Lecture 15 Interference Chp. 35

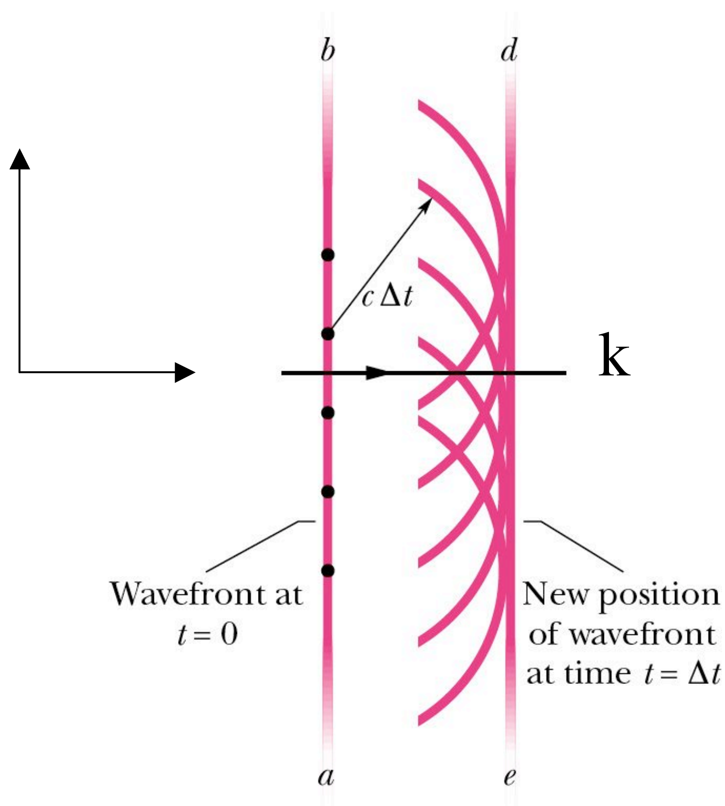
Opening Demo

- Topics
  - Interference is due to the wave nature of light
  - Huygen's principle, Coherence
  - Change in wavelength and phase change in a medium
  - Interference from thin films
  - Examples
  - Young's Interference Experiment and demo
  - Intensity in double slit experiment
  - Warm-up problem
- Demos

Huygen's Principle, Wavefronts and Coherence

$$E = E_m \sin(kx - \omega t)$$

$$E = E_m \sin(\frac{2\pi}{\lambda} - 2\pi f\, t)$$



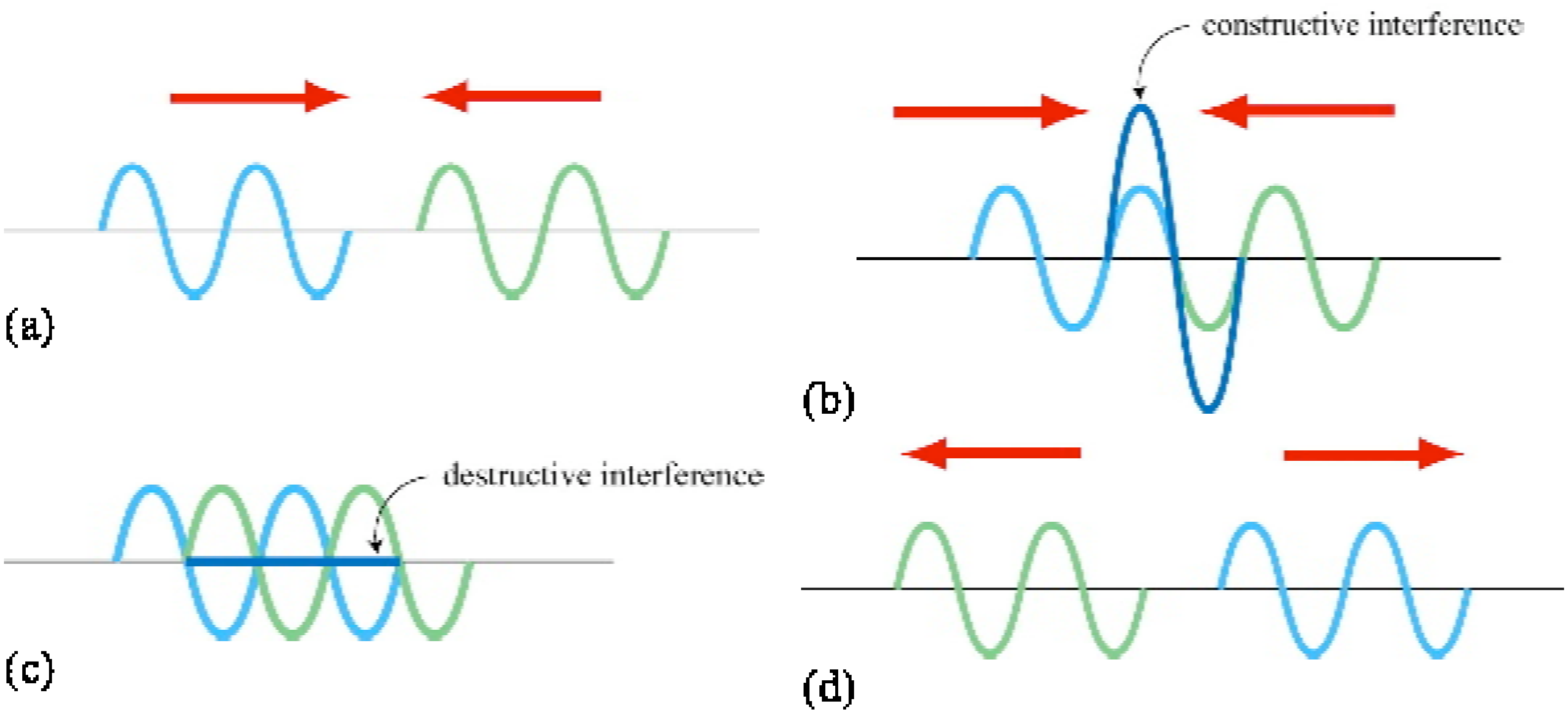
Examples of coherence are:

- Laser light
- Small spot on tungsten filament
- Wavefront

Most light is incoherent:

- Two separate light bulbs
- Two headlight beams on a car
- Sun is basically incoherent

Interference is the combination of two or more waves to form a composite wave, based on the principle of superposition



In order to form an interference pattern, the incident light must satisfy two conditions:

(i) The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation. For example, if two waves are completely out of phase with _ = π , this phase difference must not change with time.

(ii) The light must be monochromatic. This means that the light consists of just one wavelength _ = 2π/k .

Light emitted from an incandescent lightbulb is incoherent because the light consists of waves of different wavelengths and they do not maintain a constant phase relationship. Thus, no interference pattern is observed.

Once light is in phase there are three ways to get light out of phase

1. Rays go through different material with different index of refraction

2. Reflection from a medium with greater index of refraction

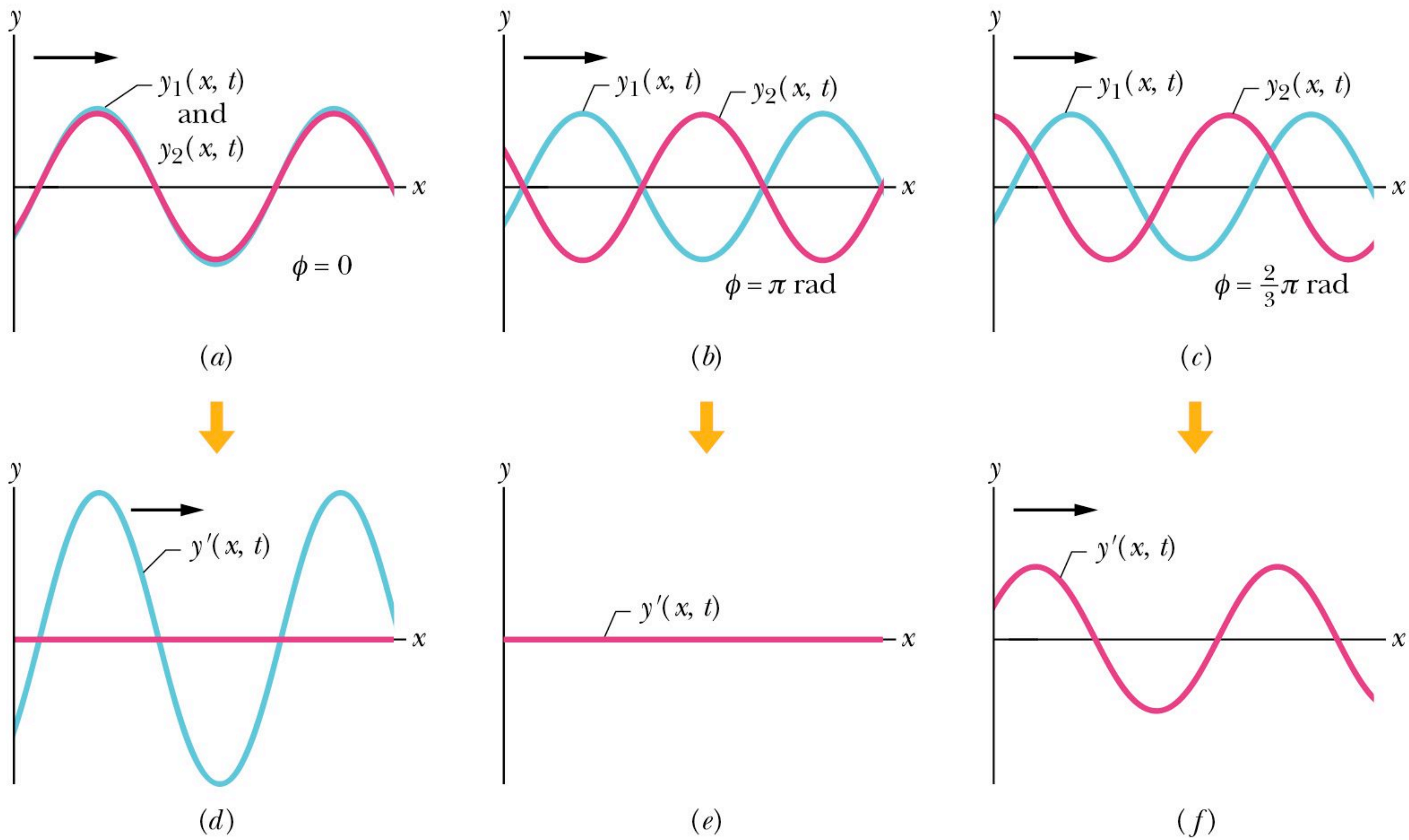
3. The selected rays travel different distances.

