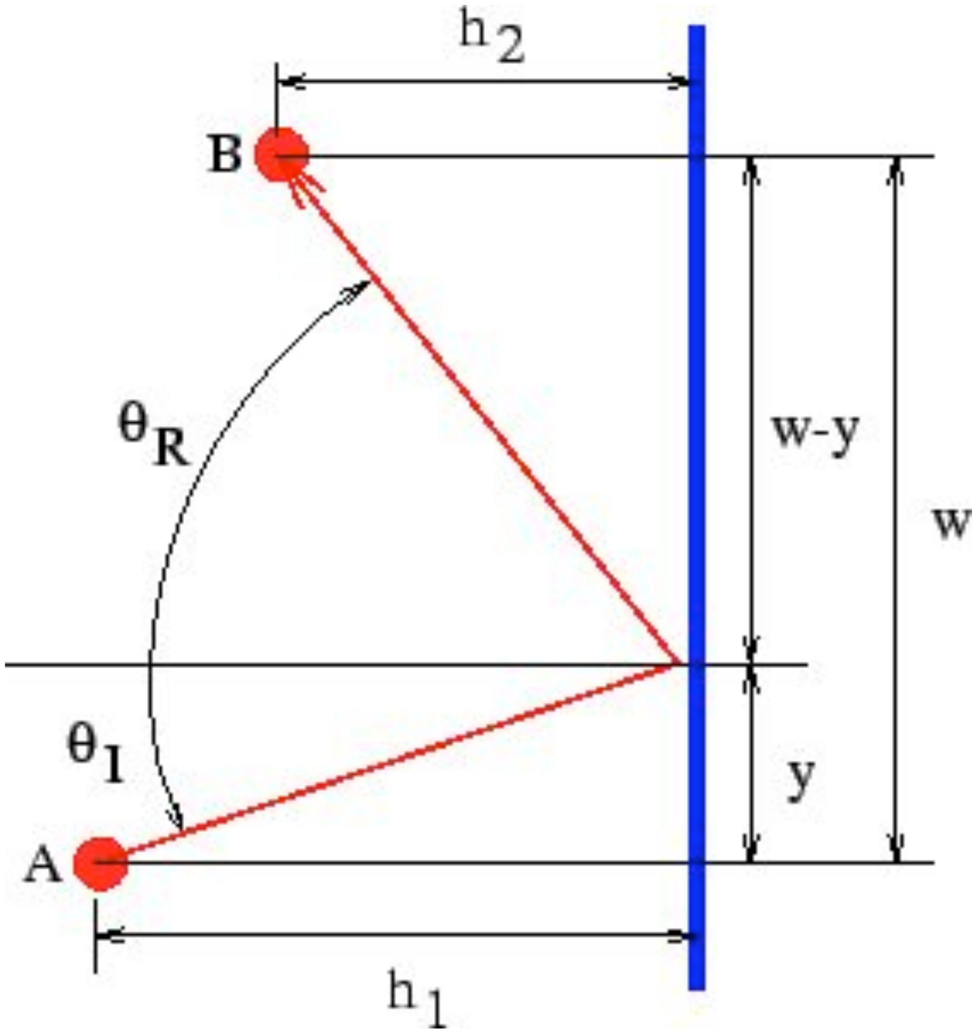
Fermat's Principle

Using Fermat's Principle you can prove the Reflection law. It states that the path taken by light when traveling from one point to another is the path that takes the shortest time compared to nearby paths.

Two light rays 1 and 2 taking different paths between points A and B and reflecting off a vertical mirror



Use calculus - method of minimization



$$t = \frac{1}{C}\left(\sqrt{h_1^2 + y^2} + \sqrt{h_2^2 + (w-y)^2}\right)$$

$$\frac{dt}{dy} = \frac{2y}{\sqrt{h_1^2 + y^2}} + \frac{-2(w-y)}{\sqrt{h_2^2 + (w-y)^2}} = 0$$

$$\frac{y}{\sqrt{h_1^2 + y^2}} = \frac{(w-y)}{\sqrt{h_2^2 + (w-y)^2}}$$

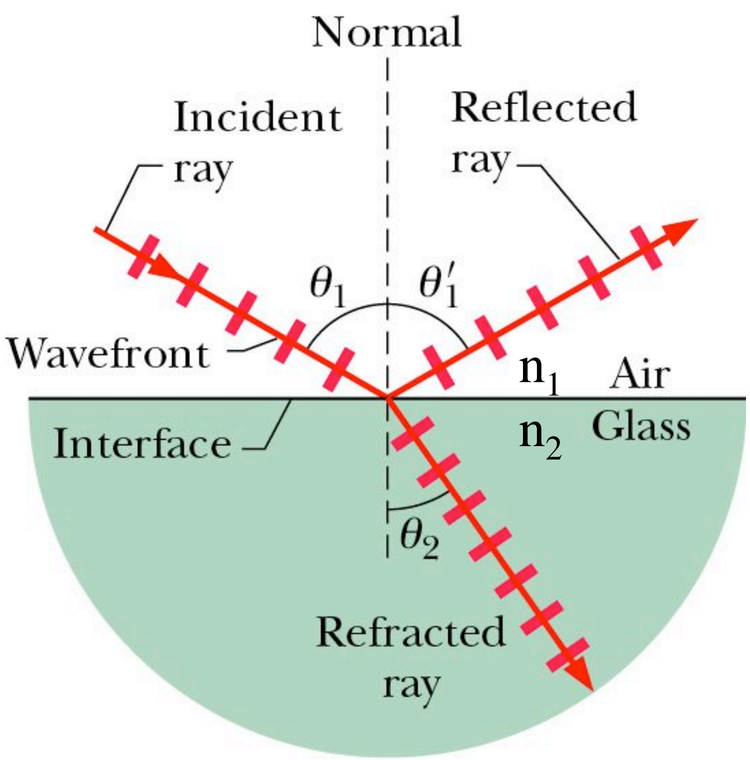
$$\sin\theta_I = \sin\theta_R$$

$$\theta_I = \theta_R$$

Write down time as a function of y and set the derivative to 0.

Law of Refraction: Snells Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

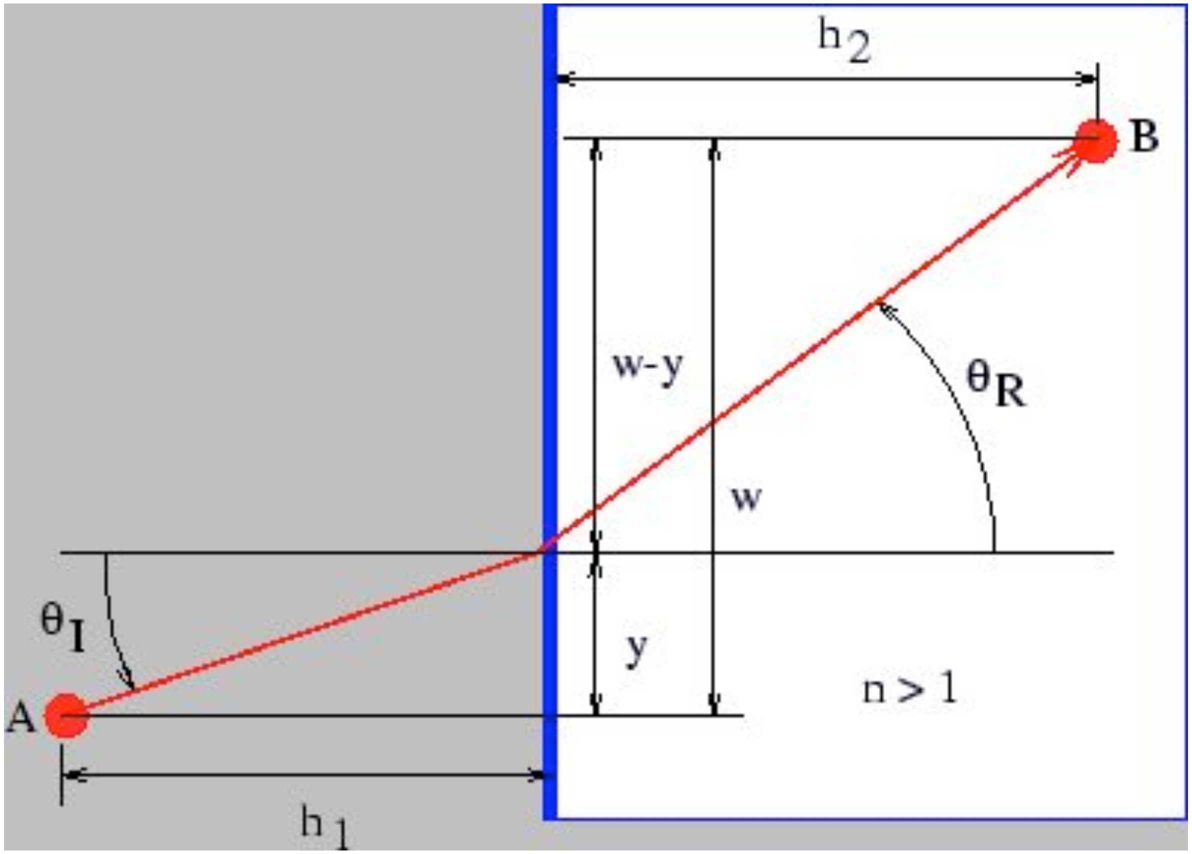


(*b*)

How do we prove it?

$$\frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2$$

Air	1.0
Glass	1.33



$$t = \left(\frac{1}{v_1}\sqrt{h_1^2 + y^2} + \frac{1}{v_2}\sqrt{h_2^2 + (w-y)^2}\right)$$

$$\frac{dt}{dy} = 0$$

$$\frac{1}{v_1}\sin\theta_I = \frac{1}{v_2}\sin\theta_R$$

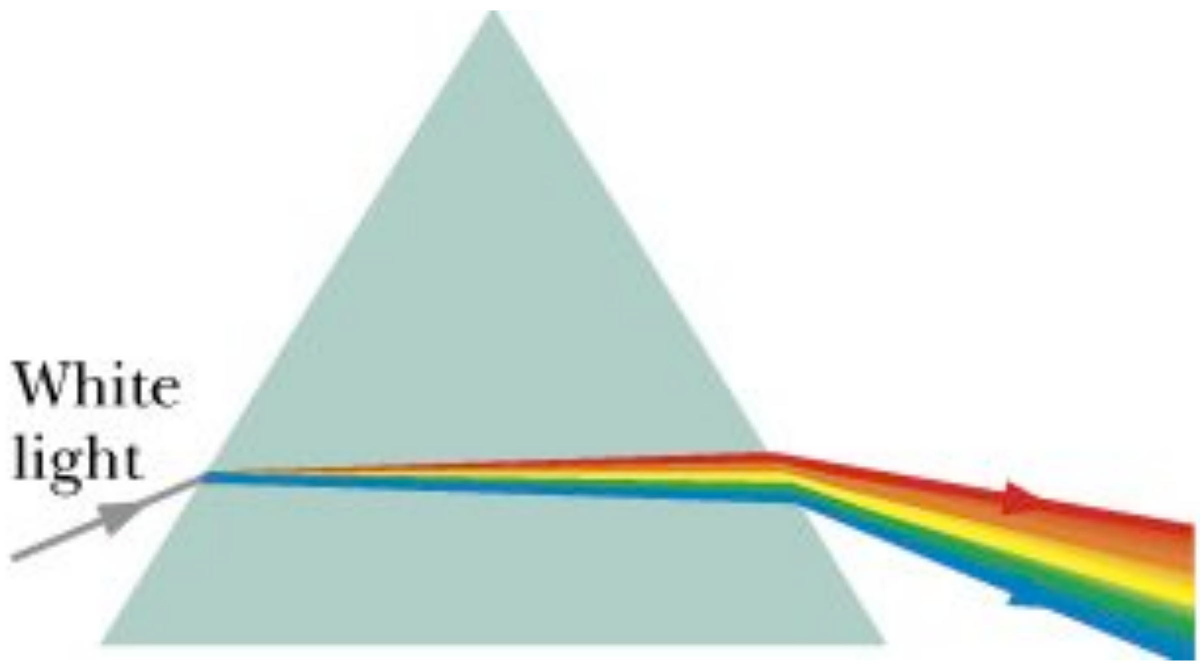
$$n_1 \sin\theta_I = \mathrm{n}_2 \sin\theta_R$$

