Lecture 2 Electric Fields Chp. 22 Ed. 7

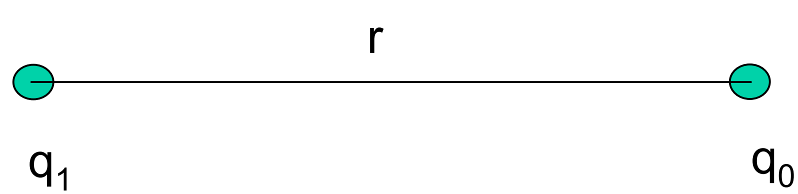
- Cartoon - Analogous to gravitational field
- Warm-up problems , Physlet
- Topics
  - Electric field = Force per unit Charge
  - Electric Field Lines
  - Electric field from more than 1 charge
  - Electric Dipoles
  - Motion of point charges in an electric field
  - Examples of finding electric fields from continuous charges
- List of Demos
  - Van de Graaff Generator, workings,lightning rod, electroscope,
  - Field lines using felt,oil, and 10 KV supply.,
    - One point charge
    - Two same sign point charges
    - Two opposite point charges
    - Slab of charge
  - Smoke remover or electrostatic precipitator
  - Kelvin water drop generator
  - Electrophorus

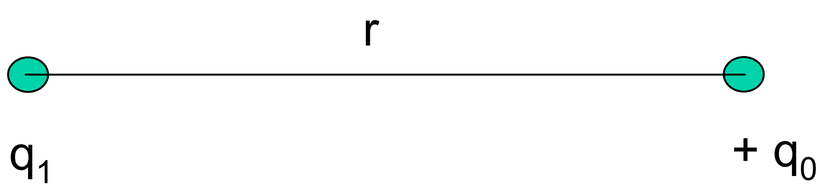
Concept of the Electric Field

- Definition of the electric field. Whenever charges are present and if I bring up another charge, it will feel a net Coulomb force from all the others. It is convenient to say that there is field there equal to the force per unit positive charge. **$E=F/q_0$.**

- The question is how does charge q_0 know about $_1$charge q_1 if it does not "touch it"? Even in a vacuum! We say there is a field produced by q_1 that extends out in space everywhere.

- The direction of the electric field is along r and points in the direction a positive test charge would move. This idea was proposed by Michael Faraday in the 1830's. The idea of the field replaces the charges as defining the situation. Consider two point charges:



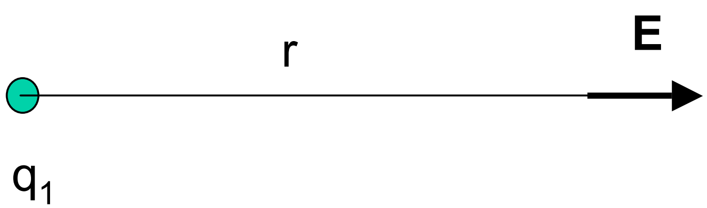


The Coulomb force is $\mathbf{F= kq_1q_0/r^2}$

The force per unit charge is $\mathbf{E = F/q_0}$

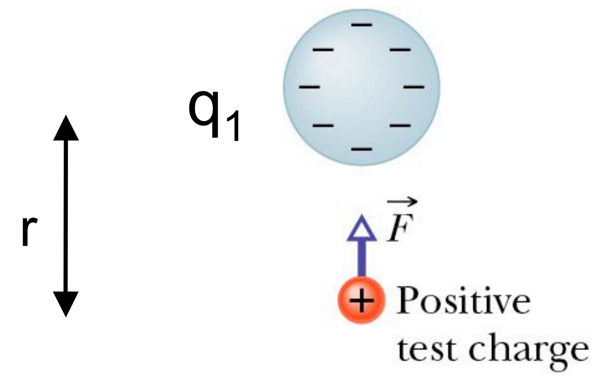
Then the electric field at r is $\mathbf{E = kq_1/r^2}$ due to the point charge q_1 .

The units are Newton/Coulomb. The electric field has direction and is a vector. How do we find the direction.? The direction is the direction a unit positive test charge would move. This is called a field line.



If q_1 were positive

Example of field lines for a point negative charge. Place a unit positive test chare at every point r and draw the direction that it would move



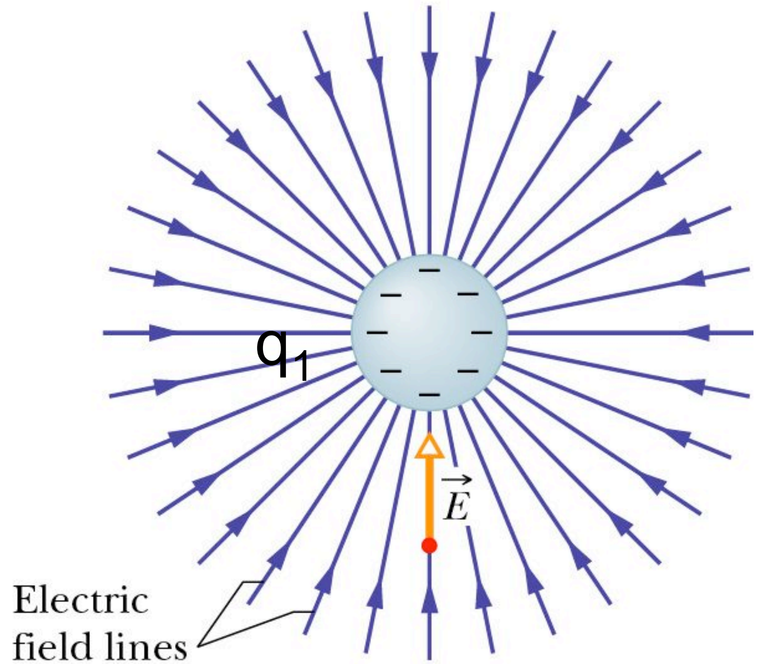
(a)

The blue lines are the field lines.
The magnitude of the electric field is

$$E = kq_1/r^2$$

The direction of the field is given by the line itself

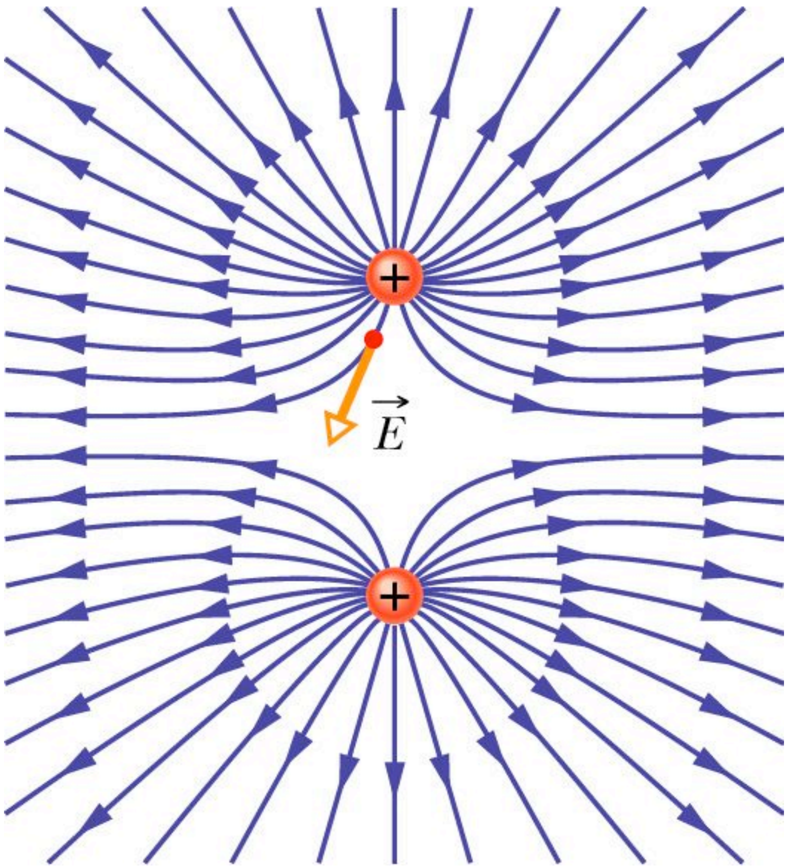
Important $F = Eq_0$, then $ma = q_0E$, and then $a = q_0E/m$



