Lecture 3 Gauss's Law Chp. 24

- Cartoon - Electric field is analogous to gravitational field
- Opening Demo -
- Warm-up problem
- Physlet /webphysics.davidson.edu/physletprob
- Topics
  - Flux
  - Electric Flux and Example
  - Gauss' Law
  - Coulombs Law from Gauss' Law
  - Isolated conductor and Electric field outside conductor
  - Application of Gauss' Law
    - Charged wire or rod
    - Plane of charge
    - Conducting Plates
    - Spherical shell of charge

- List of Demos
  – Faraday Ice pail: metal cup, charge ball, teflon rod, silk,electroscope

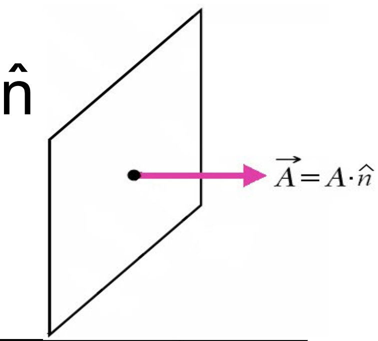
Electric Flux

Flux is a measure of the number of field lines passing through an area.

Electric flux is the number of Electric field lines penetrating a surface or an area.

In general, $\text{Flux} = \Phi = \text{normal component of the field} \times \text{area}$

$$\text{Electric Flux} = \Phi = (E\cos\theta)A = \vec{E}\cdot\vec{A} \quad \text{where } \vec{A} = |\vec{A}|\cdot\hat{n}$$



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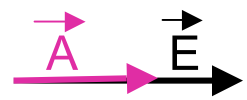
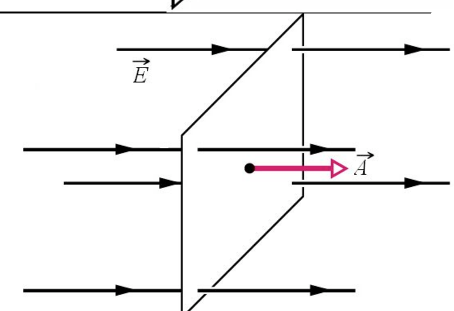
a $\quad \vec{E} \parallel \vec{A} \quad \Rightarrow \theta = 0$

So, $\quad \Phi = EA\cos 0 = EA$

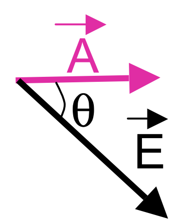


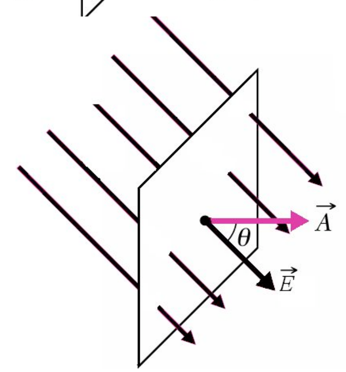


b $\quad \vec{E} \nparallel \vec{A} \quad \Rightarrow \theta \neq 0$

Let $\quad \theta = 45^\circ \quad$ Then,

$\Phi = EA\cos 45^\circ = 0.707EA$



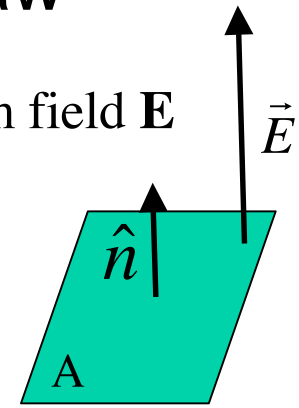


Gauss's Law

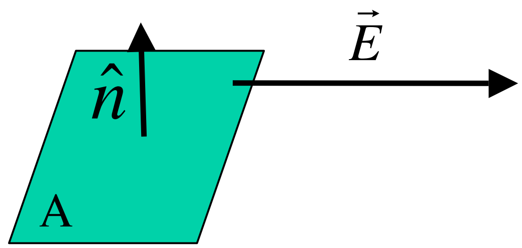
- Gauss's law makes it possible to find the electric field easily in highly symmetric situations.
- Drawing electric field lines around charges leads us to Gauss' Law
- The idea is to draw a closed surface like a balloon around any charge distribution, then some field line will exit through the surface and some will enter or renter. If we count those that leave as positive and those that enter as negative, then the net number leaving will give a measure of the net positive charge inside.