JAVA APPLET

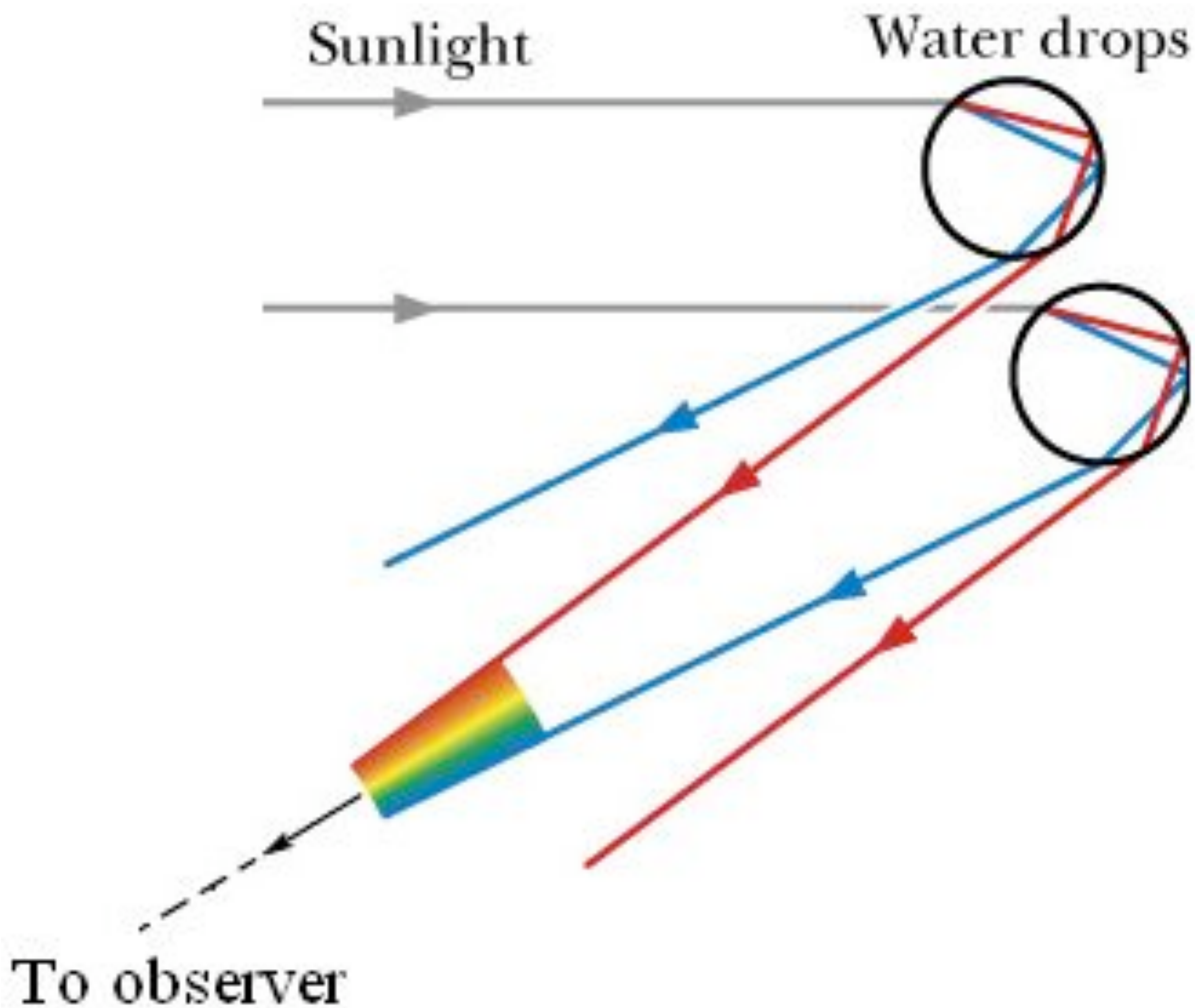
Show Fermat’s principle simulator

Dispersion

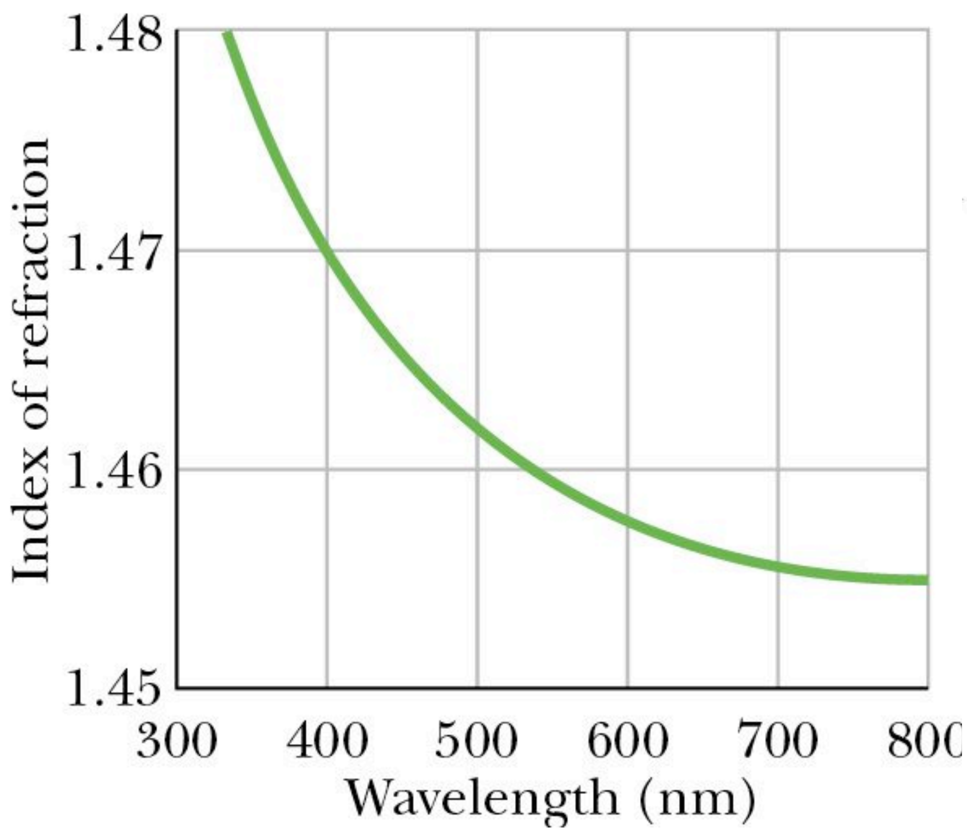
What allows you to see various colors when white light passes through a prism

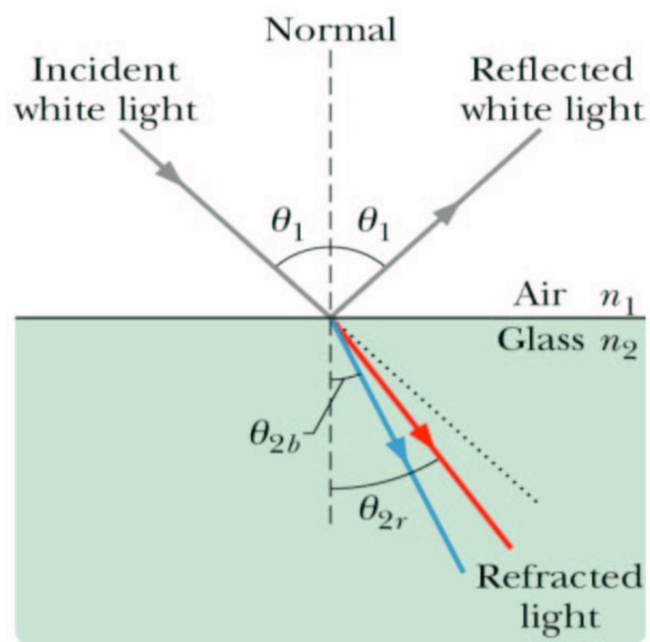


How does a Rainbow work?



Dispersion: Different wavelengths have different velocities and therefore different indices of refraction. This leads to different refractive angles for different wavelengths. Thus the light is dispersed. The frequency dose not change when n changes.





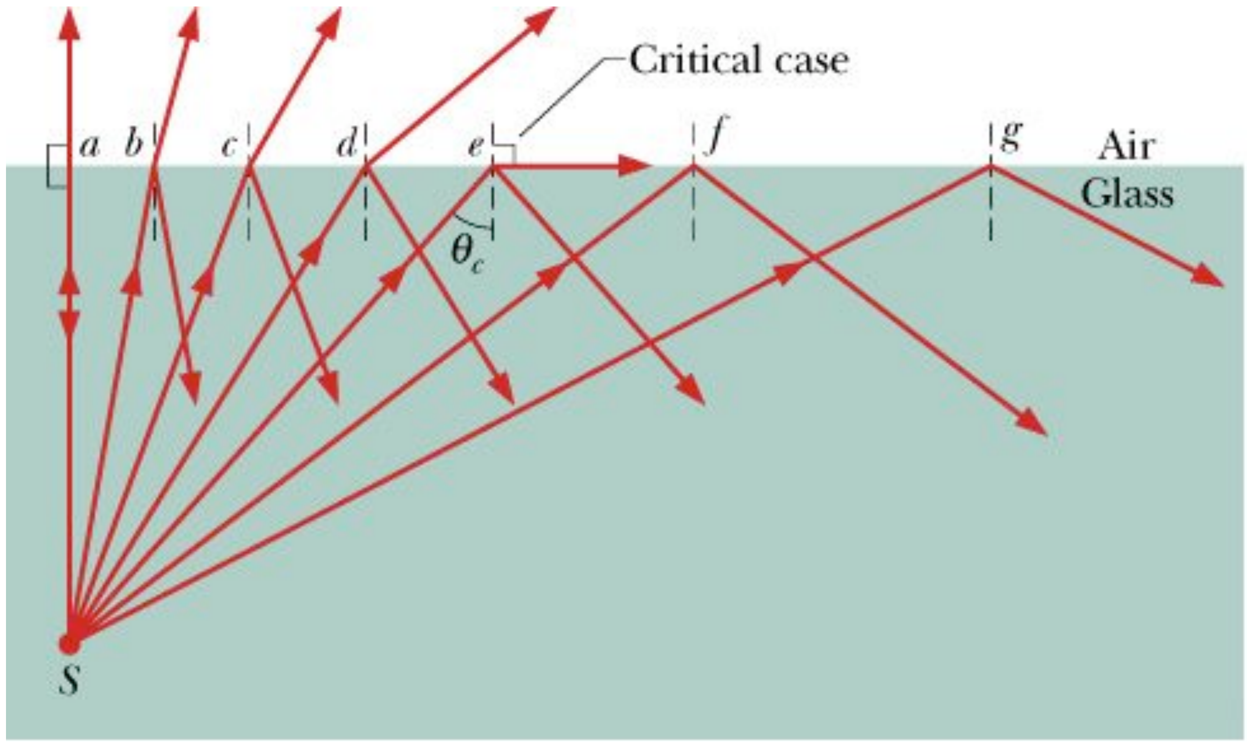
(a)

$v = f\lambda$

λ changes when medium changes

f does not change when medium changes

Why is light totally reflected inside a fiber optics cable? Internal reflection



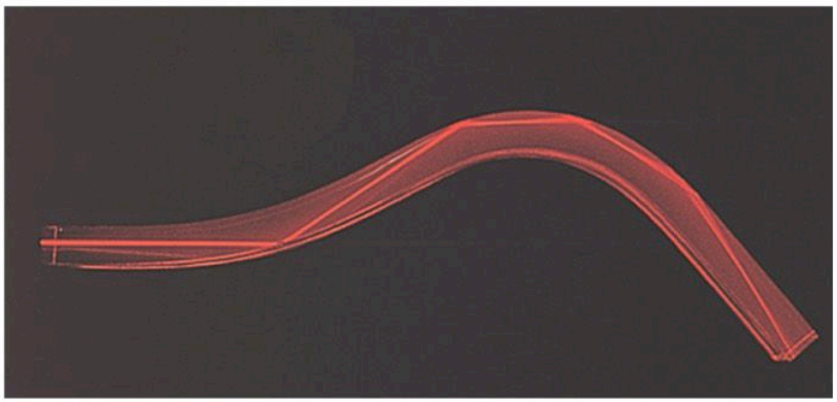
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$(1.33)\sin\theta_1 = (1.00)\sin 90 \;\; = 1.00$$