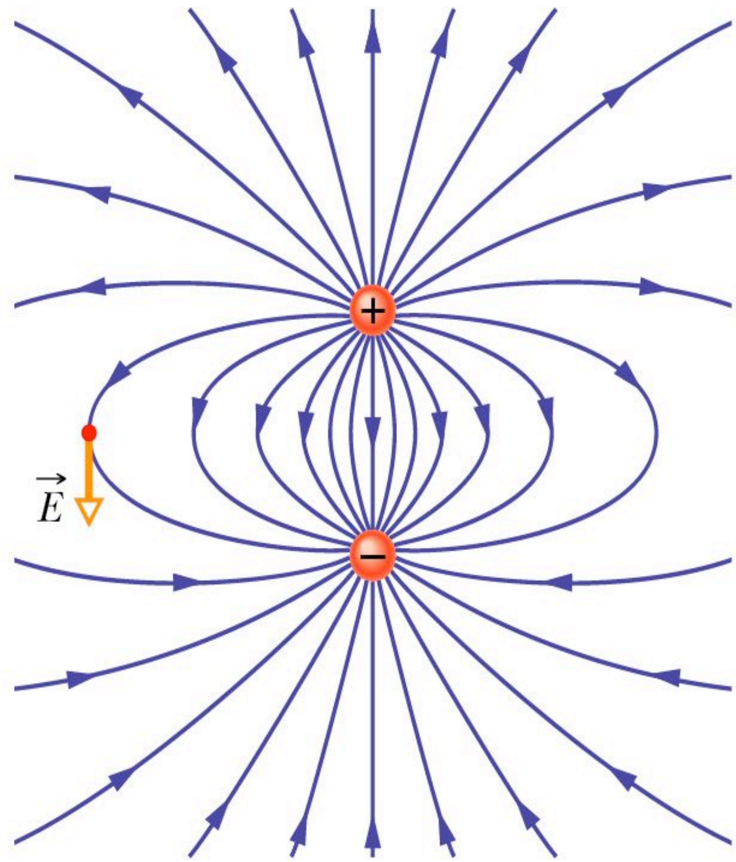
(b)

Electric Field Lines

Like charges (++)



Opposite charges (+ -)



This is called an electric dipole.

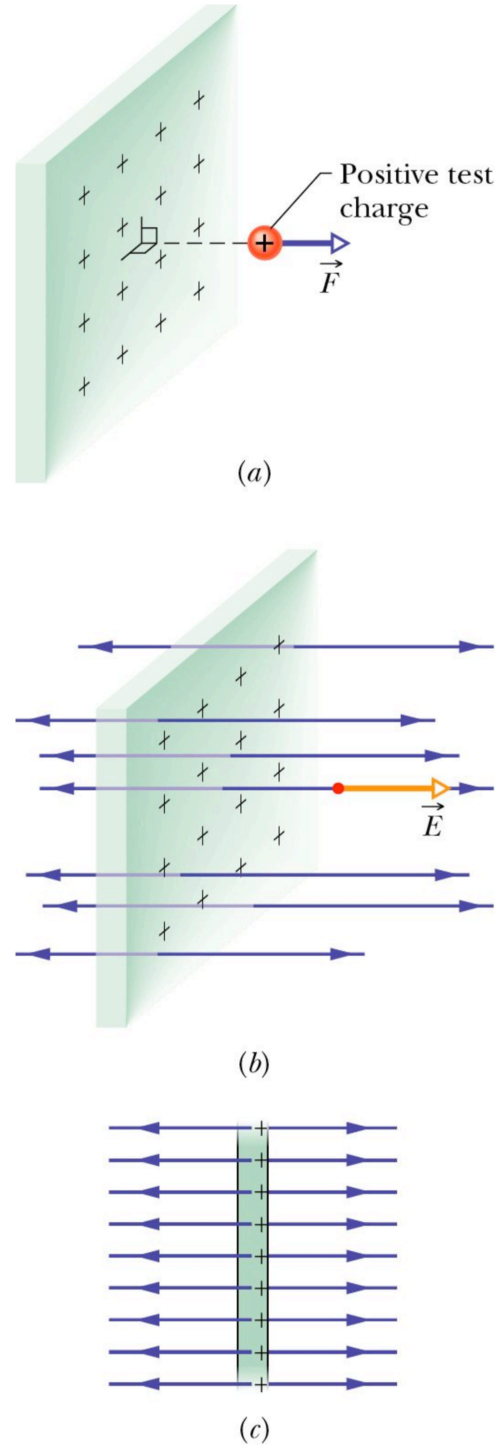
Electric Field Lines: a graphic concept used to draw pictures as an aid to develop intuition about its behavior.

The text shows a few examples. Here are the drawing rules.

- E-field lines begin on + charges and end on - charges. (or infinity).
- They enter or leave charge symmetrically.
- The number of lines entering or leaving a charge is proportional to the charge
- The density of lines indicates the strength of E at that point.
- At large distances from a system of charges, the lines become isotropic and radial as from a single point charge equal to the net charge of the system.
- No two field lines can cross.

Example of field lines for a uniform distribution of positive charge on one side of a very large nonconducting sheet.

This is called a uniform electric field.



In order to get a better idea of field lines try this Physlet.

- http://webphysics.davidson.edu/Applets/Applets.html
- Click on problems
- Click on Ch 9: E/M
- Play with Physlet 9.1.4, 9.1.7

Demo: Show field lines using felt, oil, and 10 KV supply

- One point charge
- Two point charges of same sign
- Two point charges opposite sign
- Wall of charge

Methods of evaluating electric fields

- Direct evaluation from Coulombs Law for a single point charge

$$E = \frac{kq_1}{r_1^2}$$

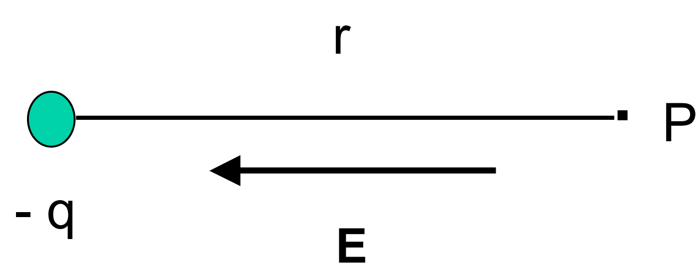
- For a group of point charges, perform the vector sum

$$E = \sum_{i=1}^{N} \frac{kq_i}{r_i^2}$$

- This is a vector equation and can be complex and messy to evaluate and we may have to resort to a computer. The principle of superposition guarantees the result.