Electric lines of flux and Gauss's Law

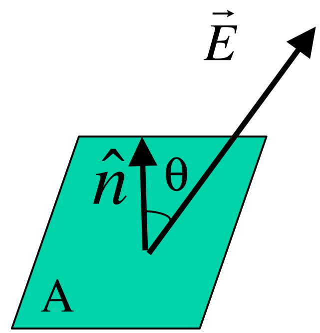
- The flux ϕ through a plane surface of area **A** due to a uniform field **E** is a simple product:
  $\Phi = EA$ where E is normal to the area A **.**

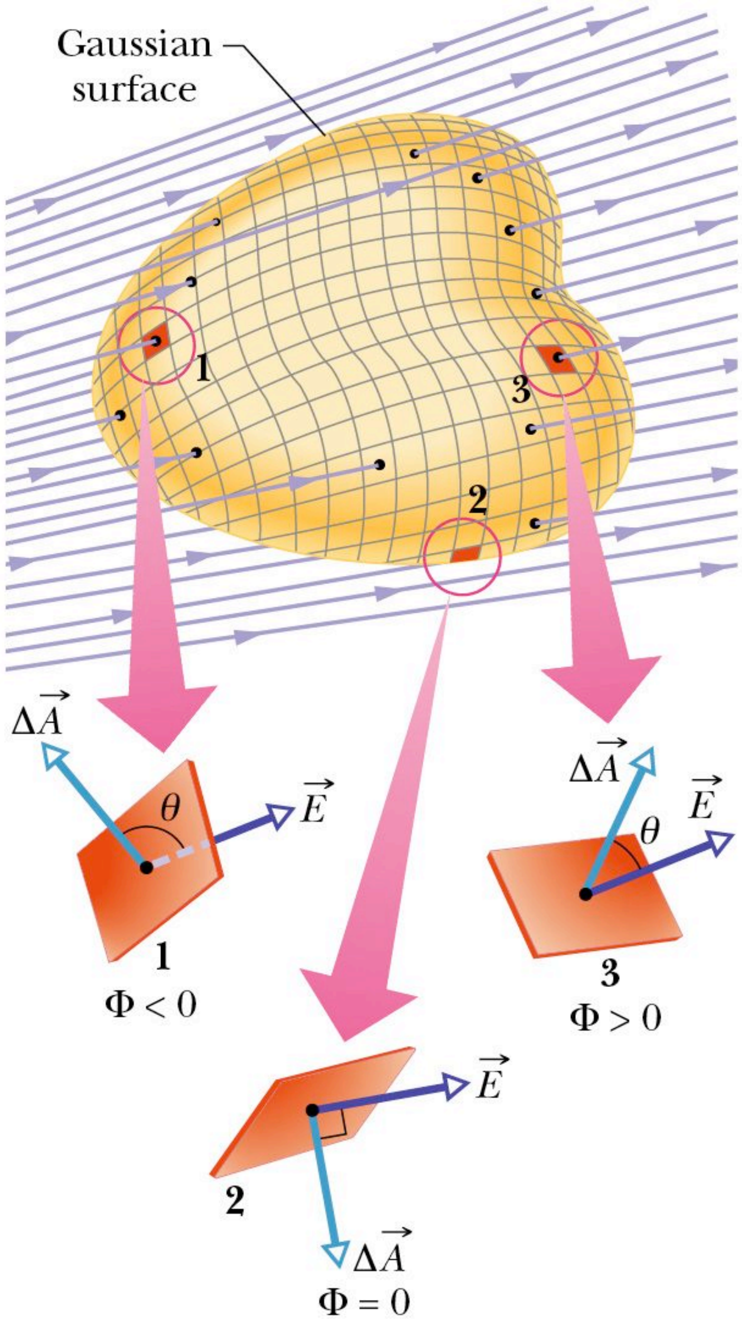


- $\Phi = E_n A = 0 \times A = 0$ because the normal component of E is 0



$\Phi = E_n A = E \cos\theta \cdot A$





Approximate Flux

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}$$

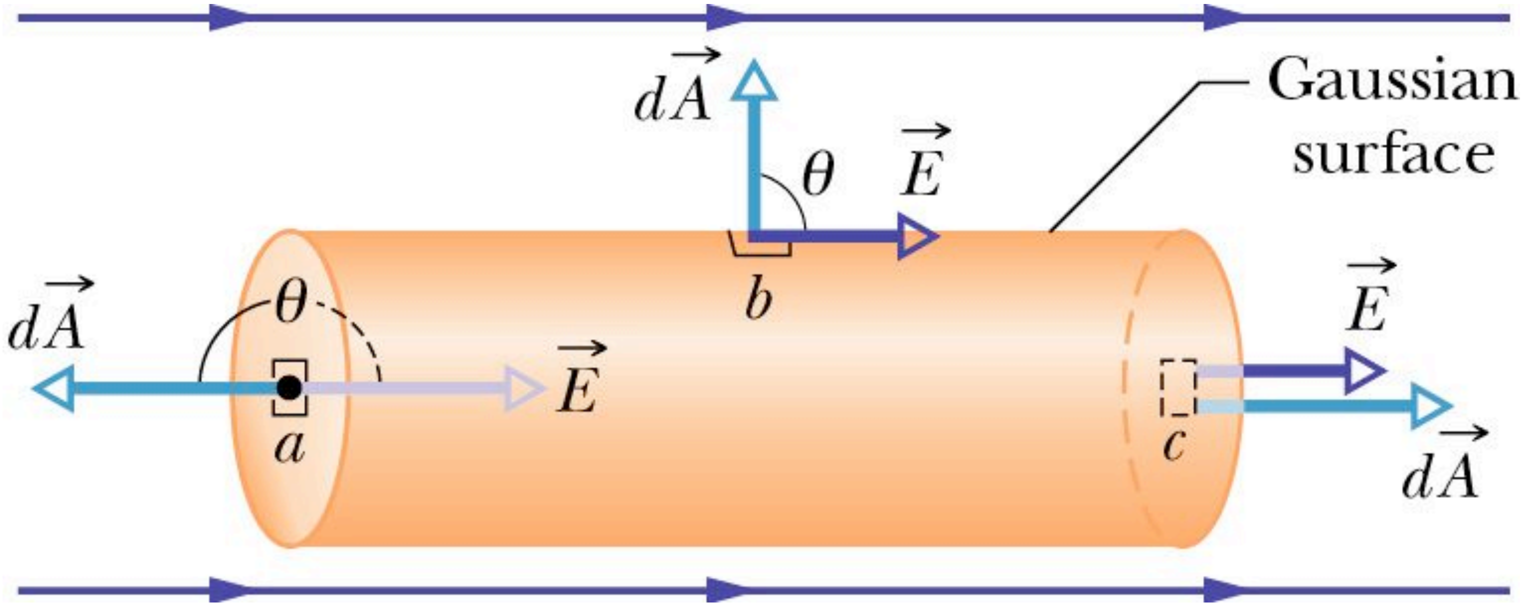
Exact Flux

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$d\vec{A} = \hat{n} dA$$

Circle means you integrate over a closed surface.

Find the electric flux through a cylindrical surface in a uniform electric field E



$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad = \oint E\cos\theta dA \qquad d\vec{A} = \hat{n} dA$$

a. $\Phi = \int E\cos 180 dA = -\int EdA = -E\pi R^2$