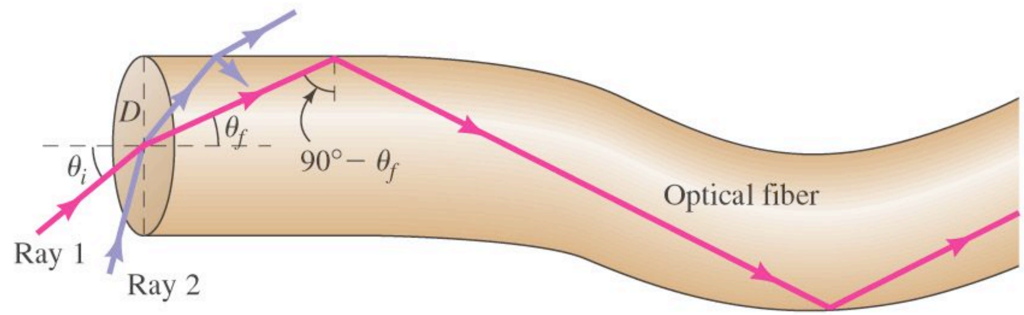
When $\theta_1 \geq \sin^{-1}\frac{1}{1.33} \geq 48.75$ deg

light won't get out of the material

Fiber Cable



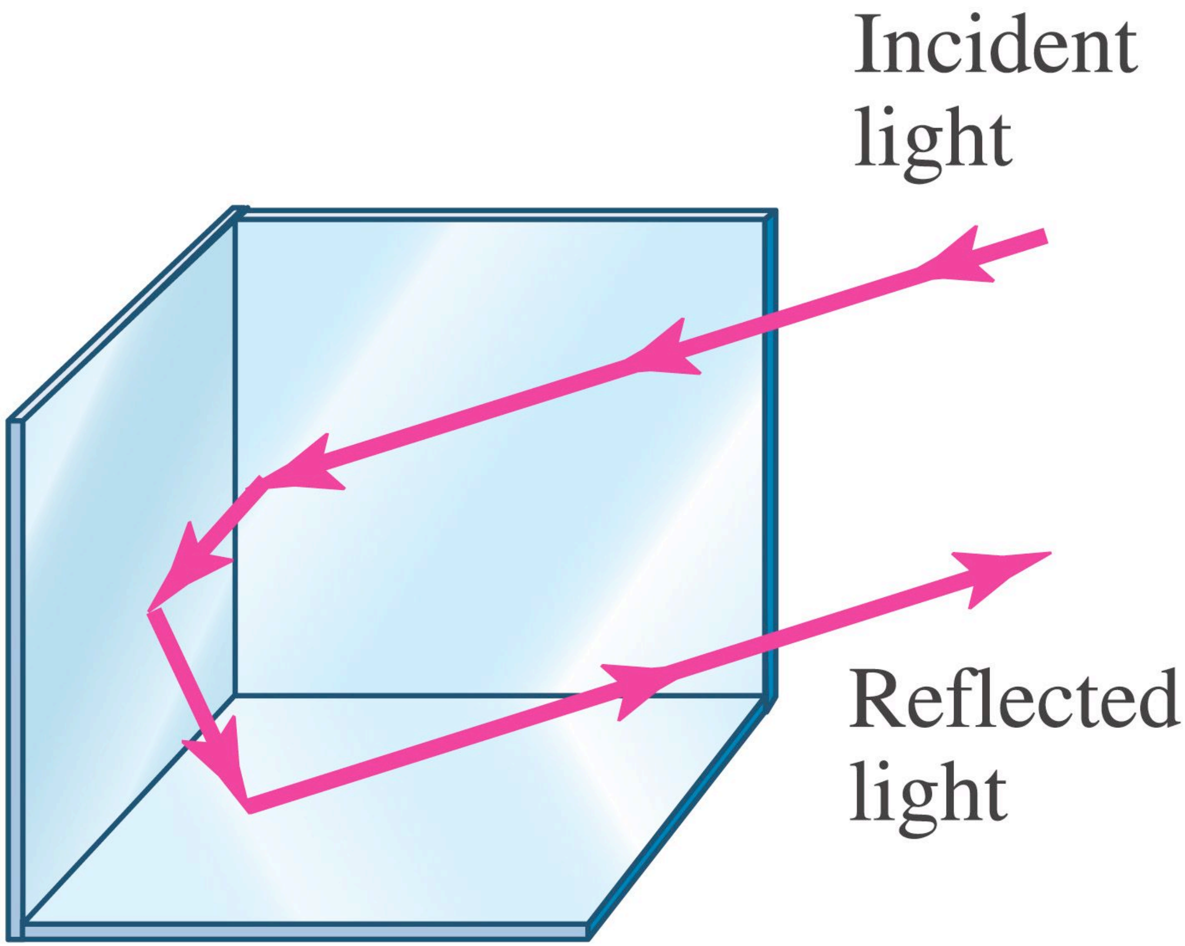
(a)



(b)

Same here

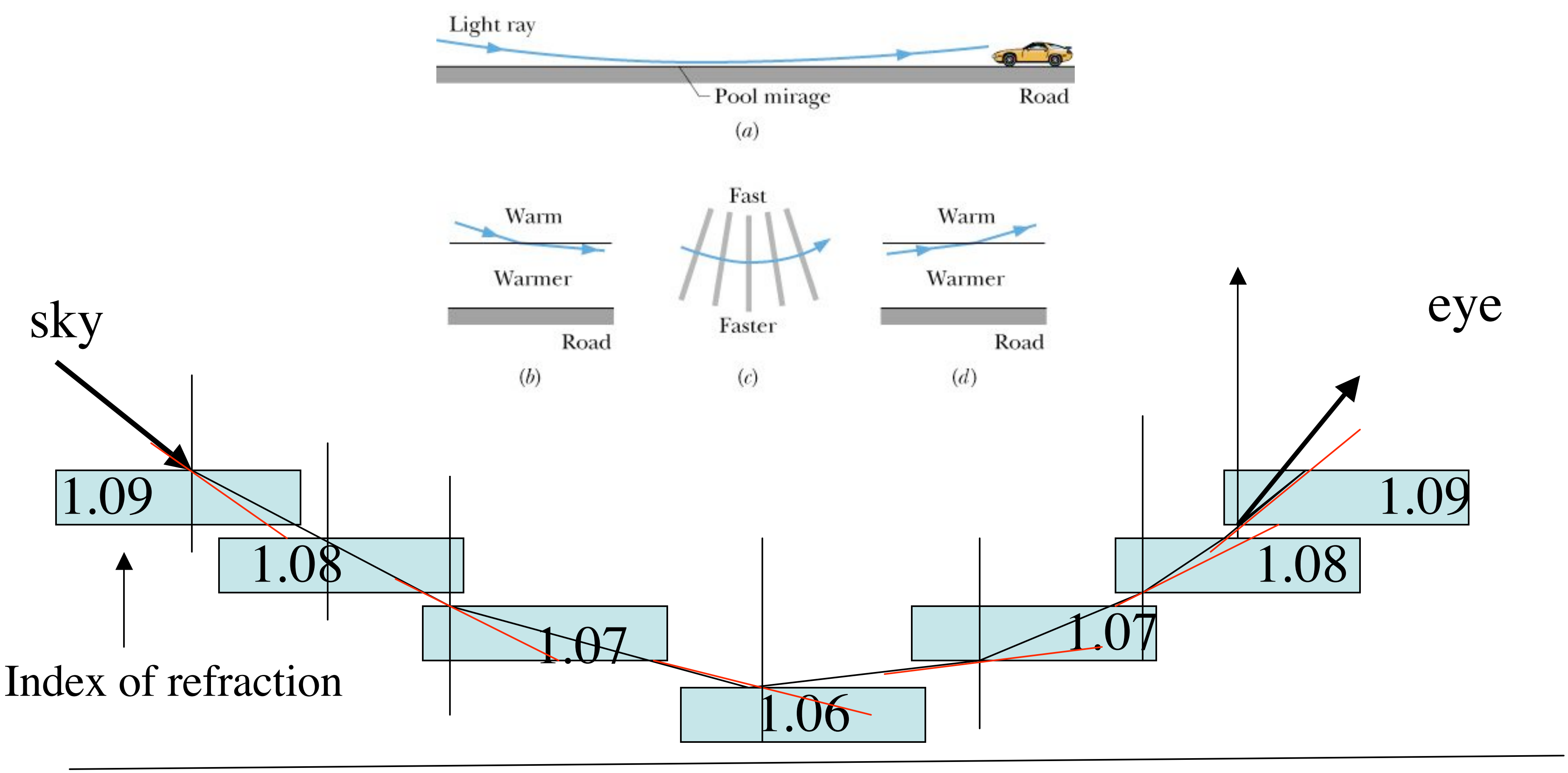
Corner Reflector?



Show Total Internal reflection simulator

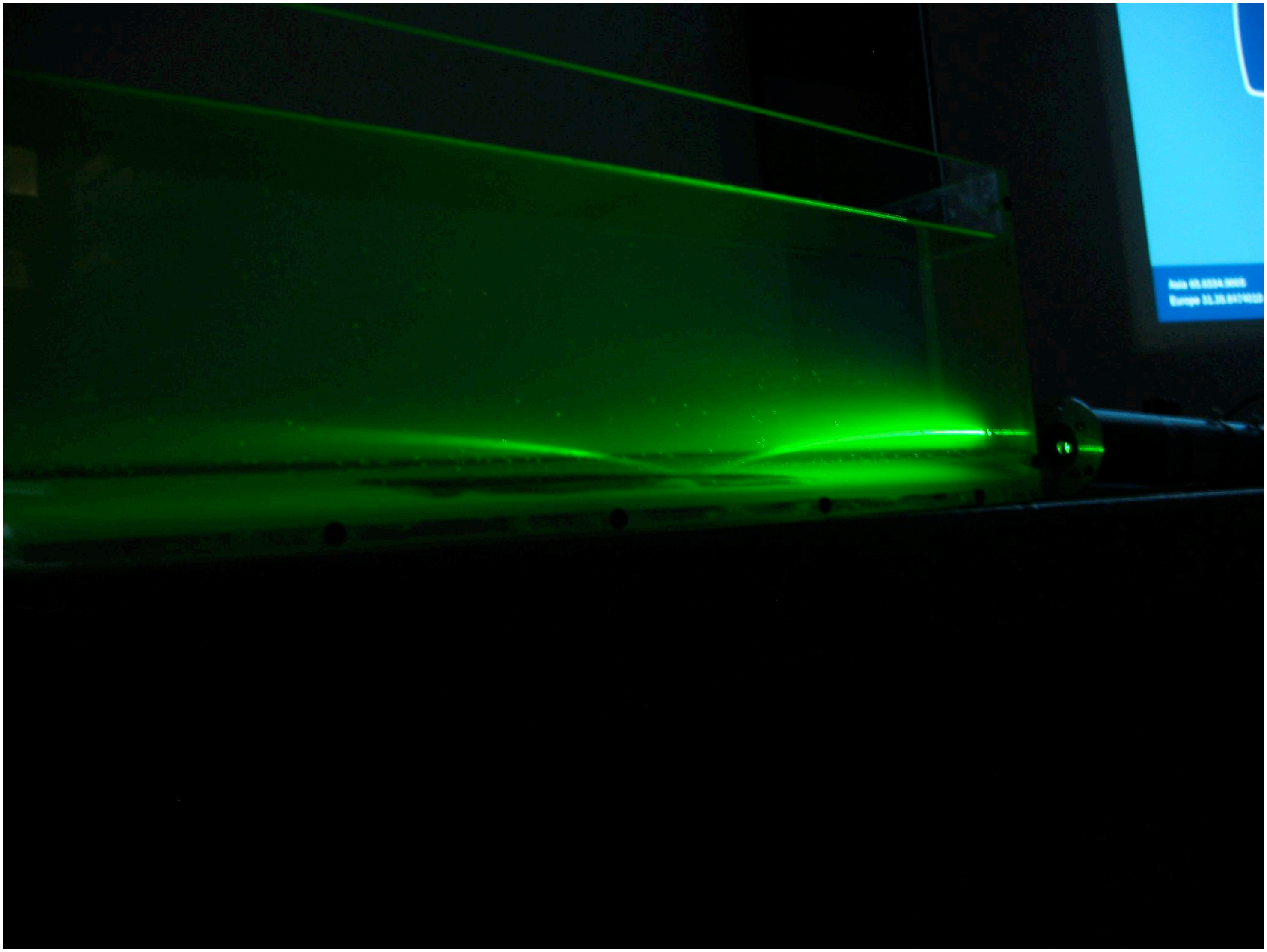
Halliday, Resnick, Walker: Fundamentals of Physics, 7th Edition - Student Companion Site