Typical Electric Fields (SI Units)

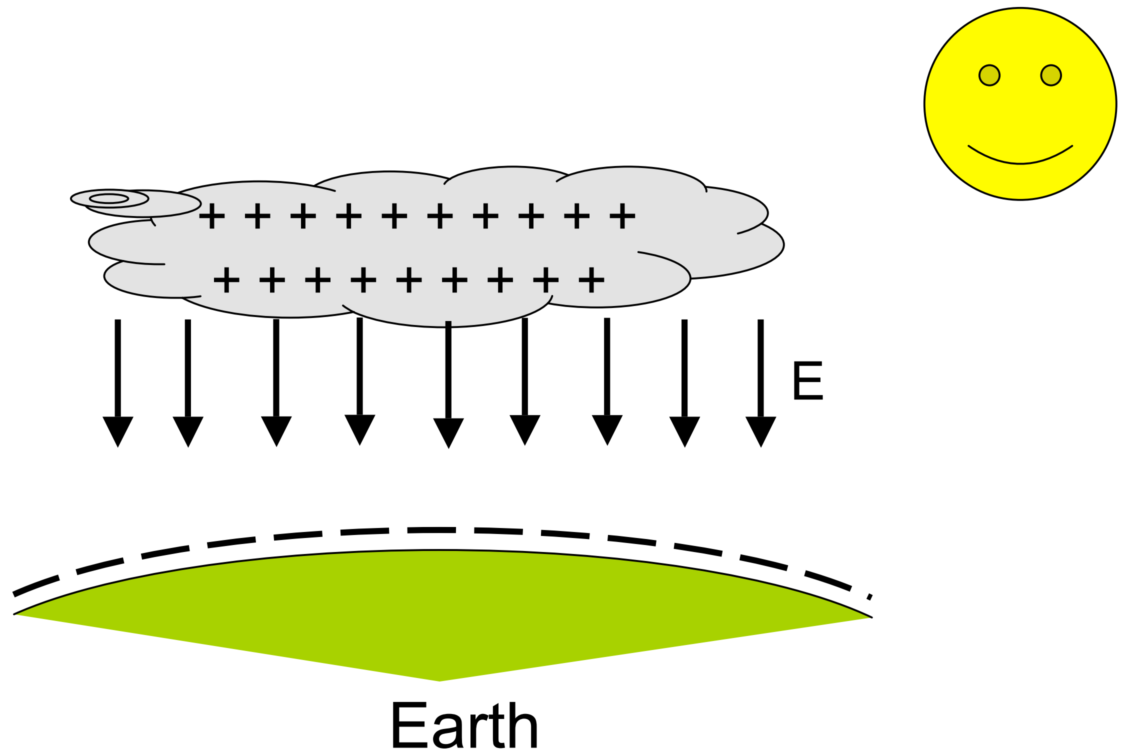
1 cm away from 1 nC of negative charge



$$E = \frac{kq_1}{r^2} = \frac{\left(10^{10}\,\mathrm{Nm^2}/\mathrm{C^2}\right)\cdot 10^{-9}\,\mathrm{C}}{10^{-4}\,\mathrm{m^2}} = 10^5\,\frac{\mathrm{N}}{\mathrm{C}}$$

Typical Electric Fields

Fair weather atmospheric electricity = $100 \frac{N}{C}$ downward 100 km high in the ionosphere

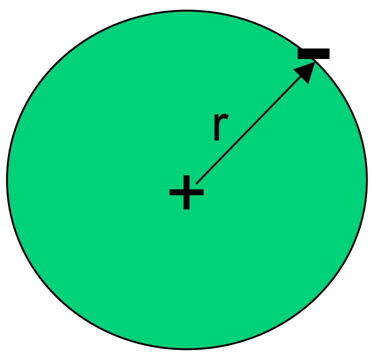


Typical Electric Fields

Field due to a proton at the location of the electron in the H atom. The radius of the electron orbit is 0.5×10^{-10}m.

$$E=\frac{kq_1}{r^2}=\frac{\left(10^{10}\,\mathrm{Nm^2}/\mathrm{C^2}\right)\cdot 1.6\times10^{-19}\,\mathrm{C}}{(0.5\times10^{-10}\,\mathrm{m})^2}=4\times10^{11}\,\frac{\mathrm{N}}{\mathrm{C}}$$

Hydrogen atom

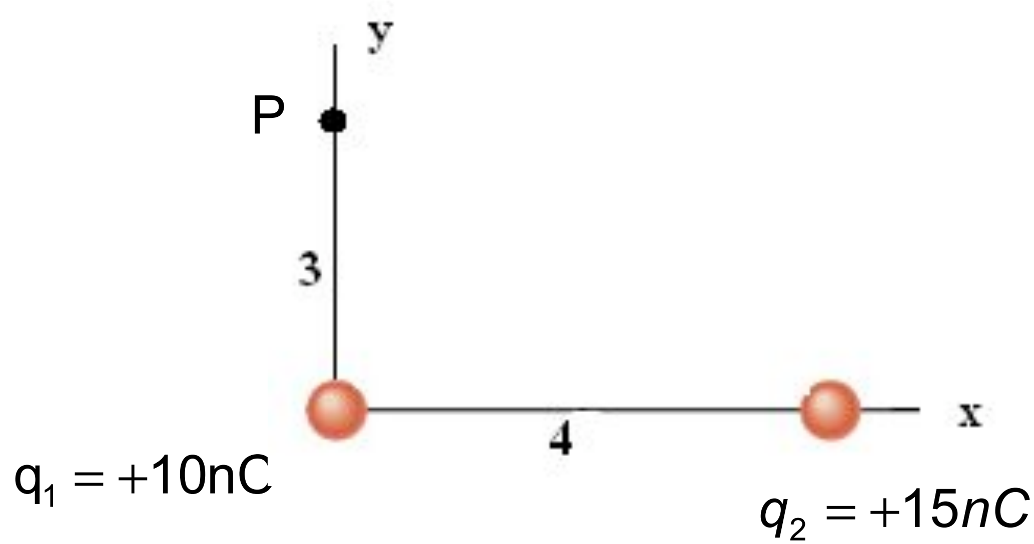


$$\text{Note}:\frac{\mathrm{N}}{\mathrm{C}}=\frac{\text{Volt}}{\text{meter}}$$

Example of finding electric field from two charges lying in a plane

We have $q_1 = +10nC$ at the origin, $q_2 = +15nC$ at $x = 4m$

What is E at $y = 3m$ and $x = 0$ (at point P)?



Use principle of superposition

Find x and y components of electric field due to both charges and add them up

Example continued

Recall $E = \frac{kq}{r^2}$

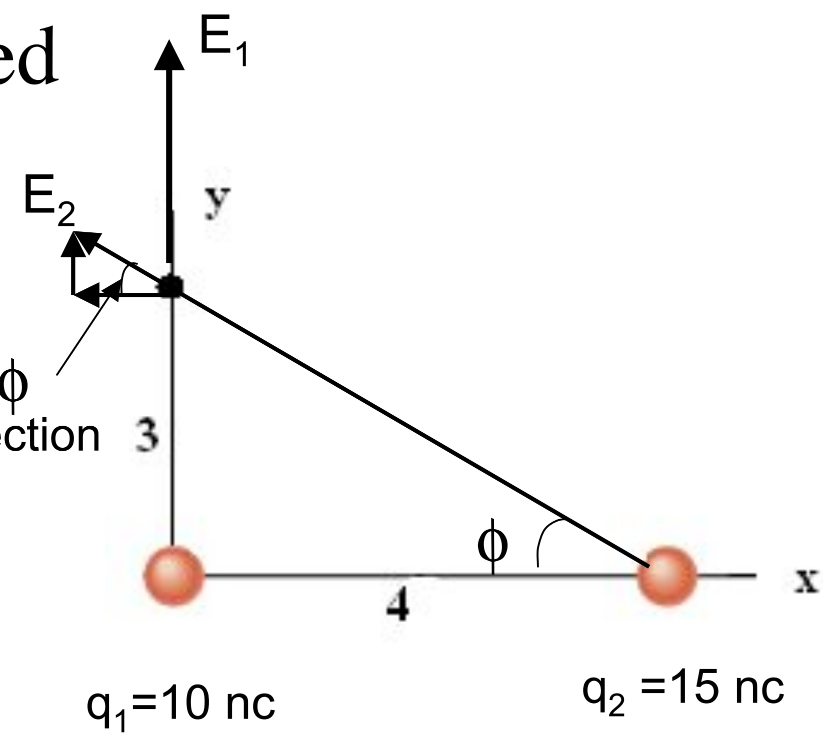
and $k = 8.99 \times 10^9 \left(Nm^2 / C^2 \right)$

