Coulombs Law
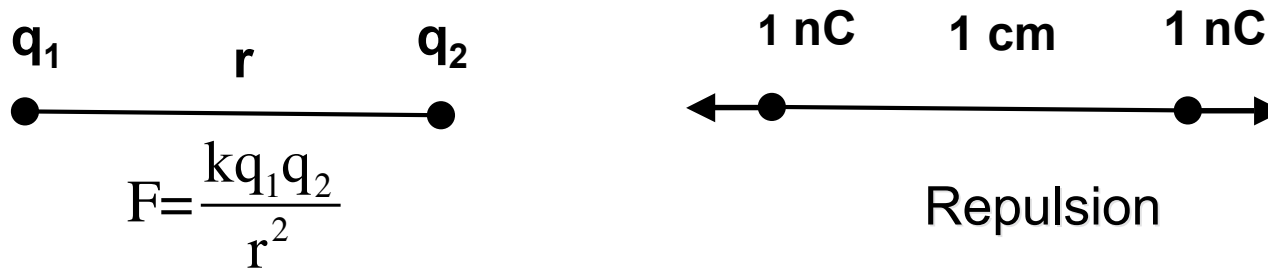# Two point Charges

•What is the force between two positive charges each 1 nanoCoulomb 1cm apart in a typical demo?

q_1 r q_2

1 nC   1 cm   1 nC

$$F = \frac{kq_1q_2}{r^2}$$

Repulsion

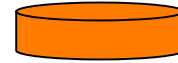$$F = \frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)\left(10^{-9}c\right)^2}{\left(10^{-2}m\right)^2} = 10^{-4}N$$

(equivalent to a weight of something with a mass of 10^{-5} kg = 10^{-2} gm or 10 mg - long strand of hair)

Coulombs Law
Two Pennies

What is the force between two 3 gm pennies one meter apart if we remove all the electrons from the copper atoms? (Modeling)

$$F = \frac{kq_1 q_2}{r^2} = \frac{\left(10^{10} \; \frac{Nm^2}{C^2}\right) q^2}{(1m)^2}$$

The force is
$$F = \frac{\left(10^{10} \; \frac{Nm^2}{C^2}\right)\left(1.4 \times 10^5 C\right)^2}{1m^2} = 2 \times 10^{20} N$$

The atom Cu has 29 protons and a 3 gm penny has

$$= \left(\frac{3gm}{63.5\,gm}\right) \times 6 \times 10^{23}\,atoms = 3 \times 10^{22}\,atoms$$

The total charge is $q = 29 \times 3 \times 10^{22}\,atoms \times 1.6 \times 10^{-19} C = 1.4 \times 10^5 C$

What is their acceleration as they separate?

$$a = \frac{F}{m} = \frac{2x10^{20} N}{3x10^{-3} kg} = 7x10^{22} \; \frac{m}{s^2}$$

Principle of Superposition
Three Charges in a Line

- In the previous example we tacitly assumed that the forces between nuclei simply added and did not interfere with each other. That is the force between two nuclei in each penny is the same as if all the others were not there. This idea is correct and is referred to as the Principle of Superposition.
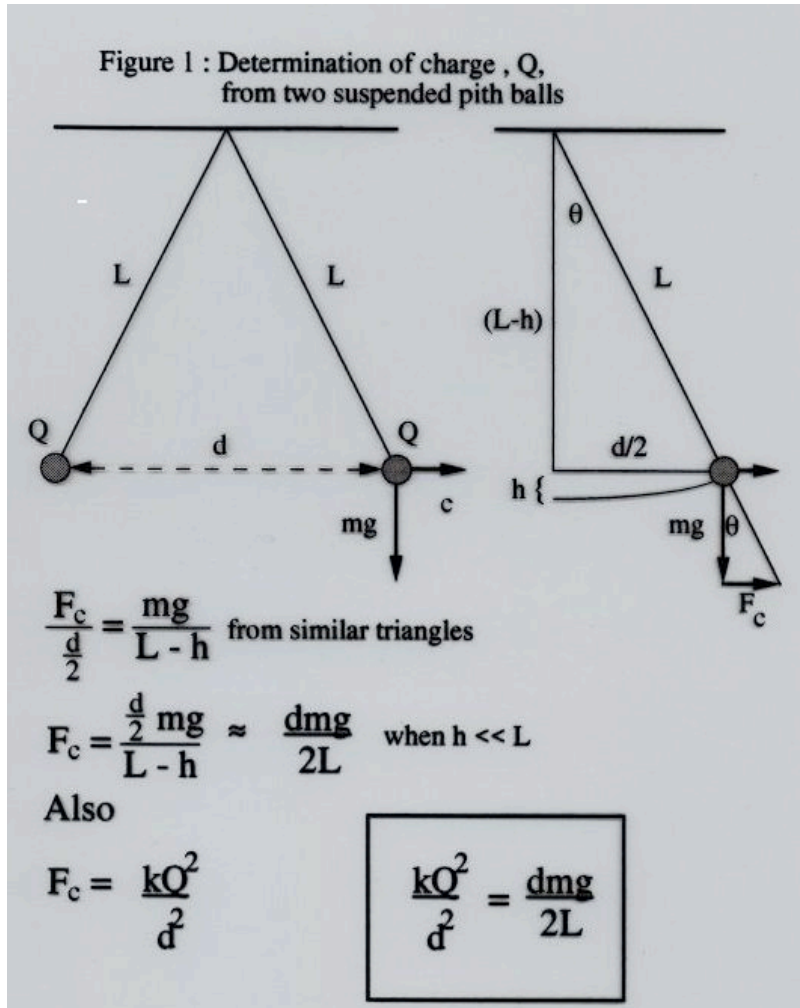
- Example of charges in a line