Now lets look at examples

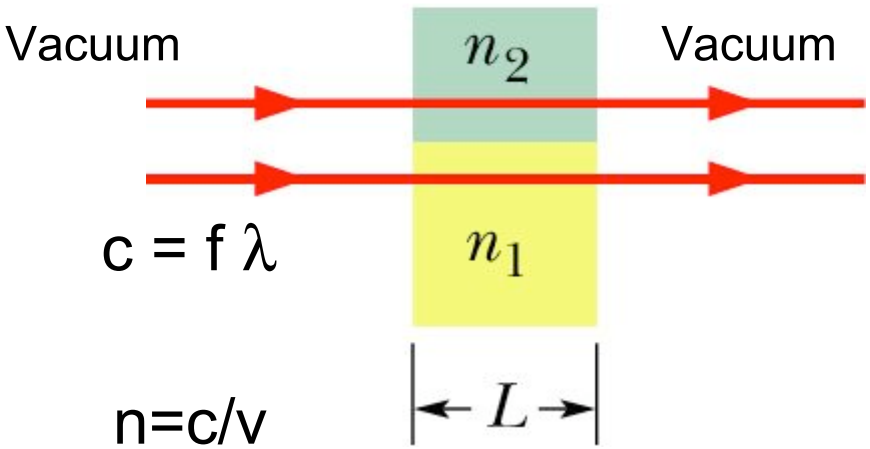
In Phase

Out of Phase by 180 degrees or π radians or λ/2

In between



Concept of path length difference, phase and index of refraction



c = f λ

n=c/v

$v_n = f\,\lambda_n = f\,\lambda/n$

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}$$

$$N_2 = \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda}$$

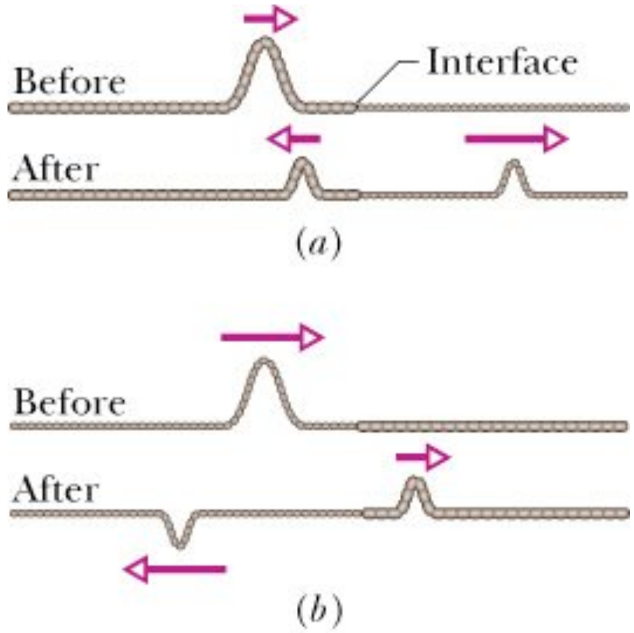
$$N_2 - N_1 = \frac{L(n_2 - n_1)}{\lambda}$$