What causes a Mirage



Hot road causes gradient in the index of refraction that increases as you increase the distance from the road

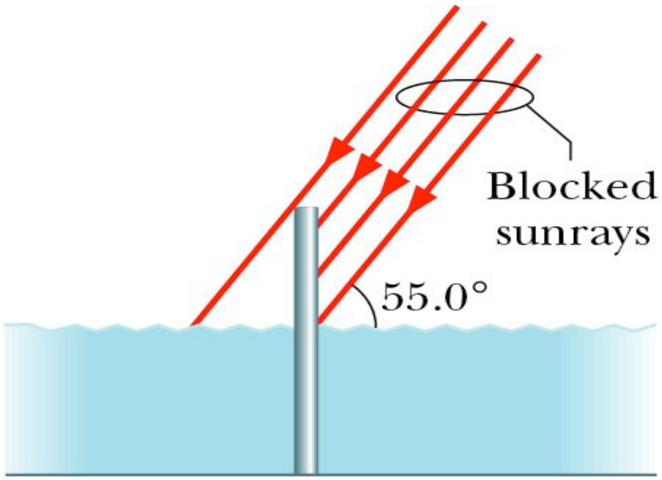
Inverse Mirage Bend



Snells Law Example

47. In the figure, a 2.00-m-long vertical pole extends from the bottom of a swimming pool to a point 50.0 cm above the water. What is the length of the shadow of the pole on the level bottom of the pool?

Consider a ray that grazes the top of the pole, as shown in the diagram below. Here $\theta_1 = 35^o$, $l_1 = 0.50$ m, and $l_2 = 1.50$ m.



The length of the shadow is x + L.

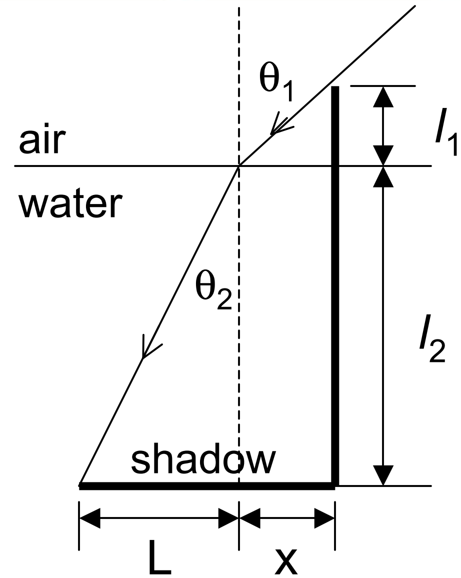
x is given by

$$x = l_1 \tan\theta_1 = (0.50\text{m})\tan 35^o = 0.35\text{ m.}$$

L is given by

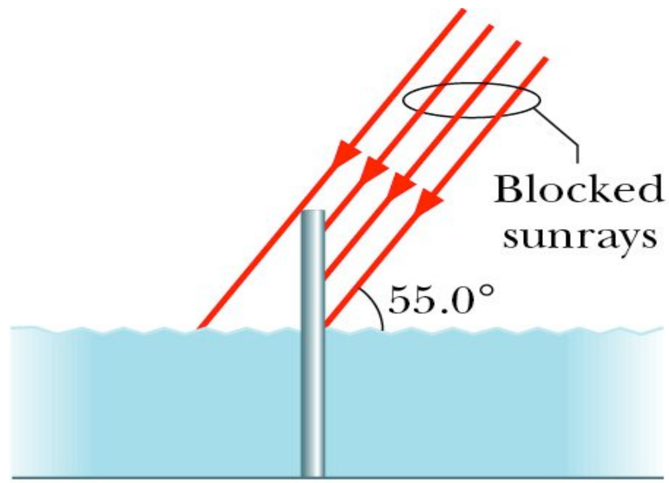
$$L = l_2 \tan\theta_2$$

Use Snells Law to find θ_2



Calculation of L

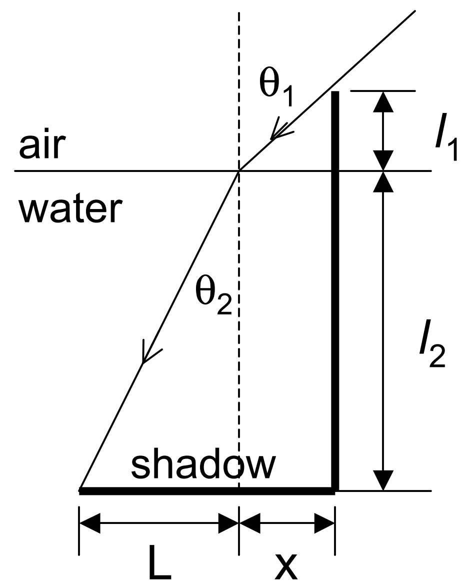
According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$



$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35^o}{1.33}\right) = 25.55^o$$

L is given by

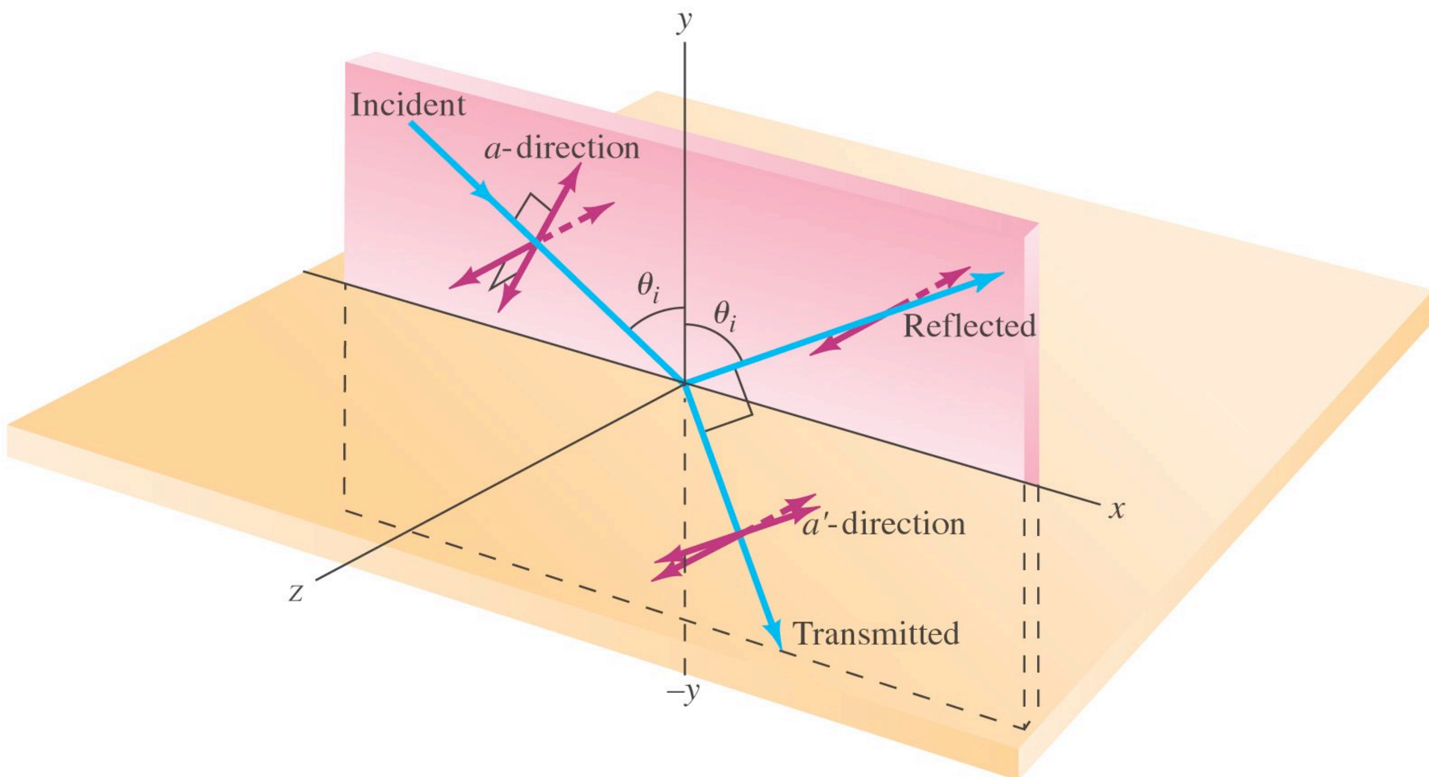
$$L = l_2 \tan\theta_2 = (1.50m)\tan 25.55^o = 0.72m.$$



The length of the shadow is L+x.

L+x = 0.35m + 0.72 m = 1.07 m.

Polarization by Reflection   Brewsters Law



$$(1.)\sin\theta_B = n\sin\theta_r$$

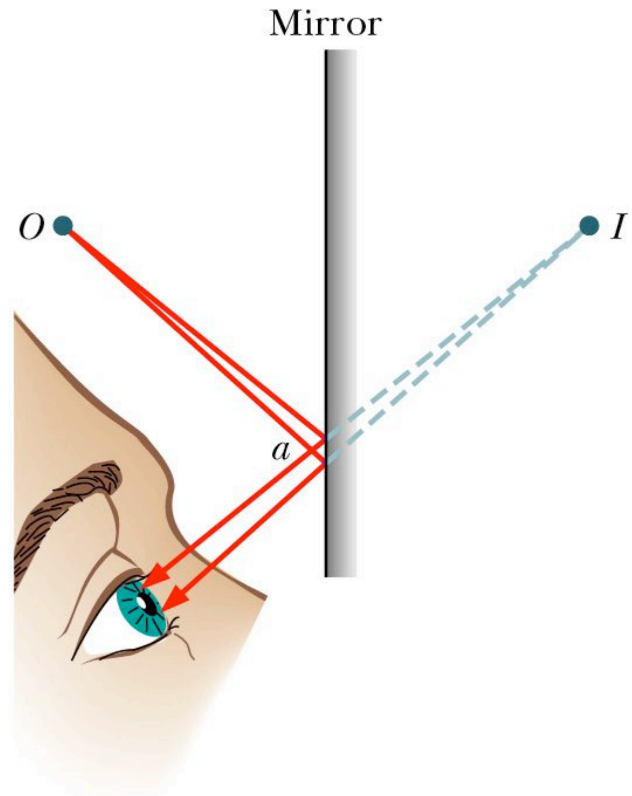
$\theta_B + \theta_r = 90$ we get 100% polarized reflected wave