Field due to q_1

$$E_1 = \frac{\left(10^{10} \mathrm{Nm}^2 / \mathrm{C}^2 \right) \times 10 \times 10^{-9} \mathrm{C}}{(3\mathrm{m})^2} = 11 \frac{\mathrm{N}}{\mathrm{C}} \text{ in the y direction}$$

$$E_{1y} = 11 \frac{\mathrm{N}}{\mathrm{C}}$$

$$E_{1x} = 0$$



Field due to $q_2 \left| E_2 \right|$

$$\left| E_2 \right| = \frac{\left(10^{10} \, Nm^2 / C^2 \right) \times 15 \times 10^{-9} \, C}{(5m)^2} = 6 \frac{N}{C} \text{ at some angle } \phi$$

Resolving it in x and y components :

$$E_{2y} = E \sin \varphi = 6 \frac{\mathrm{N}}{\mathrm{C}} \times \frac{3}{5} = \frac{18}{5} = 3.6 \frac{\mathrm{N}}{\mathrm{C}}$$

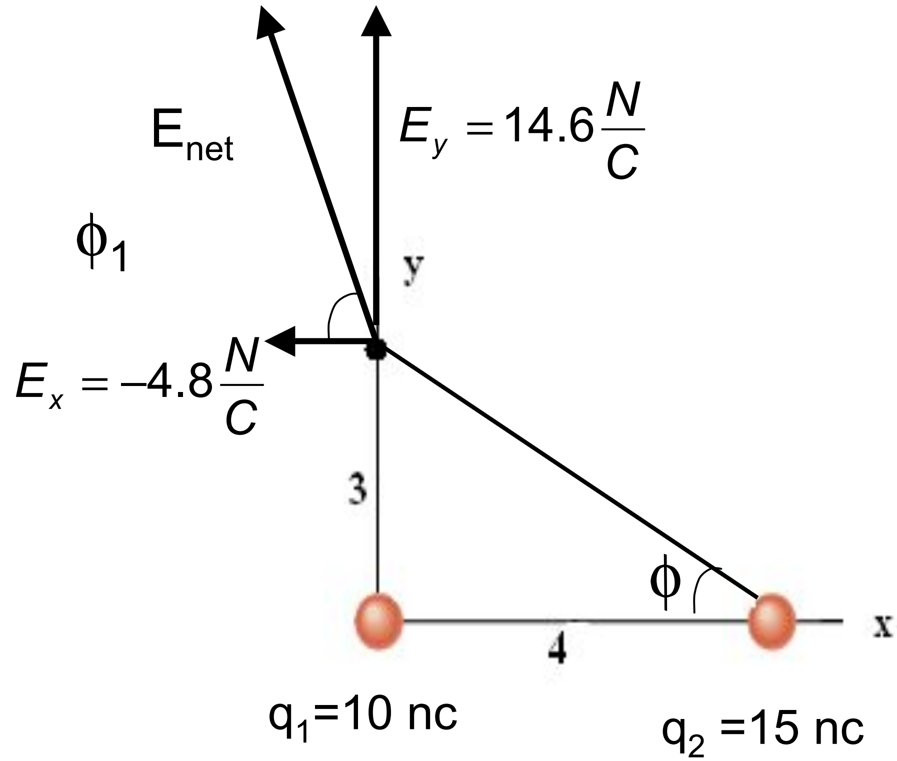
$$E_{2x} = E \cos \varphi = 6 \frac{\mathrm{N}}{\mathrm{C}} \times \frac{-4}{5} = \frac{-24}{5} = -4.8 \frac{\mathrm{N}}{\mathrm{C}}$$

Now add all components

$$E_{y_{net}} = (11 + 3.6) \frac{\mathrm{N}}{\mathrm{C}} = 14.6 \frac{\mathrm{N}}{\mathrm{C}}$$

$$E_{x_{net}} = -4.8 \frac{\mathrm{N}}{\mathrm{C}}$$

Example cont



Magnitude of total electric field is

$$E = \sqrt{E_x^2 + E_y^2}$$

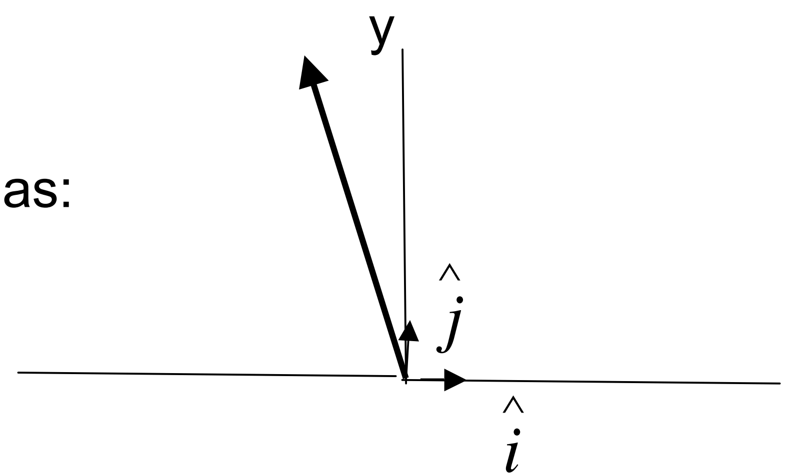
$$E = \sqrt{(14.6)^2 + (-4.8)^2} = 15.4 N/C$$

Direction of the total electric field is

$$\phi_1 = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{14.6}{-4.8}\right) = 107.2 \; or \; 72.8° \text{ (degrees)}$$

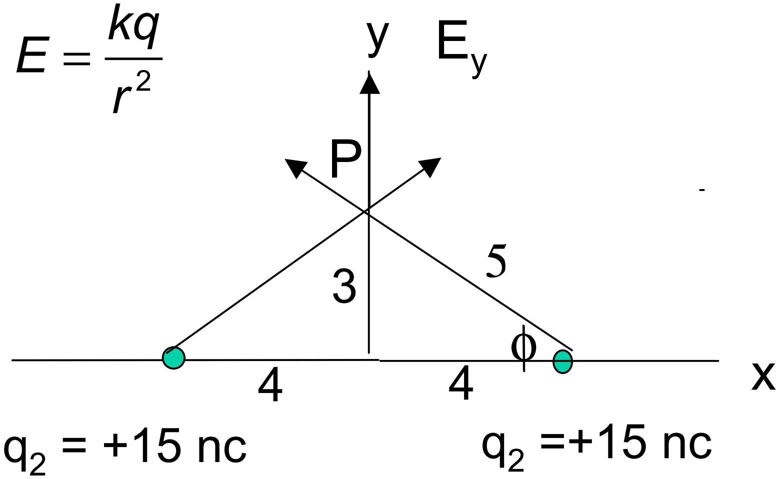
Using unit vector notation we can also write the electric field vector as:

$$\vec{E} = -4.8\hat{i} + 14.6\hat{j}$$

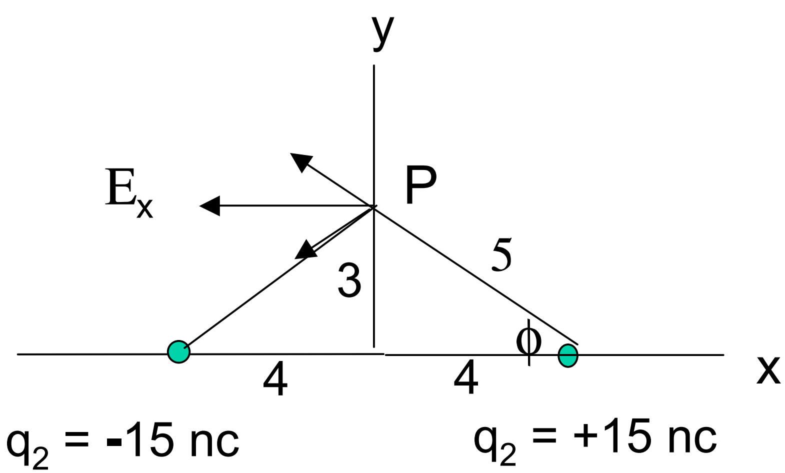


Example of two identical charges on the x axis. What is the field on the y axis at P?

$$E = \frac{kq}{r^2}$$



Example of two opposite charges on the x axis. What is the field on the y axis at P?



$E = 10^{10}$ N.m^2/C^2 15 X10^{-9} C/(5m)^2 = 6 N/C

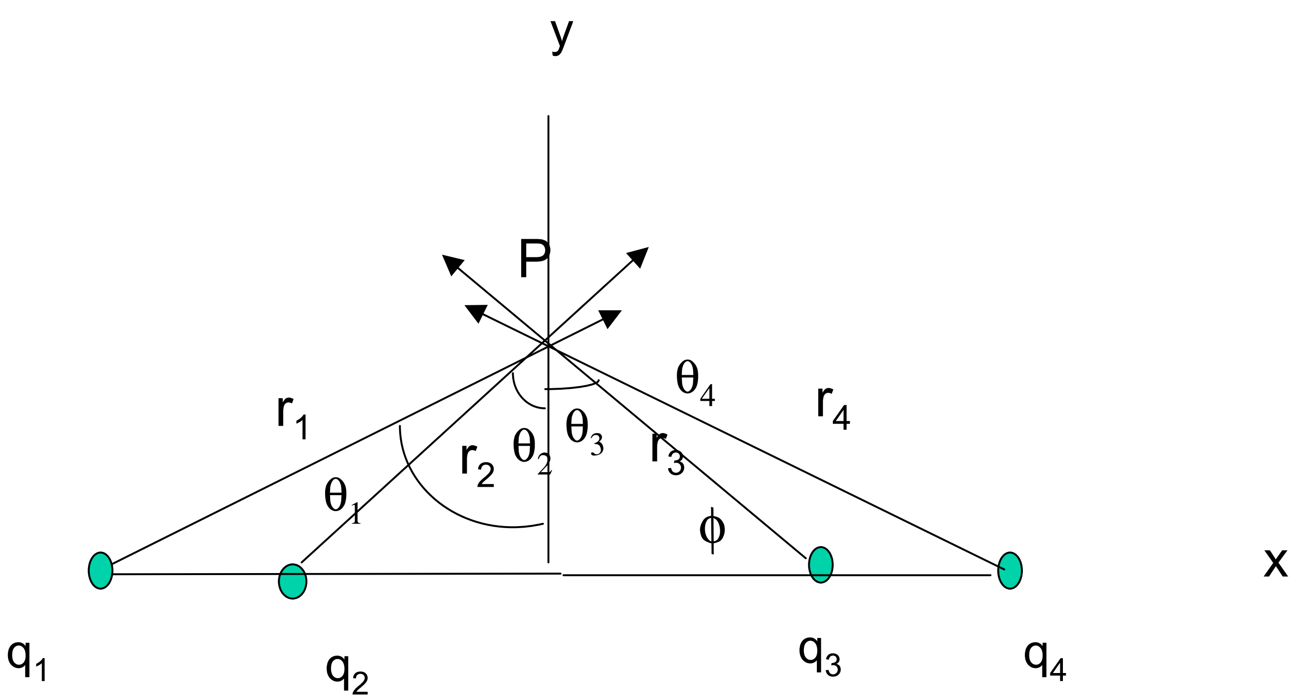
E_y=2*E sin $\phi = 2*6*3/5$ = 7.2 N/C

E_x=0

E_y=0

E_x=2*E cos $\phi = 2*6*4/5$ = - 9.6 N/C

4 equal charges symmetrically spaced along a line. What is the field at point P? (y and x = 0)



$$E_y = k\sum_{i=1}^{4} q_i \cos\theta_i / r_i^2 \qquad E_y = k\sum_{i=1}^{\infty} \Delta q_i \cos\theta_i / r_i^2$$

Continuous distribution of charges

- Instead of summing the charge we can imagine a continuous distribution and integrate it. This distribution may be over a volume, a surface or just a line.

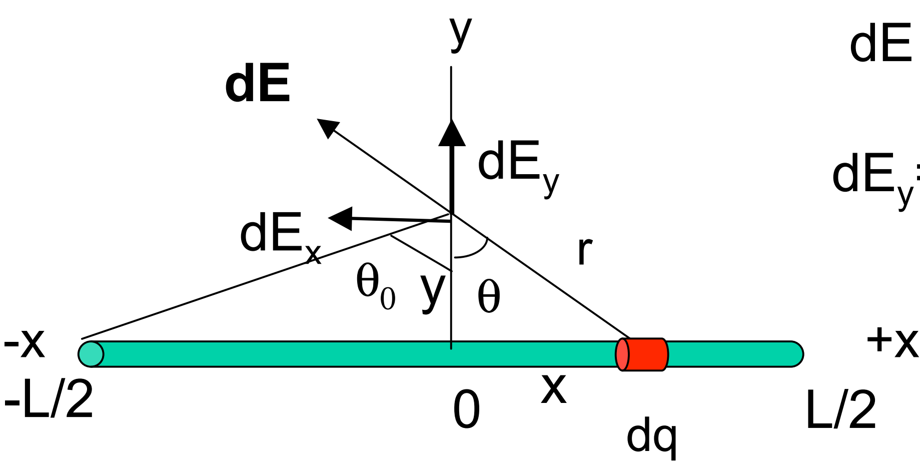
$$E = \int dE = \int kdq / r^2$$

$$dq = \rho dV \quad volume$$

$$dq = \sigma dA \quad \text{area}$$

$$dq = \lambda dl \quad \text{line}$$

Find electric field due to a line of uniform + charge of length L with linear charge density equal to λ