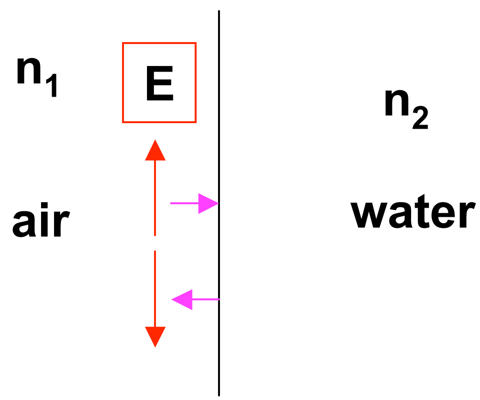
Path length difference = Phase difference = $\dfrac{L(n_2 - n_1)}{\lambda}$

Rays are in phase if $\dfrac{L(n_2 - n_1)}{\lambda} = m\lambda$ where m$=1, 2, 3..$

Rays are out of phase if $\dfrac{L(n_2 - n_1)}{\lambda} = \left(m + \dfrac{1}{2}\right)\lambda$ where m=1,2,3

1 λ is the same as 2π radian (rad), $\lambda/2$ is the same as π rad, etc.

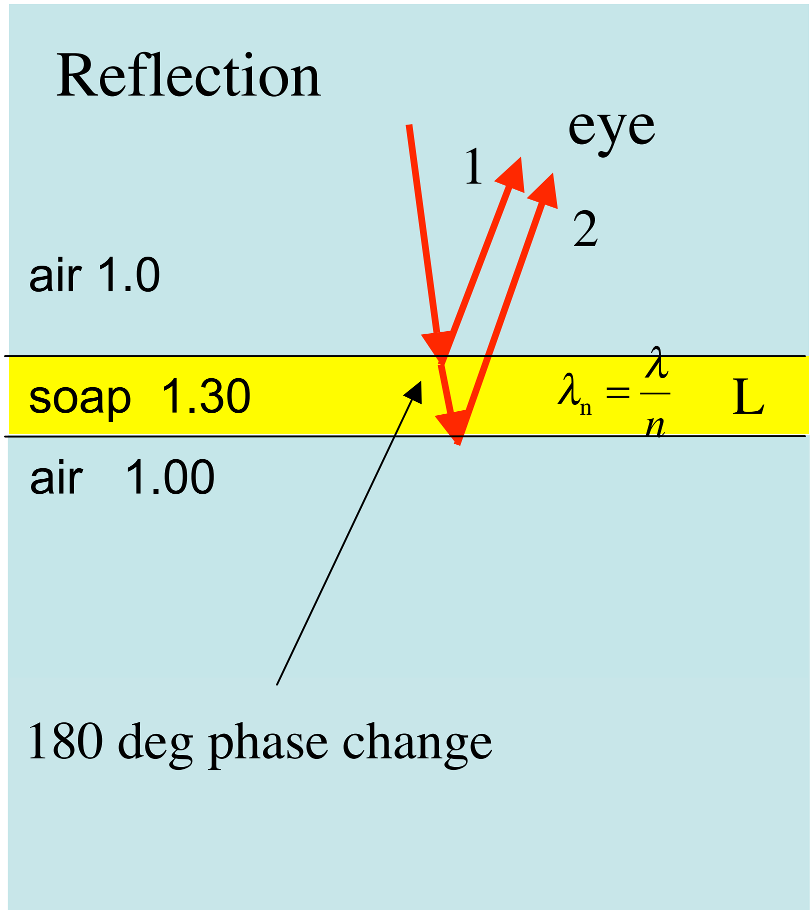




Wave reflects 180 degrees out of phase when
$n_1 < n_2$

Thin film Interference Phenomenon: Reflection

$$E = E_m \sin(kx - \omega t)$$



First consider phase change upon reflection

$$E_1 = E_m \sin(kx - \omega t + \pi)$$
$$E_1 = -E_m \sin(kx - \omega t)$$

Show wave demo

Now consider the path length differences

$$E_2 = E_m \sin(k(x + 2L) - \omega t)$$

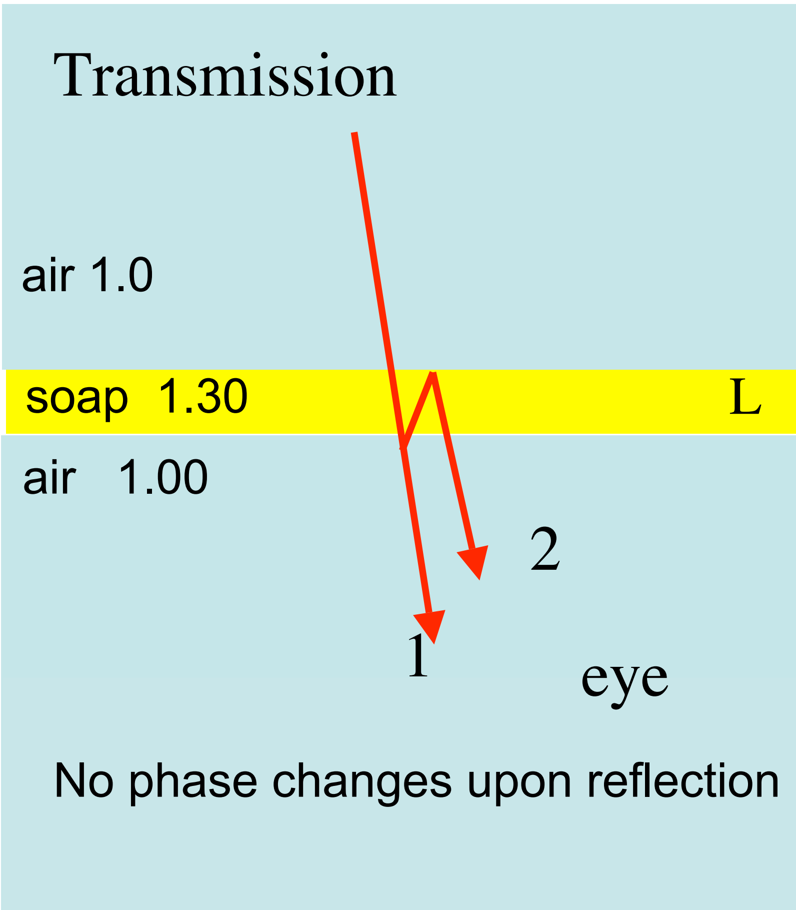
Suppose the soap thickness L is such that