  1   2   3
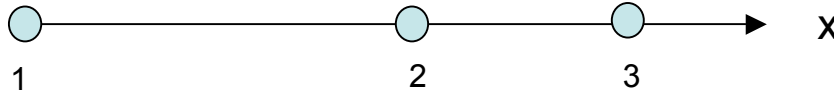
  - Three charges lie on the x axis: q_1=+25 nC at the origin, q_2= -12 nC at x =2m, q_3=+18 nC at x=3 m. What is the net force on q_1? We simply add the two forces keeping track of their directions. Let a positive force be one in the + x direction.

$$F = -kq_1 \left(\frac{q_2}{(2m)^2} + \frac{q_3}{(3m)^2} \right)$$

$$= -\left(10^{10} \, \tfrac{Nm^2}{C^2}\right)(25 \times 10^{-9} C)\left(\frac{-12 \times 10^{-9} C}{(2m)^2} + \frac{18 \times 10^{-9} C}{(3m)^2} \right)$$

$$= 2.5 \times 10^{-7} \, N$$

Two hanging equally charged pith balls

Find charge Q on two pith balls separated by distance d.
Approximate Solution

Figure 1 : Determination of charge , Q, from two suspended pith balls

$$\frac{F_c}{\frac{d}{2}} = \frac{mg}{L-h} \quad \text{from similar triangles}$$

$$F_c = \frac{\frac{d}{2}mg}{L-h} \approx \frac{dmg}{2L} \quad \text{when } h << L$$

Also

$$F_c = \frac{kQ^2}{d^2}$$

$$\boxed{\frac{kQ^2}{d^2} = \frac{dmg}{2L}}$$

$$Q = \sqrt{mgd^3/(2Lk)}$$

Suppose d =2 cm, L=20 cm, and
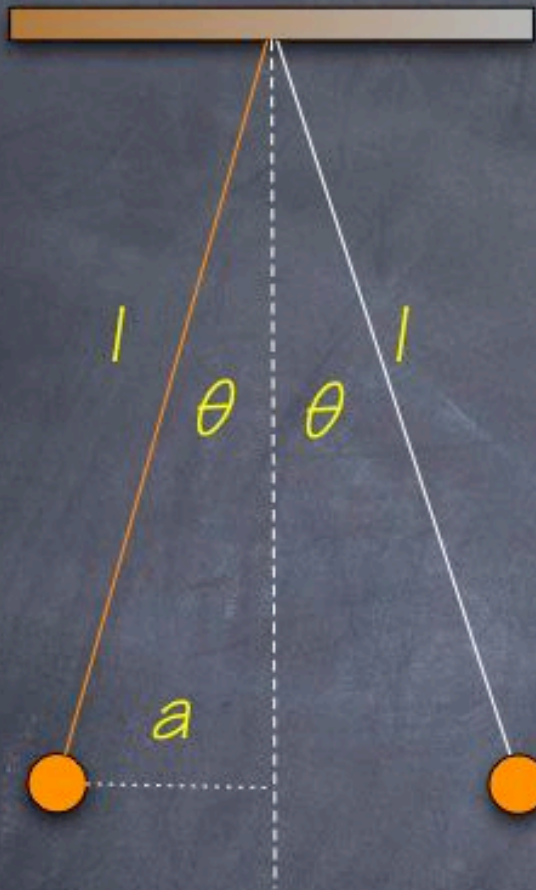m = 0.20 g, Find Q in nanoCoulombs.