$dE = k\, dq / r^2$

$dE_y = dE \cos\theta$

$$E_x = \int_{-L/2}^{L/2} dE_x = 0$$

$dq = \lambda dx$

$dE_y = k\,\lambda\, dx \cos\theta / r^2$

$E_y = k\,\lambda\, q \cos\theta / r^2$ for a point charge

$$E_y = k\lambda \int_{-L/2}^{L/2} \cos\theta\, dx / r^2$$

$x = y \tan\theta \quad dx = y \sec^2\theta\, d\theta$

$r = y \sec\theta \quad r^2 = y^2 \sec^2\theta$

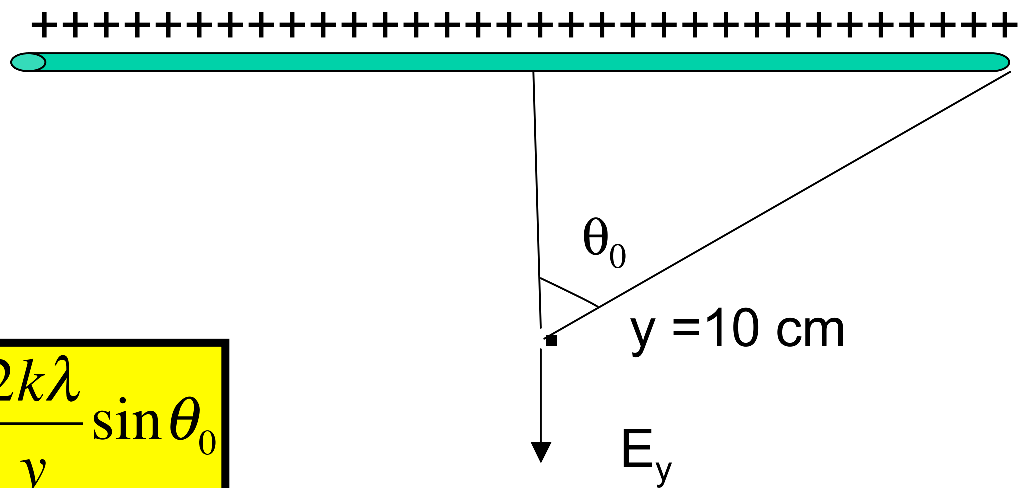
$dx/r^2 = d\theta/y$

$$E_y = k\lambda / y \int_{-\theta_0}^{\theta_0} \cos\theta\, d\theta$$

$$E_y = \frac{2k\lambda}{y} \sin\theta_0$$

$$\sin\theta_0 = \frac{L/2}{\sqrt{y^2 + L^2/4}}$$

What is the electric field from an infinitely long wire with linear charge density of +100 nC/m at a point 10 away from it. What do the lines of flux look like?



Θ_0=90 for an infinitely long wire

$$E_y = \frac{2k\lambda}{y}\sin\theta_0$$

$$E_y = \frac{2*10^{10}Nm^2*100*10^{-9}C/m}{0.1m}\sin 90 = 2*10^4 N/C$$

Typical field for the electrostatic smoke remover

Example of two opposite charges on the x axis. What is the field on the y axis?

Electric field at point P at a distance r due to the two point charges L distance across is the sum of the electric fields due to ***+q*** and ***–q*** (superposition principle).

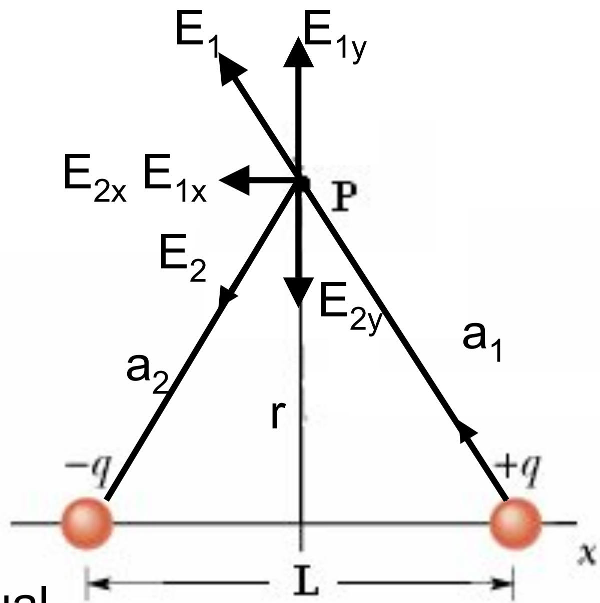
Electric field due to the ***+q*** is

$$E_1 = \frac{k(+q)}{a_1^{\,2}}$$

Electric field due to the ***-q*** is

$$E_2 = \frac{k(-q)}{a_2^{\,2}}$$

Now, $a_1 = \sqrt{r^2 + \left(\frac{L}{2}\right)^2} = a_2 = a$



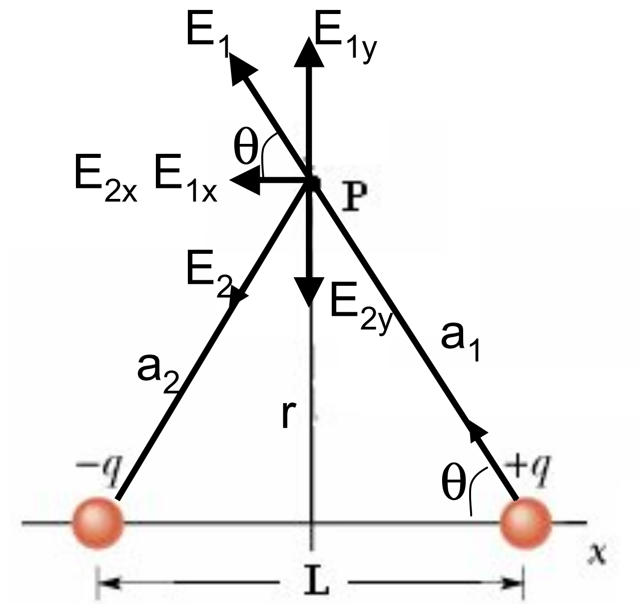
Notice that the magnitude of E_1 and E_2 are equal. Also we can see that the y-component of both the fields are equal and in opposite direction, so they cancel out. The x-component of both the fields are equal and are in the same direction, so they add up.

$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = E$ and $\vec{E}_{net} = (E_{1x} + E_{2x})\hat{i} + (E_{1y} + E_{2y})\hat{j}$

But $E_{1y} + E_{2y} = 0$

$E_{1x} + E_{2x} = -2E\cos\theta$

So, $\vec{E}_{net} = -2E\cos\theta\,\hat{i}$ Now, $E = \frac{kq}{a^2}$



So, $\vec{E}_{net} = -2\frac{kq}{a^2}\cos\theta\,\hat{i}$

But $\cos\theta = \frac{\frac{L}{2}}{a} = \frac{L}{2a}$

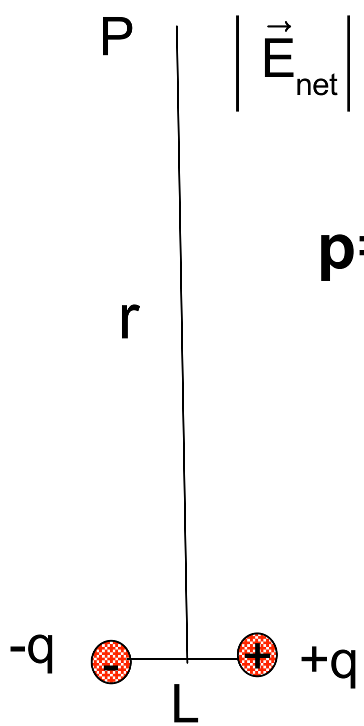
So, $\vec{E}_{net} = -2\frac{kq}{a^2}\frac{L}{2a}\hat{i}$

$\vec{E}_{net} = \frac{kqL}{a^3}(-\hat{i})$ as $a = \sqrt{r^2 + \left(\frac{L}{2}\right)^2}$ and $|p| = |qL|$

$\left|\vec{E}_{net}\right| = k\frac{p}{\left(r^2 + (\frac{L}{2})^2\right)^{\frac{3}{2}}} = k\frac{p}{r^3}\left(\frac{1}{(1 + (\frac{L}{2r})^2}\right)^{\frac{3}{2}}$ Now when r>>L, the term $\left(\frac{L}{2r}\right)^2 \to 0$

$\left|\vec{E}_{net}\right| \simeq \frac{kp}{r^3}$

This is called an Electric Dipole: A pair of equal and opposite charges q separated by a displacement L. It has an electric dipole moment p=qL.