$$2L = \left(\frac{1}{2}\right)\lambda_n$$

Constructive Interference

$$E_2 = E_m \sin(kx + k2L - \omega t) =$$
$$E_m \sin(kx + \frac{2\pi}{\lambda}\frac{\lambda}{2} - \omega t) =$$
$$E_2 = E_m \sin(kx - \omega t + \pi)$$
$$2L = \left(m + \frac{1}{2}\right)\lambda_{\mathrm{n}_2} \quad \text{where} \quad \mathrm{m} = 0,\ 1,\ 2,\ \ldots$$

Thin film Interference Phenomenon
Transmission



Constructive interference

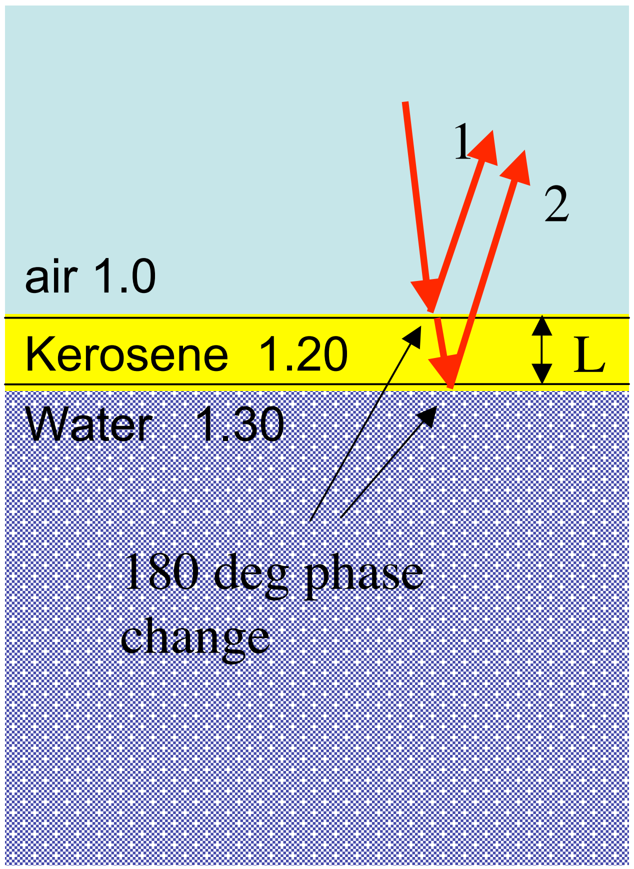
$$2L = (m)\frac{\lambda}{n} \qquad \lambda = \frac{2nL}{m}$$

m = 1,2,3,4….

n =1.30

39. A disabled tanker leaks kerosene (n = 1.20) into the Persian Gulf, creating a large slick on top of the water (n = 1.30).

(a) If you are looking straight down from an airplane while the Sun is overhead at a region of the slick where its thickness is L=460 nm, for which wavelength(s) of visible light is the reflection brightest because of constructive interference?