Change to proper
units:m,kg,s,N,C

$$Q = \sqrt{.0002 * 9.81 * (0.02)^3 /(2 * 0.20 * 10^{10})}$$

$$Q = \sqrt{2 * 10^{-4} * 9.81 * (2 * 10^{-2})^3 /(2 * 0.20 * 10^{10})}$$

$$Q = \sqrt{16 * 10^{-10} * 9.81 * /(0.4 * 10^{10})}$$

$$Q = \sqrt{40 * 10^{-20} * 9.81} = \sqrt{400 * 10^{-20}} = \sqrt{4 * 10^{-18}} C$$
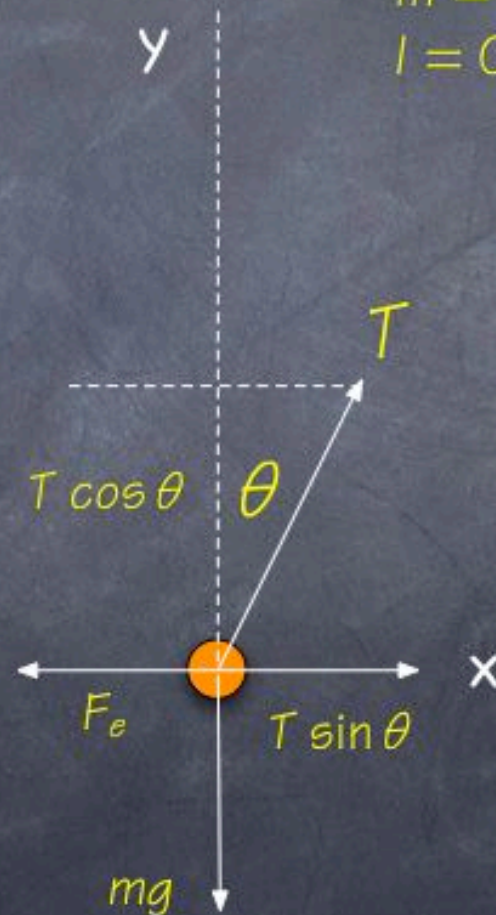
$$Q = 2 \times 10^{-9} C = 2 \text{ nanoC}$$

2 spheres are in equilibrium and have equal charge. What is the the charge on each sphere if:

$\theta = 5°$
$m = 3(10)^{-2} kg$
$l = 0.15m$

Exact Solution

$$\sum F_x = F_e - T \sin \theta = 0$$

$$\sum F_y = T \cos \theta - mg = 0$$

$$\sum F_x = F_e - T\sin\theta = 0 \quad \text{Eq. 1}$$

$$\sum F_y = T\cos\theta - mg = 0 \quad \text{Eq. 2}$$
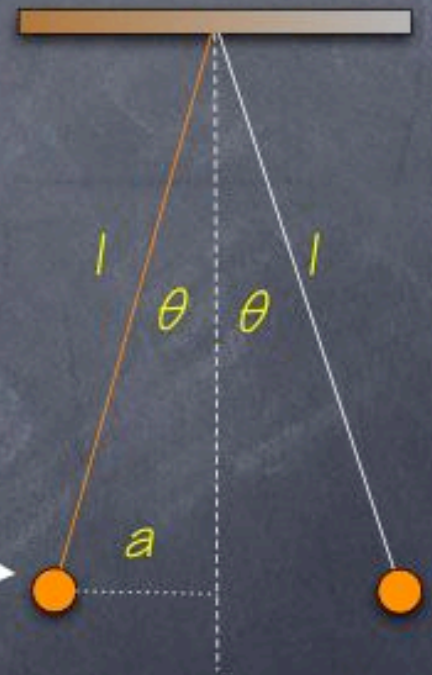
Eq. 2 tells us that : $T = mg/\cos\theta$