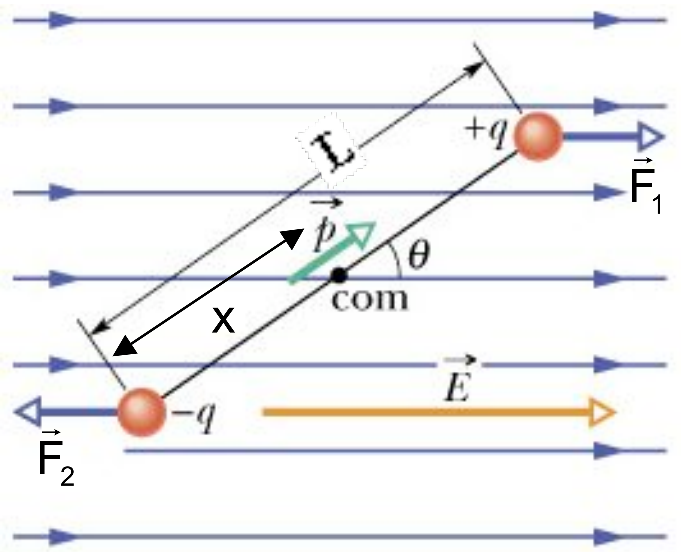
$$\left|\vec{E}_{net}\right| \simeq \frac{kp}{r^3}$$ when r is large compared to L

p=qL = the electric dipole moment

Note inverse **cube** law

Electric Dipoles in Electric fields

A uniform external electric field exerts no net force on a dipole, but it does exert torque that tends to rotate the dipole in the direction of the field (align $\vec{p}$ with $\vec{E}_{ext}$)



Torque about the com = τ

$$= Fx\sin\theta + F(L - x)\sin\theta = FL\sin\theta$$

$$= qEL\sin\theta = pE\sin\theta = \vec{p}\times\vec{E}$$

So, $\vec{\tau} = \vec{p}\times\vec{E}$

When the dipole rotates through $d\theta$, the electric field does work:

Why do Electric Dipoles align with Electric fields ?

Work done equals $dW = -\tau d\theta = -pE\sin\theta d\theta$

The minus sign arises because the torque opposes any increase in θ.

Setting the negative of this work equal to the change in the potential energy, we have

$$dU = -dW = +pE\sin\theta d\theta$$

Integrating,

$$U = \int dU = -\int dW = -\int pE\sin\theta d\theta = -pE\cos\theta + U_0$$

We choose $U = 0$ when $\theta = 90°$

Potential Energy $= U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

$$\text{So,} \quad U = -\vec{p}\cdot\vec{E}$$

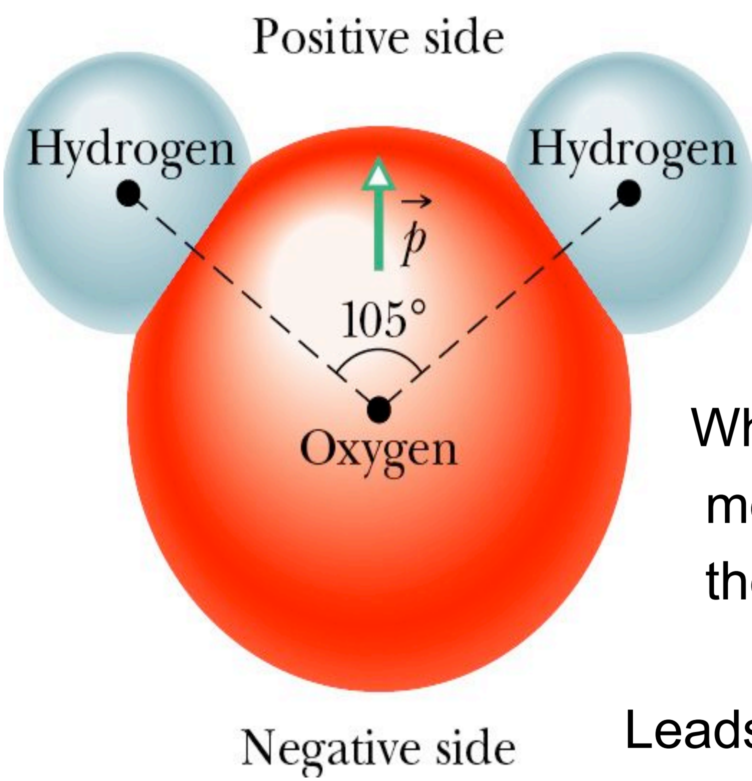
The energy is minimum when $\vec{p}$ aligns with $\vec{E}$

Water (H_2O) is a molecule that has a permanent dipole moment.

GIven $p = 6.2 \times 10^{-30}$ C m   And $q = -10$ e and $q = +10e$

What is d?

$d = p / 10e = 6.2 \times 10^{-30}$ C m / $10 * 1.6 \times 10^{-19}$ C $= 3.9 \times 10^{-12}$ m



Very small distance but still is responsible for the conductivity of water.

When a dipole is an electric field, the dipole moment wants to rotate to line up with the electric field. It experiences a torque.

Leads to how microwave ovens heat up food

Electric field gradient

- When a dipole is an electric field that varies with position, then the magnitude of the electric force will be different for the two charges. The dipole can be permanent like NaCl or water or induced as seen in the hanging pith ball. Induced dipoles are always attracted to the region of higher field. Explains why wood is attracted to the teflon rod and how a smoke remover or microwave oven works.

- Show smoke remover demo.

Smoke Remover

Negatively charged central wire has electric field that varies as 1/r (strong electric field gradient). Field induces a dipole moment on the smoke particles. The positive end gets attracted more to the wire.

In the meantime a corona discharge is created. This just means that induced dipole moments in the air molecules cause them to be attracted towards the wire where they receive an electron and get repelled producing a cloud of ions around the wire.

When the smoke particle hits the wire it receives an electron and then is repelled to the side of the can where it sticks. However, it only has to enter the cloud of ions before it is repelled.

It would also work if the polarity of the wire is reversed

Motion of point charges in electric fields

- When a point charge such as an electron is placed in an electric field E, it is accelerated according to Newton's Law:

  a = F/m = qE/m for uniform electric fields

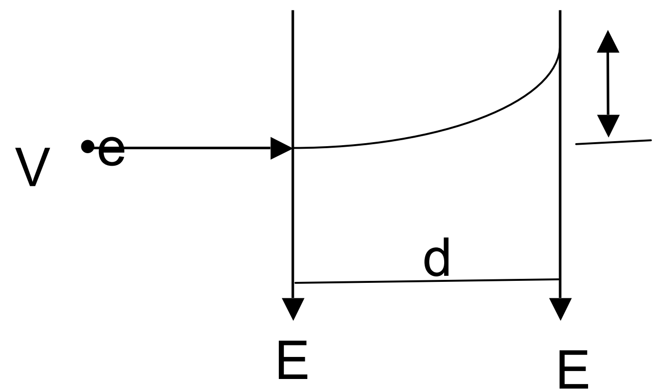
  a = F/m = mg/m = g for uniform gravitational fields

  If the field is uniform, we now have a projectile motion problem- constant acceleration in one direction. So we have parabolic motion just as in hitting a baseball, etc except the magnitudes of velocities and accelerations are different.