$$\sin\ \theta_{\mathrm{B}} = n\sin(90-\theta_B) = n\cos\theta_B$$

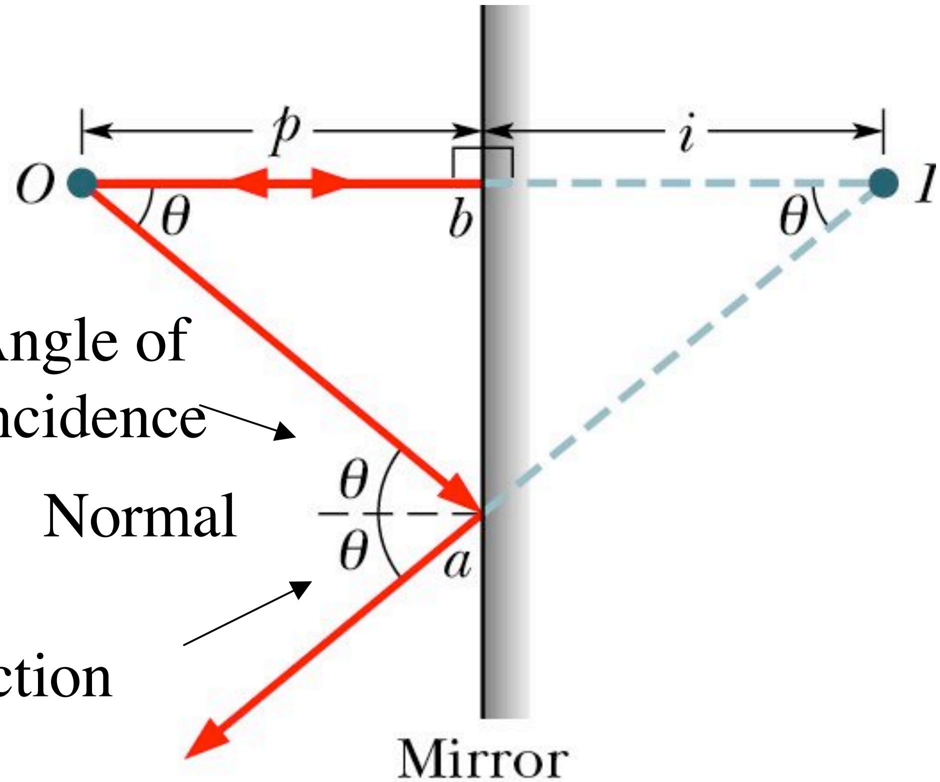
$\theta_B = \tan^{-1} n$ Brewsters Law

Mirrors and Lenses

Plane Mirrors Where is the image formed



Plane mirrors



Real side

Angle of incidence

Normal

Angle of reflection

eye

Virtual side

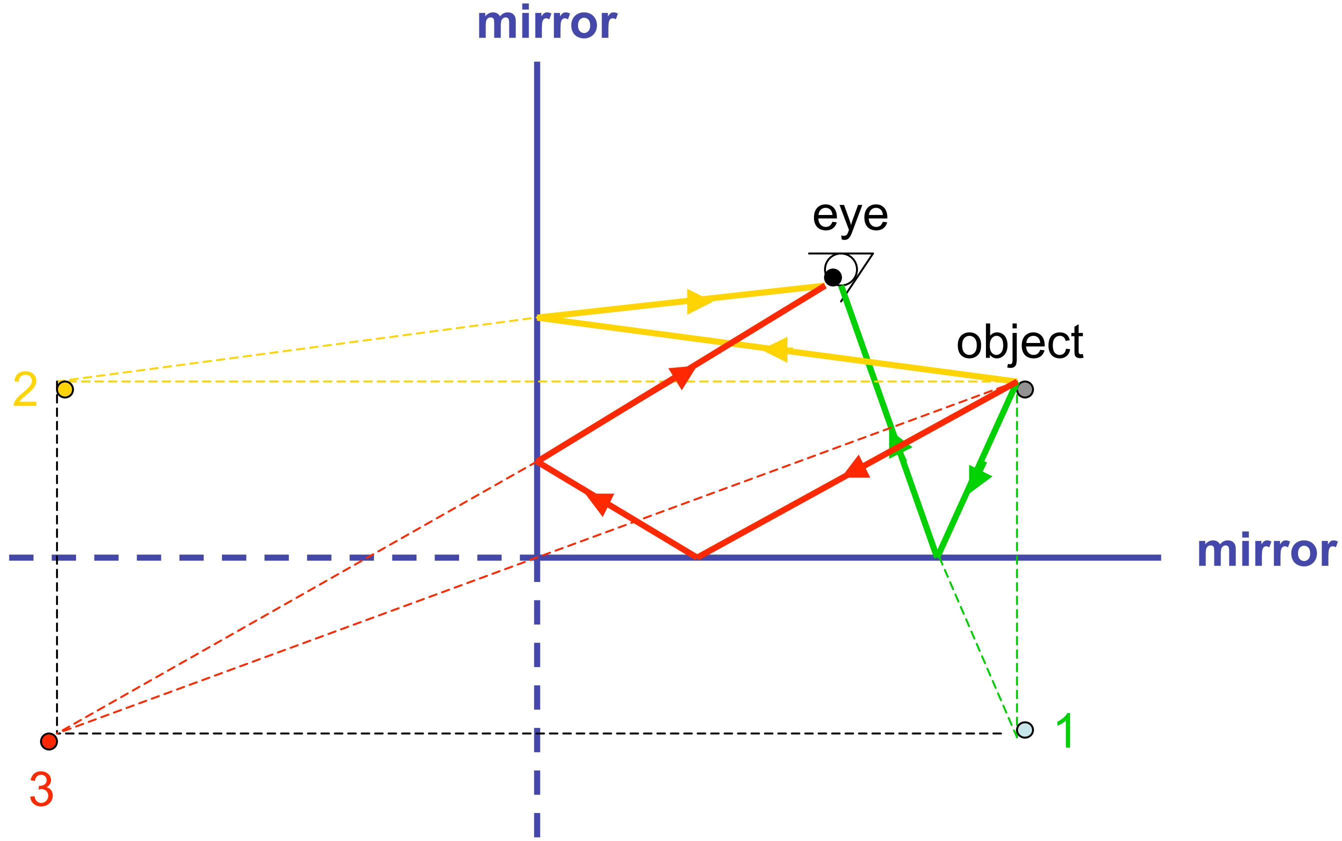
Virtual image

$$i = -p$$

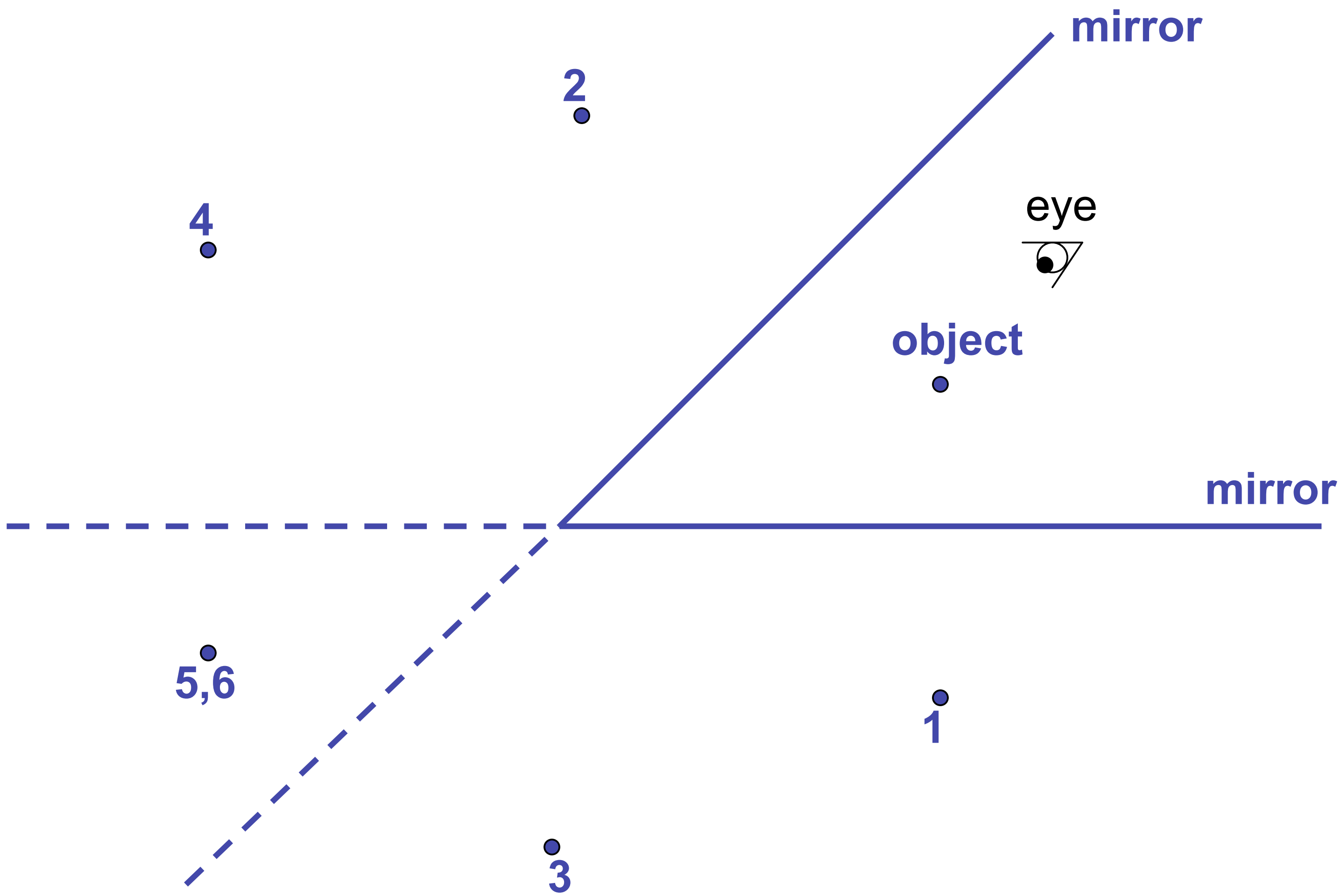
Object distance = - image distance

Image size = Object size

Problem: Two plane mirrors make an angle of 90°. How many images are there for an object placed between them?

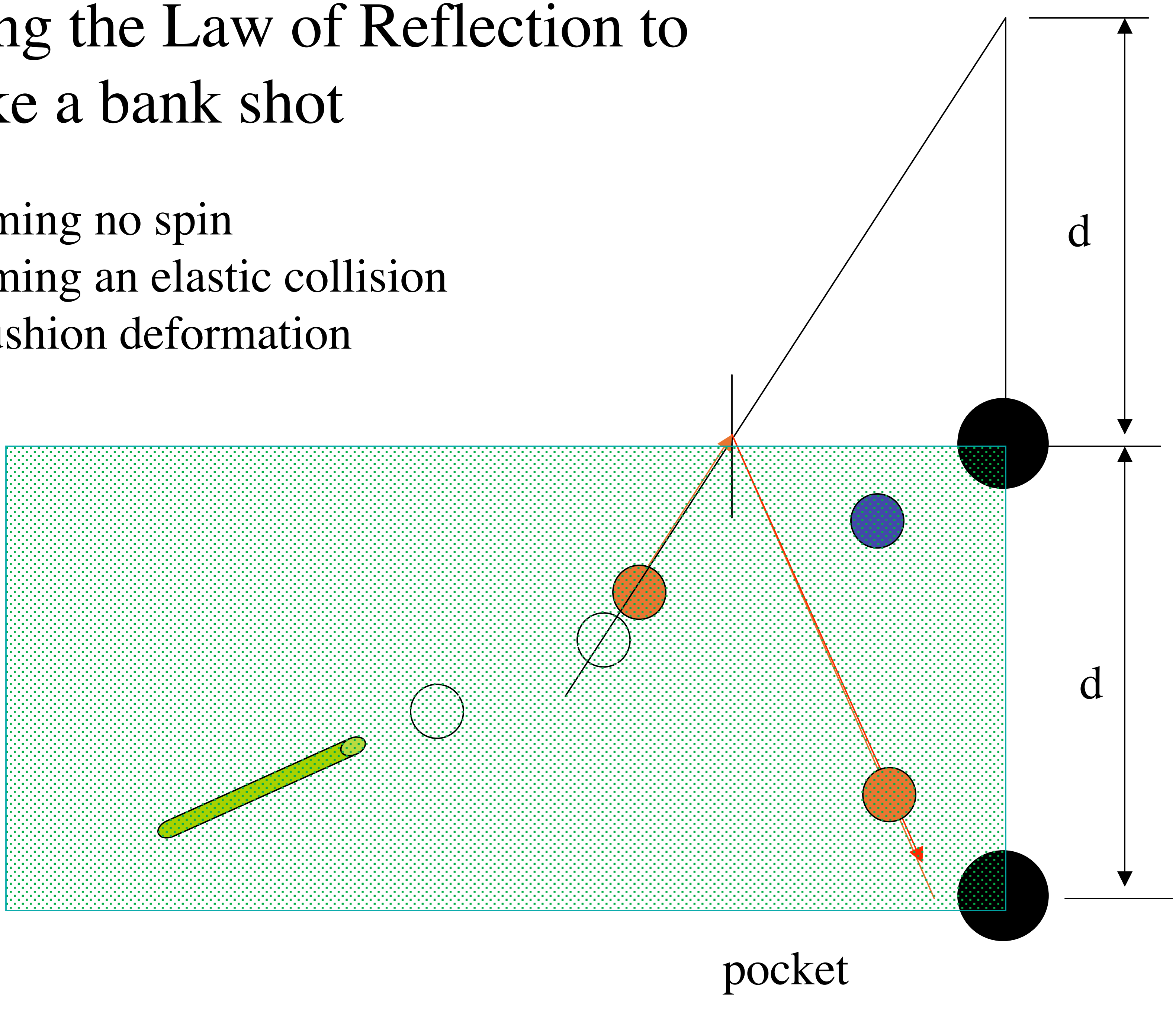


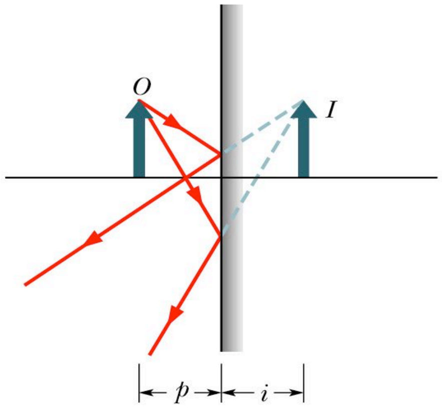
Problem: Two plan mirrors make an angle of 60°. Find all images for a point object on the bisector.