Combine with Eq. 1 to get:
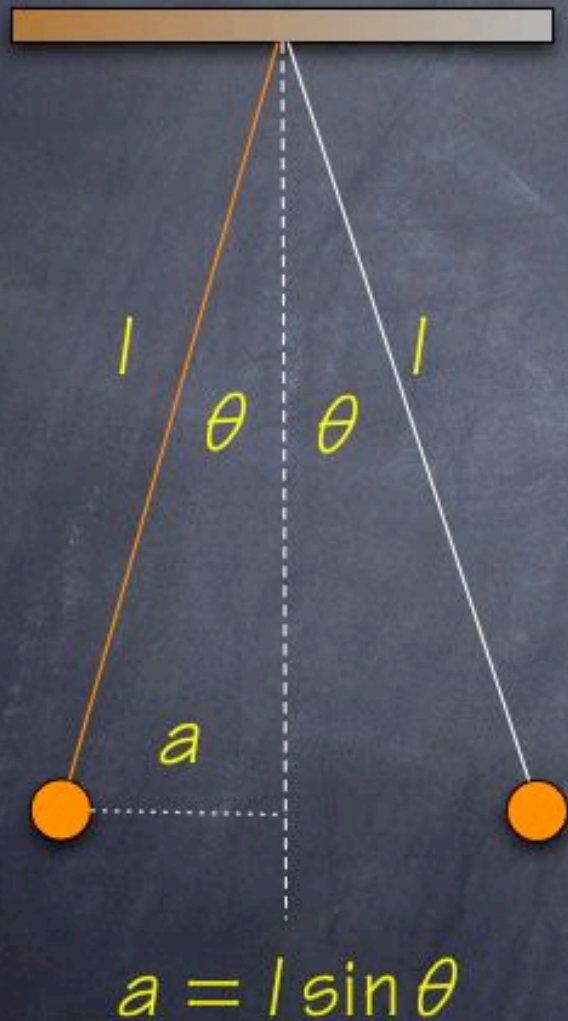
$$F_e = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$$

Using Coulomb's Law for F_e

$$F_e = \frac{1}{4\pi\epsilon_O}\frac{q^2}{(2a)^2} = mg\tan\theta$$

Solve for q: $q = \sqrt{4\pi\epsilon_O(2a)^2 mg\tan\theta}$

But what is a?

$a = l\sin\theta$

$\theta = 5°$

$m = 3(10)^{-2}kg$

$l = 0.15m$

$$k = \frac{1}{4\pi\epsilon_o}\frac{Nm^2}{C^2}$$

$$q = \sqrt{4\pi\epsilon_0(2l\sin\theta)^2 mg\tan\theta}$$

Units balance: $C = \sqrt{\frac{C^2}{Nm^2}\cdot m^2 N}$

Plug in numbers

$q = 4.4\times 10^{-8} Coulomb$

Force on a charge due to an infinitely long wire.

Charge Q is uniformly distributed on the infinite wire with charge density λ. We choose the Y-axis through the line charge and the X-axis through the point charge. Now, because the line charge is infinite and straight (on Y-axis), we can assume the X-axis is the mid-point of the infinite wire.
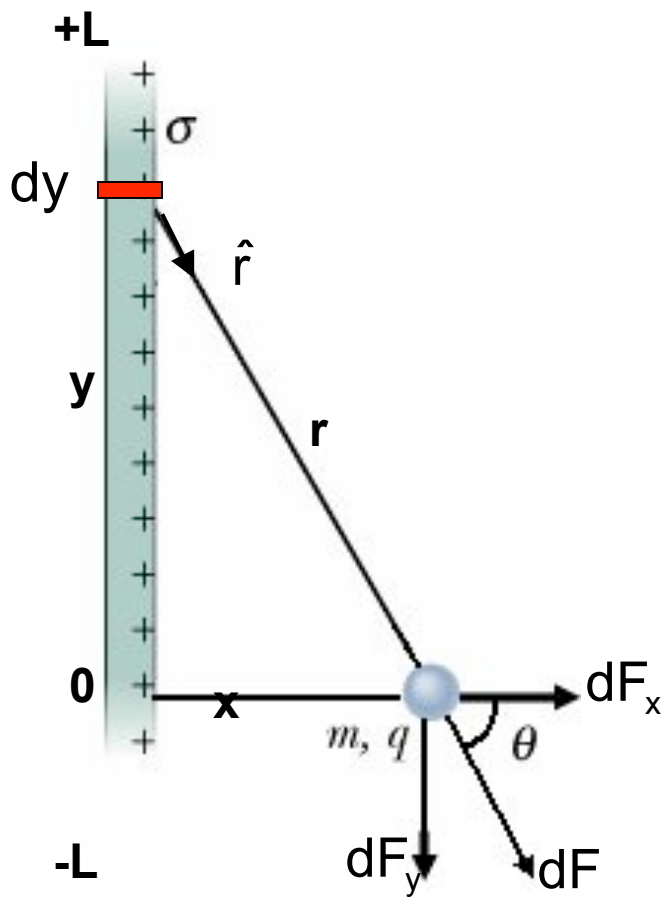
Then by Coulomb's law for the small line segment dy