Path difference between ray 1and ray 2 = 2L.
Phase changes cancel out

For constructive interference path difference must = integral number of wavelengths

$$2L = (m)\frac{\lambda}{n_2} \qquad \lambda = \frac{2n_2L}{m}$$

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460nm)}{m} = 1104,\ 552,\ 368\ nm$$

for m = 1, m = 2, and m = 3 respectively

We note that only the 552 nm wavelength falls within the visible light range.

(b) If you are scuba diving directly under this same region of the slick, for which wavelength(s) of visible light is the transmitted intensity strongest? (Hint: use figure (a) with appropriate indices of refraction.)

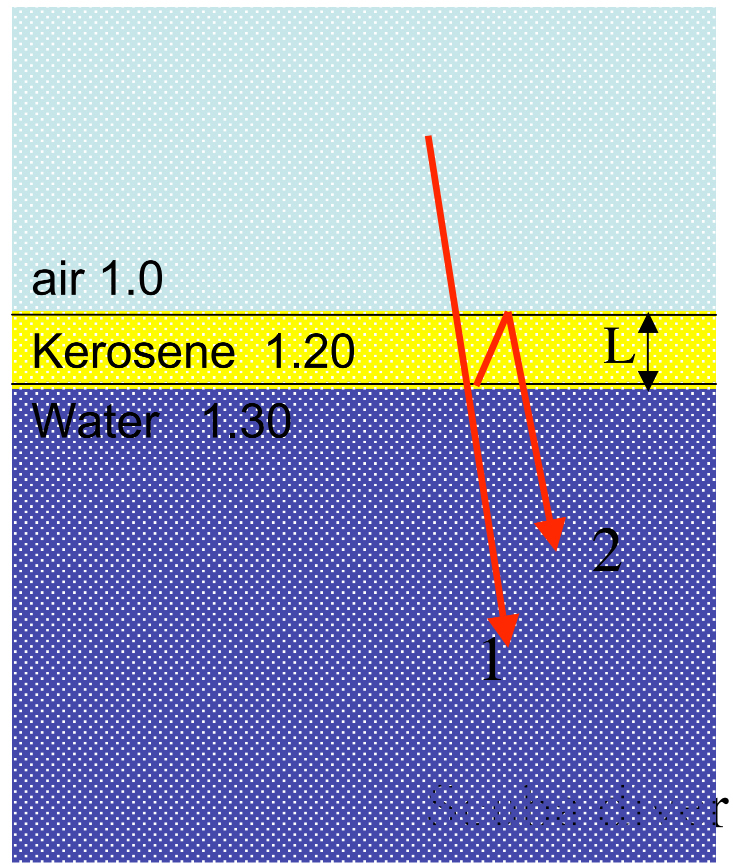
For transmission, ray 2 undergoes 180 deg phase shift upon reflection at the Kerosene-water interface. Therefore, for constructive interference 2L= integral number of wavelengths in n_2 plus half a wavelength.

$$2L = \left(m + \frac{1}{2}\right)\lambda_{n_2} \quad \text{where} \quad m = 0,\ 1,\ 2,\ \ldots$$

$$\lambda_n = \frac{\lambda}{n_2}$$

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0,\ 1,\ 2,\ \ldots$$

Solve for λ



$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{Solving for } \lambda$$

$$\lambda = \frac{4n_2L}{2m+1}$$

$$\lambda = \frac{4n_2L}{2m+1} = \frac{4(1.2)(460)}{1} = 2208\ nm \quad \mathrm{m} = 0$$

$$\lambda = \frac{4n_2L}{2m+1} = \frac{4(1.2)(460)}{3} = 736\ nm \quad \mathrm{m} = 1$$

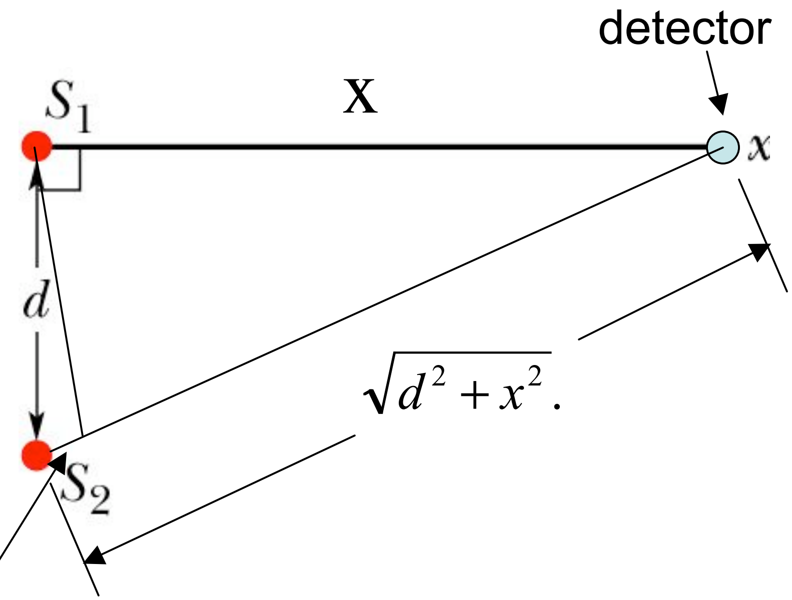
$$\lambda = \frac{4n_2L}{2m+1} = \frac{4(1.2)(460)}{5} = 441.6\ nm \quad m = 2$$