Using the Law of Reflection to make a bank shot

Assuming no spin
Assuming an elastic collision
No cushion deformation

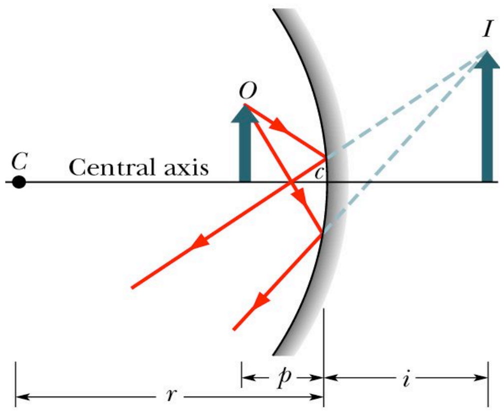




(a)

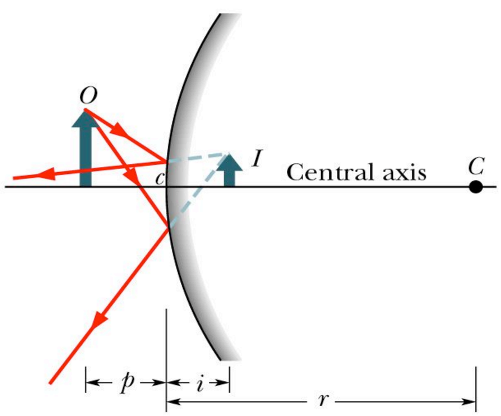
$i = -p$ magnification = 1

What happens if we bend the mirror?



(b)

Concave mirror.
Image gets magnified.
Field of view is diminished



(c)

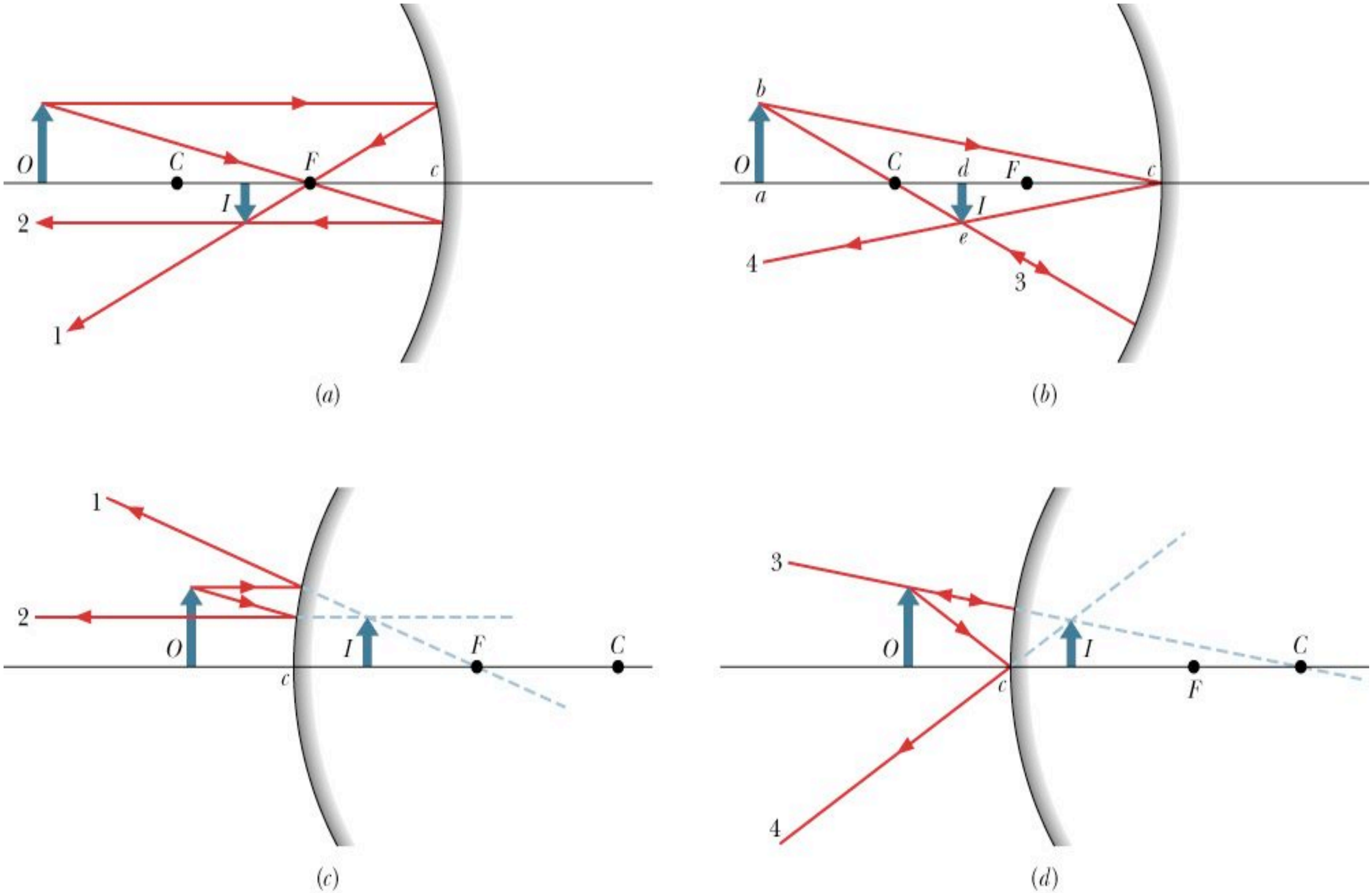
Convex mirror.
Image is reduced.
Field of view increased.

Rules for drawing images for mirrors

- Initial parallel ray reflects through focal point.
- Ray that passes in initially through focal point reflects parallel from mirror
- Ray reflects from C the radius of curvature of mirror reflects along itself.
- Ray that reflects from mirror at little point c is reflected symmetrically

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}$$

$$m=\frac{-i}{p}$$

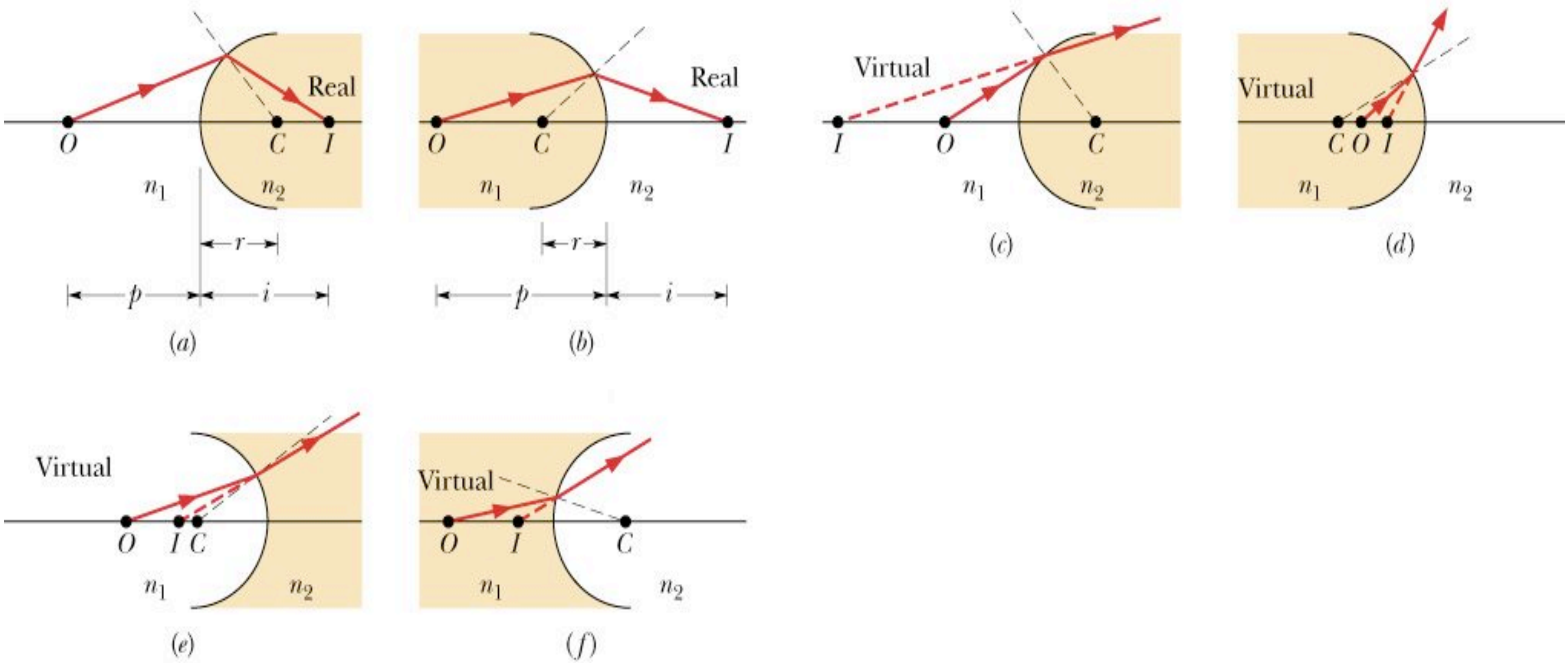


O
C
F
c
I
2
1
(a)
b
O
a
C
d
F
c
I
e
4
3
(b)
1
2
O
c
I
F
C
(c)
3
O
c
I
F
C
4
(d)

Concept Simulator/Illustrations

Halliday, Resnick, Walker: Fundamentals of Physics, 7th Edition - Student Companion Site