$$d\vec{F} = \frac{kqdQ}{r^2}\hat{r} \quad \text{but } dQ = \lambda dy \quad \Rightarrow d\vec{F} = \frac{kq\lambda dy}{r^2}\hat{r}$$

Resolving the forces into its components:

$$F = \int dF = \hat{i}\int dF_x + \hat{j}\int dF_y \quad \text{but by symmetry } \int dF_y = 0$$

As it is can be seen that the y-component of the force exerted by the top half of the infinite wire will cancel the y-component force exerted by the bottom half of the infinite wire.

+L

-L

dy

y

0

x

$\hat{r}$

r

σ

m, q

θ

dF$_x$

dF$_y$

dF

Force on a charge due to an infinitely long wire cont.
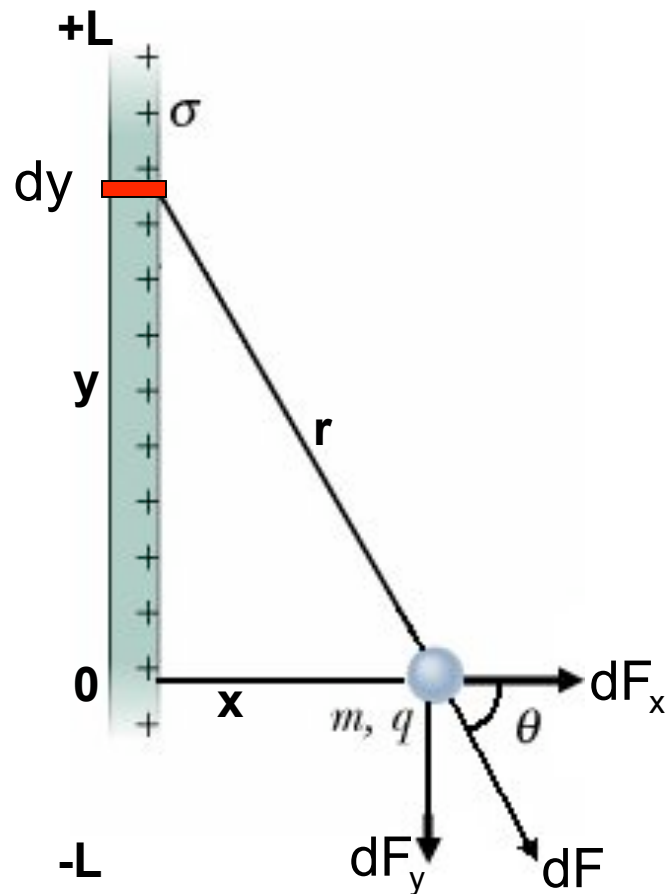
So, $F = \int dF_x = \int \cos\theta \, dF$ also $r^2 = x^2 + y^2$

$$\vec{F} = \int_{-L}^{+L} \frac{kq\lambda dy}{(x^2 + y^2)} \cos\theta \, \hat{r} \qquad \cos\theta = \frac{x}{\sqrt{(x^2 + y^2)}}$$

$$F = kq\lambda x \int_{-L}^{+L} \frac{dy}{(x^2 + y^2)^{\frac{3}{2}}}$$

Now, $\int \frac{dy}{(ay^2 + c)^{\frac{3}{2}}} = \frac{y}{c\sqrt{ay^2 + c}}$

So, $F = \dfrac{2kq\lambda Lx}{x^2\sqrt{L^2 + x^2}}$ So, $F = \dfrac{2kq\lambda}{x}\left(\dfrac{1}{\sqrt{1 + \frac{x^2}{L^2}}}\right)$

Now, for an infinite wire, $L \rightarrow \infty$

So, $F = \dfrac{2kq\lambda}{x}$

You can also use polar coordinates for this integration.