Visible spectrum is 430 nm - 690 nm

We note that only the 441.6 nm wavelength (blue) is in the visible range,

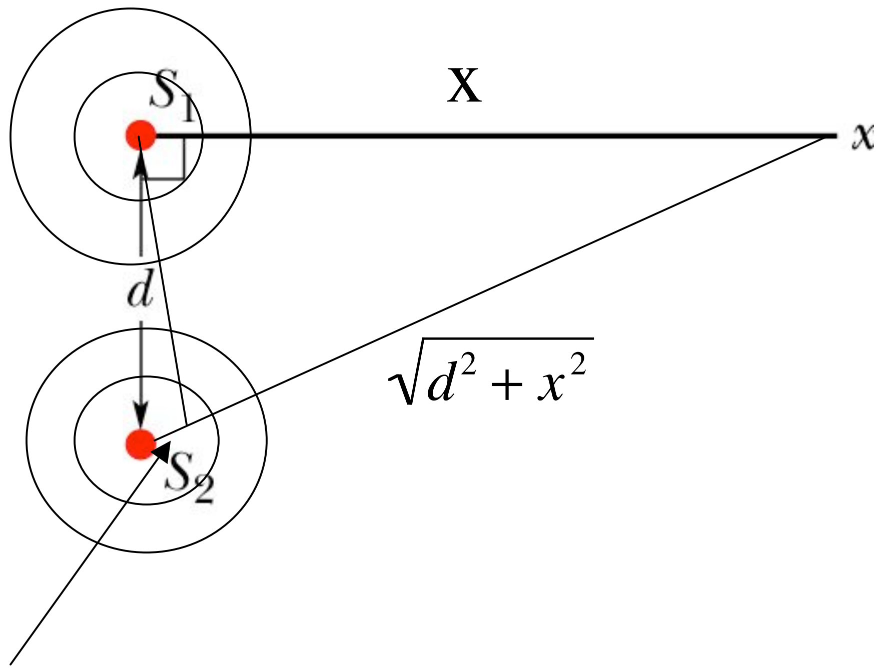
27. S_1 and S_2 in Fig. 36-29 are point sources of electromagnetic waves of wavelength 1.00 m. They are in phase and separated by d = 4.00 m, and they emit at the same power.

(a) If a detector is moved to the right along the x-axis from source S_1, at what distances from S_1 are the first three interference maxima detected?



The wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2+x^2}$.

The path difference is $\sqrt{d^2+x^2}-x$



For constructive interference we have

$$\text{path difference} = \sqrt{d^2 + x^2} - x = m\lambda \qquad m = 1,2,3..$$

The solution for x of this equation is

Solve for x

$$\sqrt{d^2 + x^2} - x = m\lambda$$

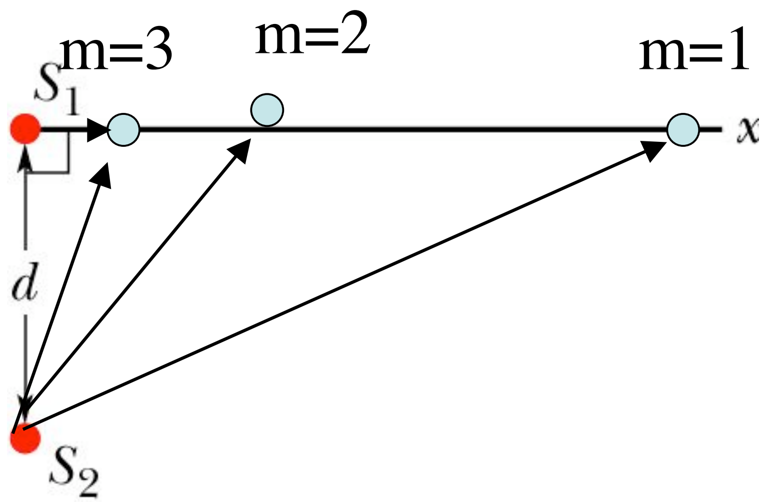
$\sqrt{\mathrm{d}^2 + \mathrm{x}^2} = \mathrm{m}\lambda + \mathrm{x}$ Now square both sides

$$d^2 + x^2 = (m\lambda + x)^2$$

$d^2 + x^2 = m^2\lambda^2 + 2m\lambda x + x^2$ Now cancel x^2

$d^2 = m^2\lambda^2 + 2m\lambda x$ solve for x

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} \text{ for m} = 1,\ 2,\ 3,..$$