Spherical refracting surfaces



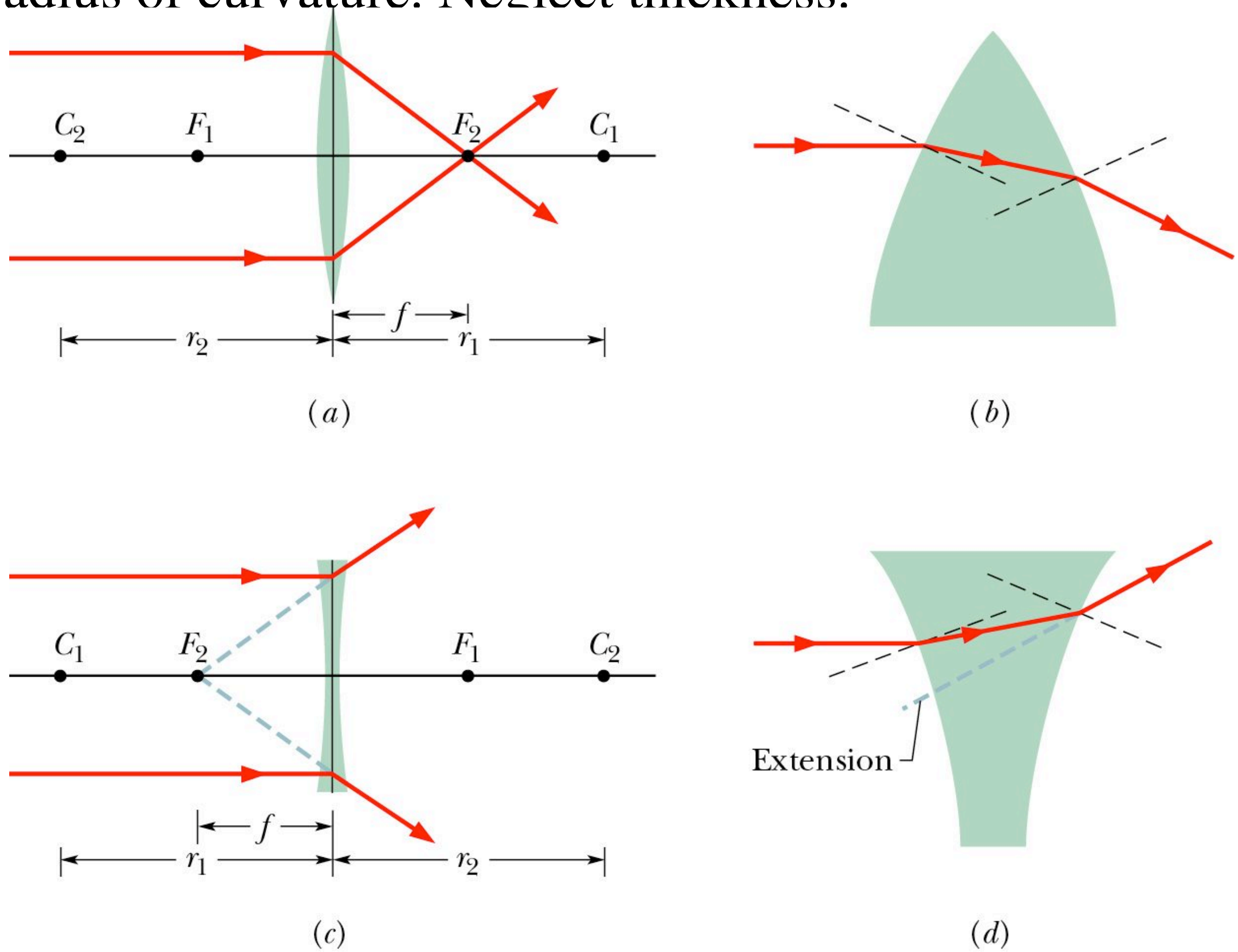
Using Snell's Law and assuming small Angles between the rays with the central axis, we get the following formula:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

Apply this equation to Thin Lenses where the thickness is small compared to object distance, image distance, and radius of curvature. Neglect thickness.

Converging lens

Diverging lens



Thin Lens Equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$$

Lensmaker Equation

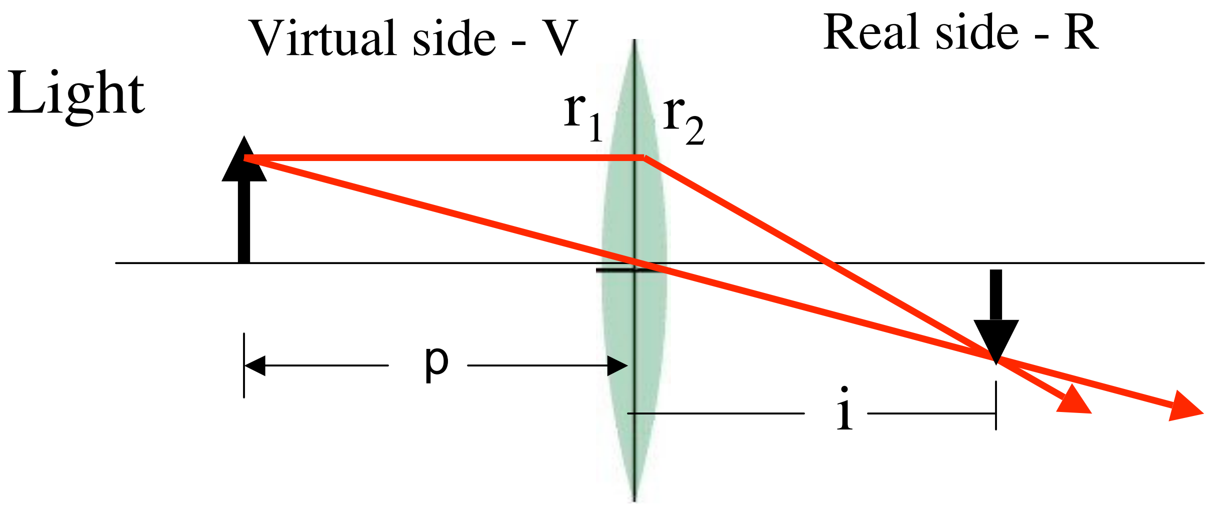
$$\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$$

Lateral Magnification for a Lens

$$m = -\frac{i}{p}$$

What is the sign convention?

Sign Convention



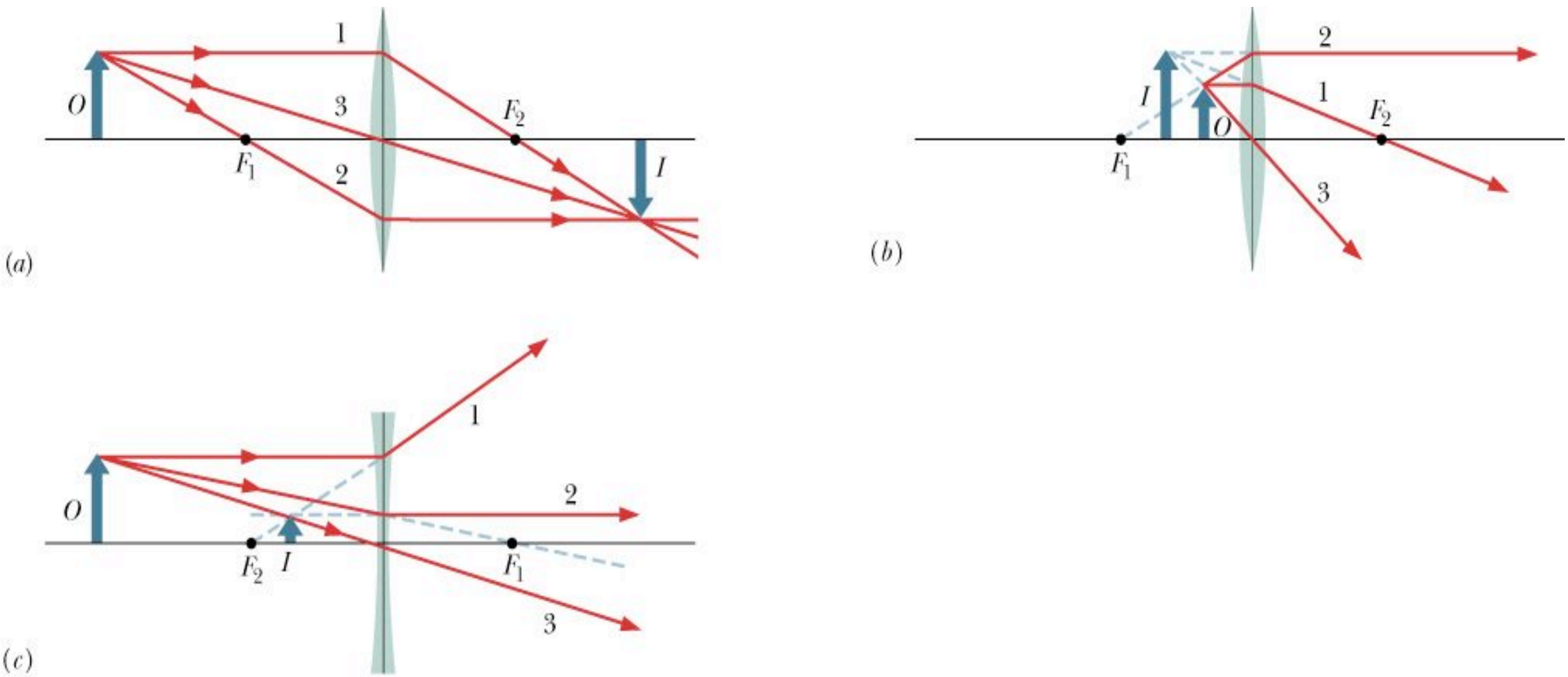
Real object - distance p is pos on V side (Incident rays are diverging)
Radius of curvature is pos on R side.
Real image - distance is pos on R side.

Virtual object - distance is neg on R side. Incident rays are converging)
Radius of curvature is neg on the V side.
Virtual image- distance is neg on the V side.

Rules for drawing rays to locate images from a lens

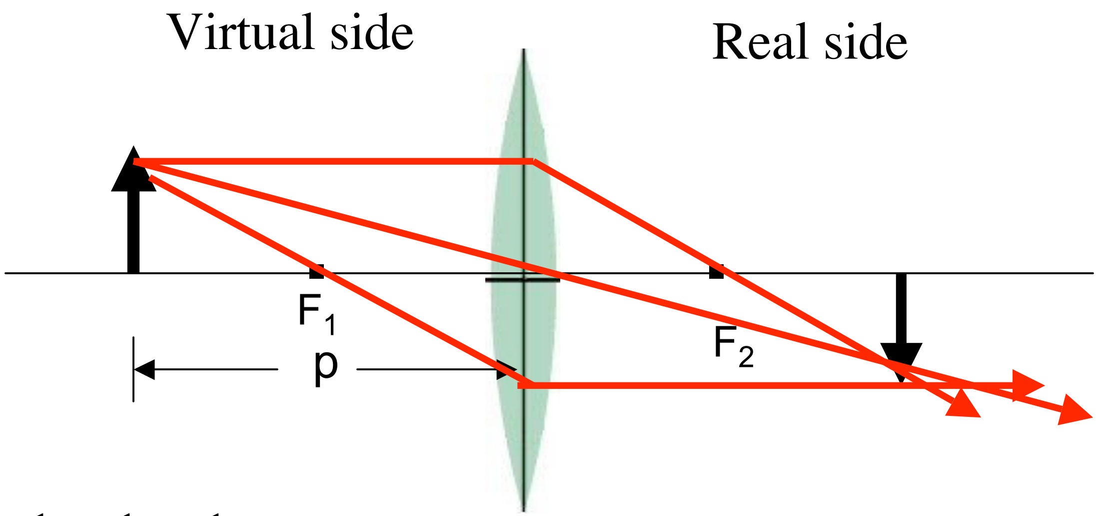
- A ray initially parallel to the central axis will pass through the focal point.
- A ray that initially passes through the focal point will emerge from the lens parallel to the central axis.
- A ray that is directed towards the center of the lens will go straight through the lens undeflected.

Example of drawing images



Example

24(b). Given a lens with a focal length f = 5 cm and object distance p = +10 cm, find the following: i and m. Is the image real or virtual? Upright or inverted? Draw 3 rays.



$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{i} = \frac{1}{5} - \frac{1}{10} = +\frac{1}{10}$$

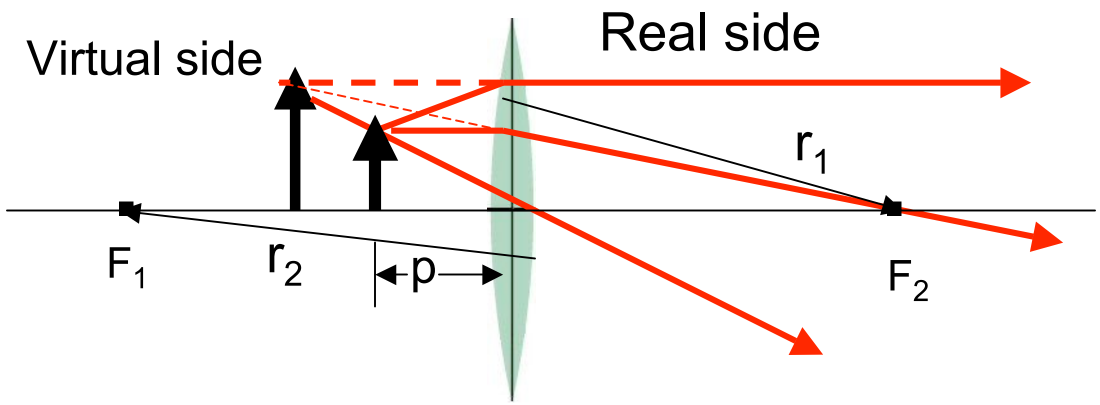
$$i = +10\ cm$$

$$m = \frac{y'}{y} = -\frac{i}{p}$$

$$m = -\frac{10}{10} = -1$$

Image is real, inverted.