  Replace g by qE/m in all equations;

  For example, In $\mathbf{y = 1/2at^2}$ **we get** $\mathbf{y = 1/2(qE/m)t^2}$

Example: An electron is projected perpendicularly to a downward electric field of **E= 2000 N/C** with a horizontal velocity **$v=10^6$ m/s.** How much is the electron deflected after traveling 1 cm.



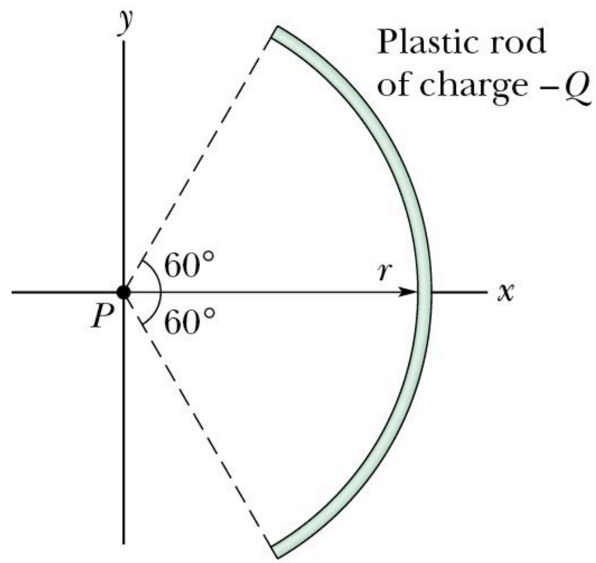
Since velocity in x direction does not change, **$t=d/v = 10^{-2}/10^6 = 10^{-8}$ sec,** so the distance the electron falls upward is
$y = 1/2at^2 = 0.5*eE/m*t^2 = 0.5*1.6*10^{-19}*2*10^3/10^{-30}*(10^{-8})^2 = 0.016$m

Back to computing Electric Fields

- Electric field due to an arc of a circle of uniform charge.
- Electric field due to a ring of uniform charge
- Electric field of a uniform charged disk
- Next time we will go on to another simpler method to calculate electric fields that works for highly symmetric situations using Gauss's Law.

What is the field at the center due to arc of charge

$$E_y = \int_{-L/2}^{L/2} dE_y = 0$$



(a)

dE$_x$= k dq cos θ/r^2

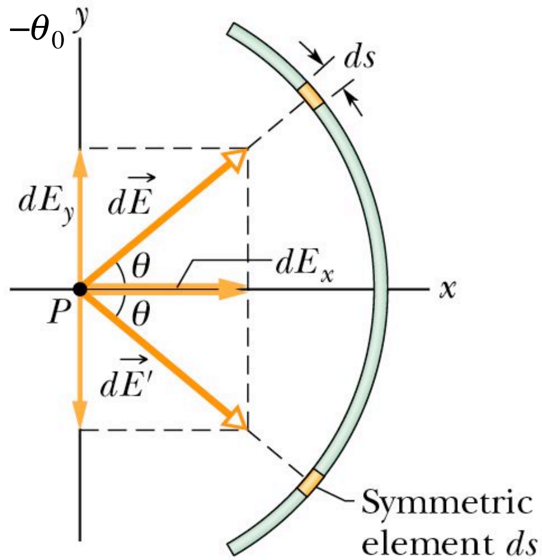
dE$_x$= k λ ds cos θ/r^2

s=r θ

ds=r dθ

$$E_x = k\lambda \int_{-L/2}^{L/2} r d\theta \cos\theta / r^2 = k\lambda / r \int_{-\theta_0}^{\theta_0} d\theta \cos\theta$$

$$E_x = \frac{2k\lambda}{r}\sin\theta_0$$



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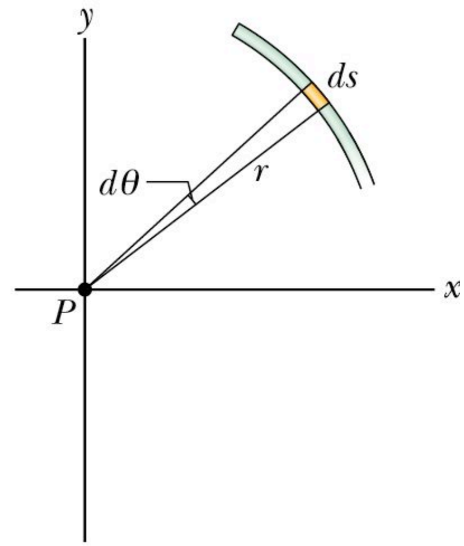
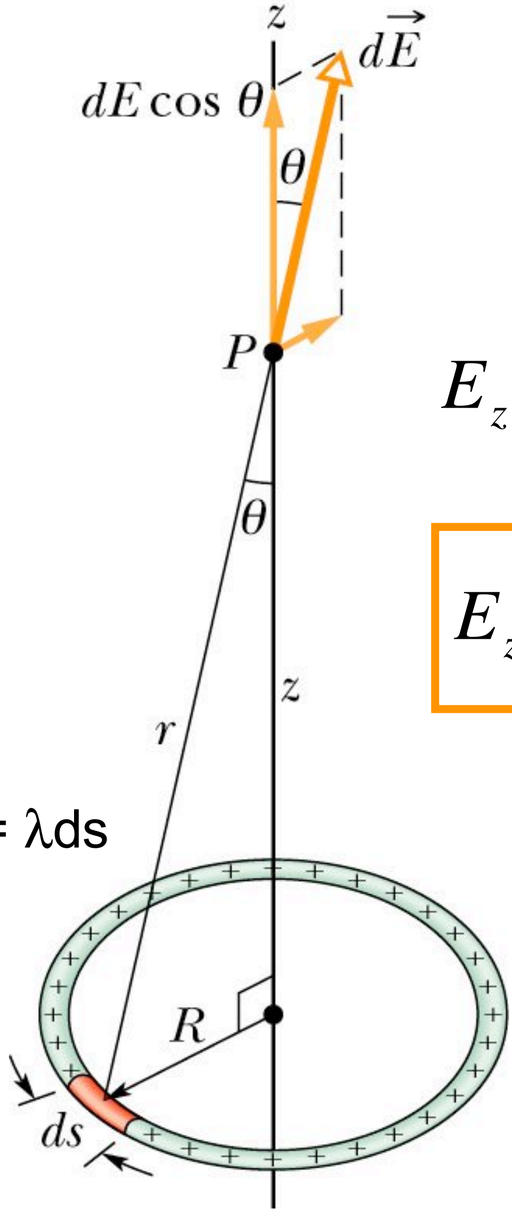


(c)

What is the field at the center of a circle of charge? θ_0=180

Find the electric field on the axis of a uniformly charged ring with linear charge density $\lambda = Q/2\pi R$.



dq = λds

$$E_z = \int dE\cos\theta$$

$$dE = k\frac{dq}{r^2} = k\frac{\lambda ds}{r^2}$$

$$E_z = \frac{k\lambda\cos\theta}{r^2}\int ds$$

$$\int ds = \int_0^{2\pi} R\,d\theta = R\int_0^{2\pi} d\theta = 2\pi R$$

$$s = R\theta$$

$$E_z = \frac{k\lambda\cos\theta}{r^2}2\pi R$$

$r^2 = z^2 + R^2$

$\cos\theta = z/r$

$$E_z = \frac{kQz}{(z^2+R^2)^{3/2}}$$

=0 at z=0
=0 at z=infinity
=max at z=0.7R