$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda}.$$

$$x = \frac{16 - m^2}{2m}$$

$$For \quad m = 3 \quad x = \frac{16-(3)^2}{(2)(3)} = 1.17m.$$

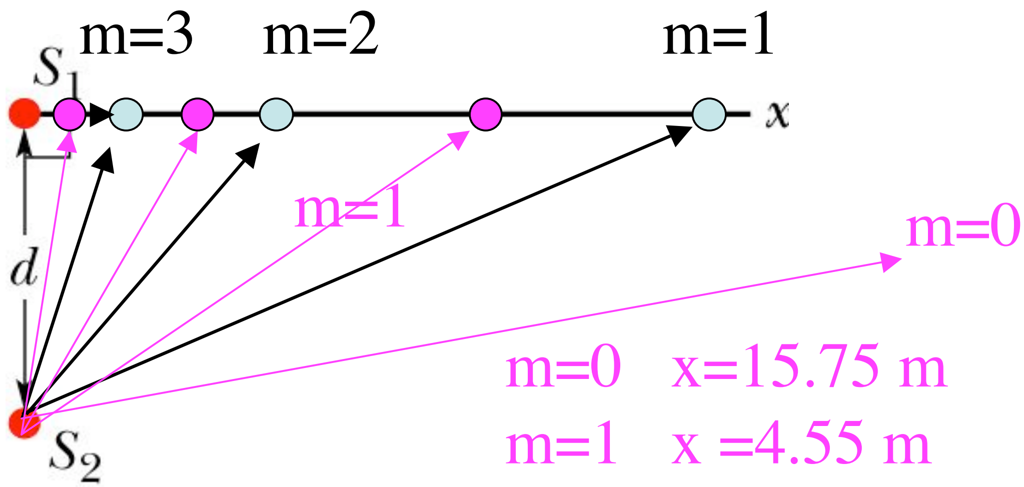
$$For \quad m = 2 \quad x = \frac{16-(2)^2}{(2)(2)} = 3.0m.$$

$$For \quad m = 1 \quad x = \frac{16-(1)^2}{(2)(1)} = 7.5m.$$

What about m = 4 ? This corresponds to x=0. Path difference =4 meters.

Where do the minima occur?

$$\text{path difference} = \sqrt{d^2 + x^2} - x = (m + \tfrac{1}{2})\lambda \qquad m = 0,1,2,3$$



$$x = \frac{d^2 - (m+\frac{1}{2})^2\lambda^2}{2(m+\frac{1}{2})\lambda}.$$

$$x = \frac{16 - (m+\frac{1}{2})^2}{2(m+\frac{1}{2})}.$$

m=0 x=15.75 m
m=1 x =4.55 m
m=2 x=1.95 m
m=3 x= 0.55 m

Although the amplitudes are the same at the sources, the waves travel different distances to get to the points of minimum intensity and each amplitude decreases in inverse proportion to the square of the distance traveled. The intensity is not zero at the minima positions.

$$I_1 = \frac{P_0}{4\pi x^2} \qquad I_2 = \frac{P_0}{4\pi(d^2 + x^2)} \qquad \frac{I_1}{I_2} = \frac{x^2}{d^2 + x^2} = \frac{(0.55)^2}{4^2 + (0.55)^2} \sim \frac{1}{64}$$

Demo with speakers using sound waves

Set oscillator frequency to 1372 Hz,
Then wavelength of sound is 343/1372 =0.25 m

Set speakers apart by 1m. Then maxima occur at $$x = \frac{d^2 - m^2\lambda^2}{2m\lambda}.$$

$$x = \frac{1 - m^2(1/16)}{2m(1/4)}$$

$$x = \frac{16 - m^2}{8 - m}$$

$m = 4 \quad x = 0$

$m = 3 \quad \mathrm{x} = \frac{16\text{-}9}{24} = \frac{7}{24} m \approx 0.33m$

$m = 2 \quad \mathrm{x} = \frac{16\text{-}4}{16} = \frac{12}{16} m \approx 0.75m$

$m = 1 \quad \mathrm{x} = \frac{16\text{-}1}{7} = \frac{15}{7} m \approx 2.0m$