24(e). Given a lens with the properties (lengths in cm) r_1 = +30, r_2 = -30, p = +10, and n = 1.5, find the following: f, i and m. Is the image real or virtual? Upright or inverted? Draw 3 rays.



$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{30} - \frac{1}{-30}\right) = \frac{1}{30}$$

$$\boxed{f = 30cm}$$

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p}$$

$$\frac{1}{i} = \frac{1}{30} - \frac{1}{10} = -\frac{1}{15}$$

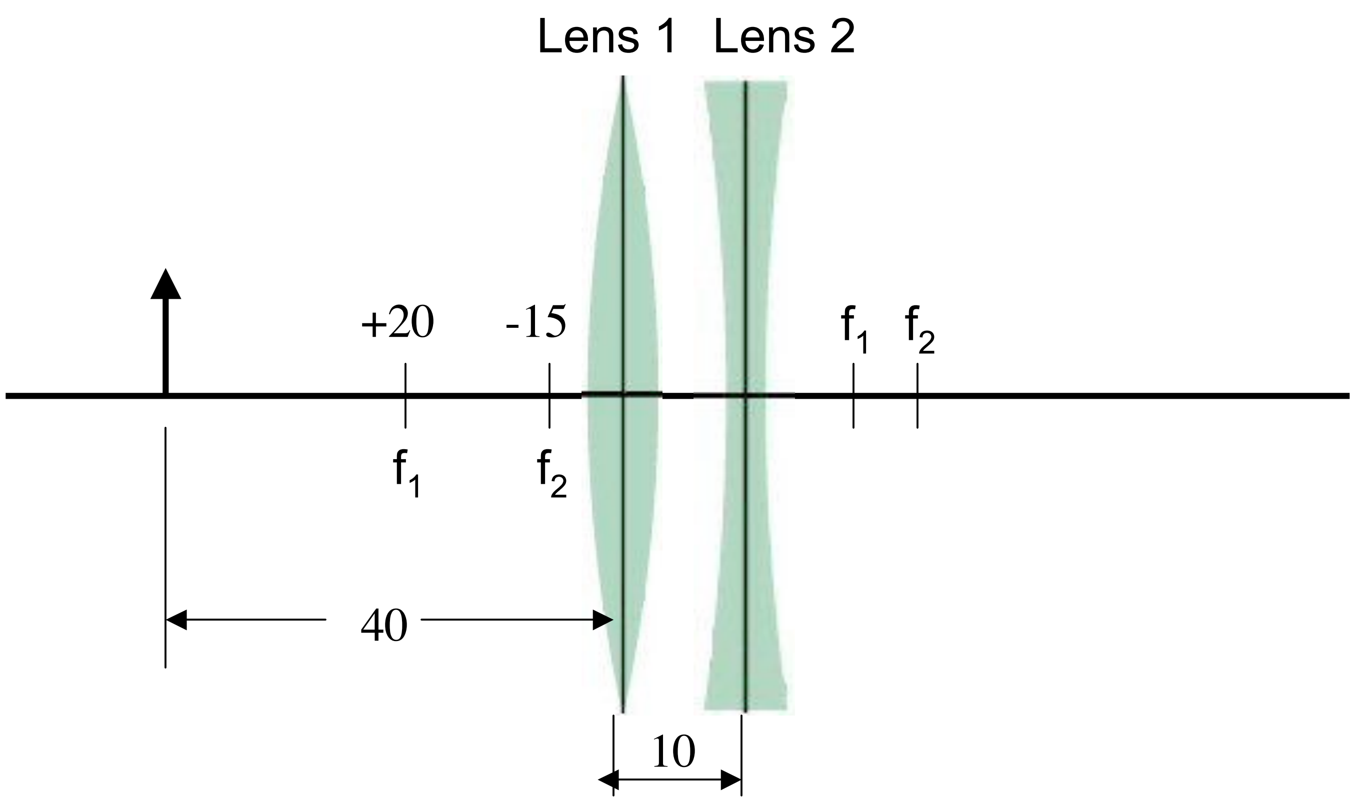
$$\boxed{i = -15cm}$$

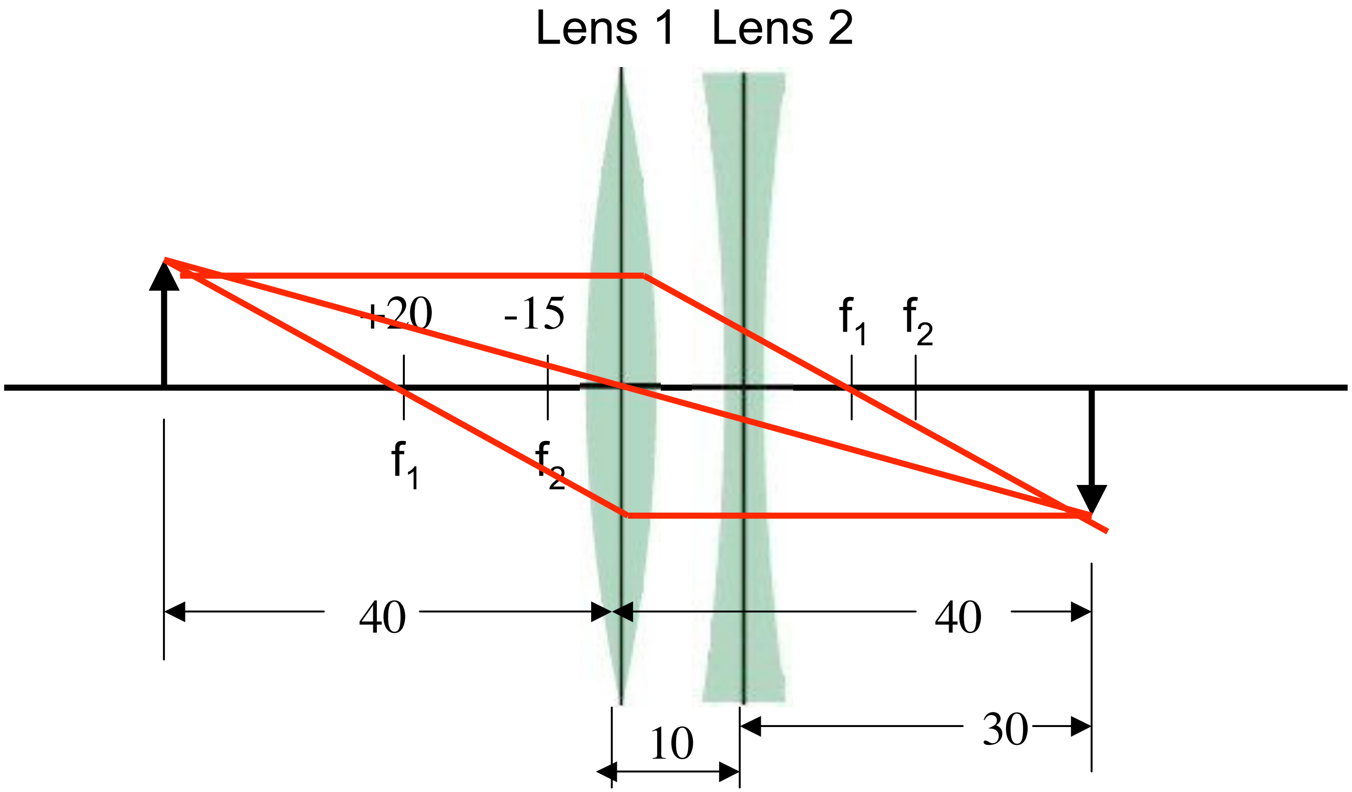
$$m = \frac{y'}{y} = -\frac{i}{p}$$

$$m = -\frac{-15}{10} = \boxed{+1.5}$$

Image is virtual, upright.

27. A converging lens with a focal length of +20 cm is located 10 cm to the left of a diverging lens having a focal length of -15 cm. If an object is located 40 cm to the left of the converging lens, locate and describe completely the final image formed by the diverging lens. Treat each lens Separately.



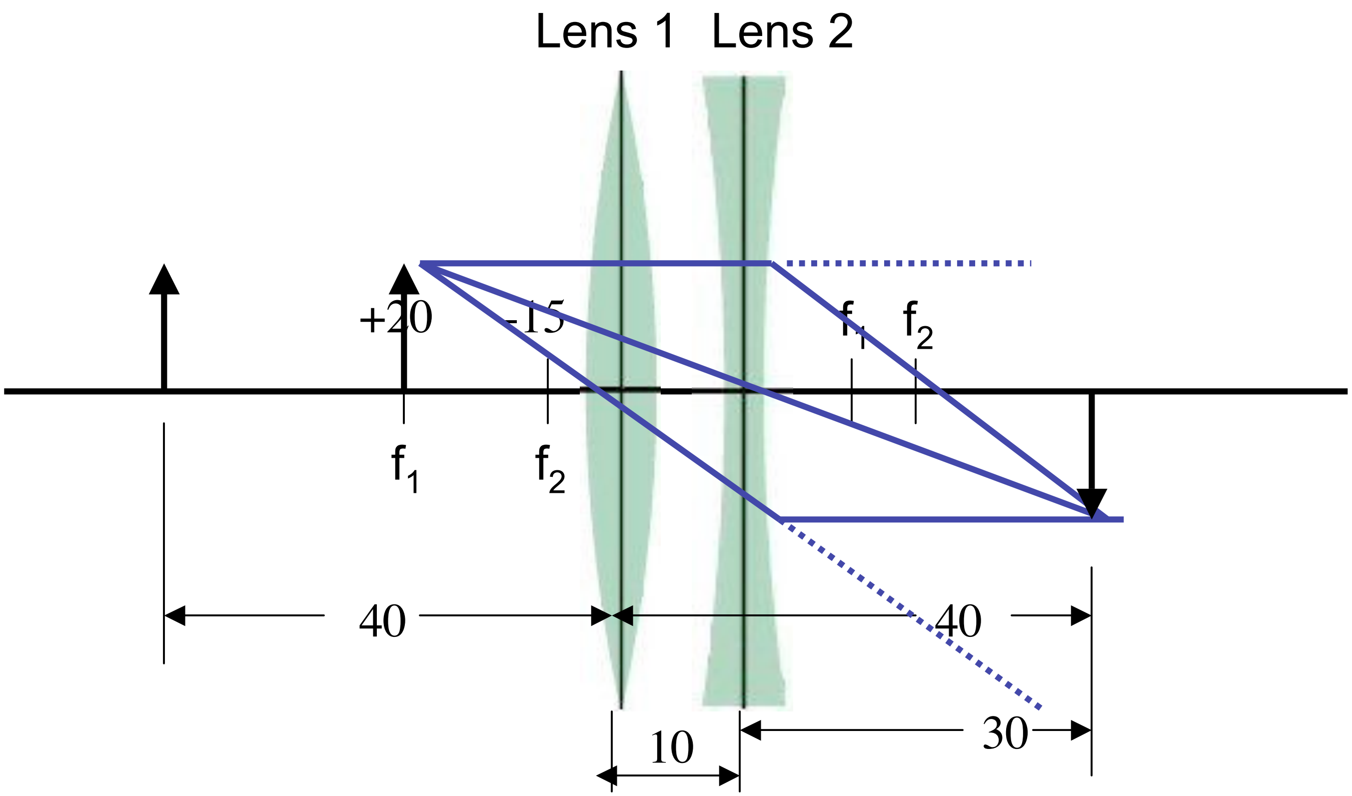


Ignoring the diverging lens (lens 2), the image formed by the converging lens (lens 1) is located at a distance

$$\frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{20cm} - \frac{1}{40cm}. \quad i_1 = 40cm$$

Since m = $-i_1/p_1$= - 40/40= - 1 , the image is inverted

This image now serves as a virtual object for lens 2, with p_2 = - (40 cm - 10 cm) = - 30 cm.



$$\frac{1}{i_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{-15cm} - \frac{1}{-30cm} \qquad i_2 = -30cm.$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2. It is virtual (since $i_2 < 0$).

The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) =$ (-40/40)x(30/-30) =+1, so the image has the same size orientation as the object.

Optical Instruments

Magnifying lens
Compound microscope
Refracting telescope