b. $\Phi = \int E\cos 90 dA = 0$

c. $\Phi = \int E\cos 0 dA = \int EdA = E\pi R^2$

What would be the flux if the cylinder were vertical ?

Suppose it were any shape?

Flux from a. + b. + c. $=0$

Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta dA = \oint EdA$

$E \parallel n$

$\cos 0° = 1$

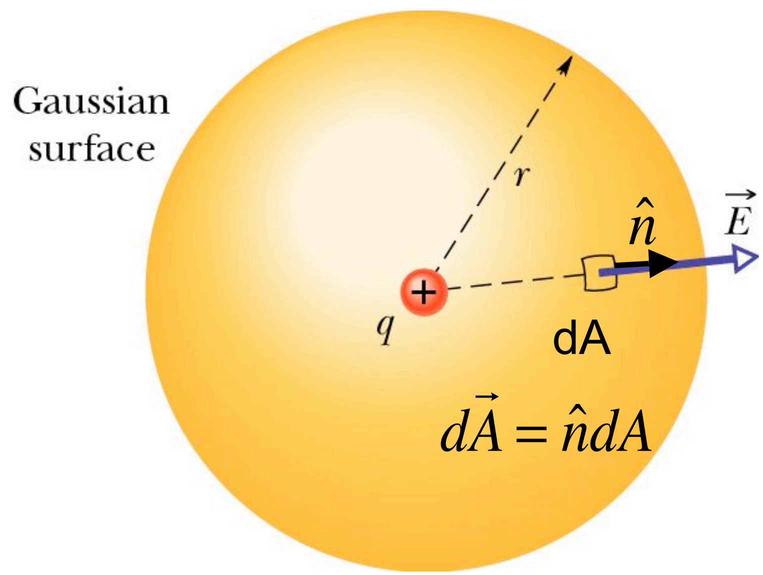
For a Point charge $E = \frac{kq}{r^2}$

$$\Phi = \oint EdA = \oint \frac{kq}{r^2} dA$$

$$\Phi = \frac{kq}{r^2} \oint dA = \frac{kq}{r^2}(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$4\pi k = \frac{1}{\varepsilon_0}$ where $\varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$



$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$ Gauss' Law

Gauss' Law

$$\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}$$

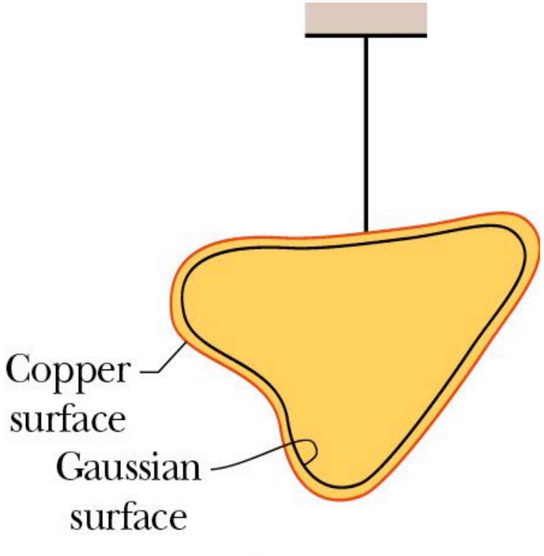
This result can be extended to any shape surface with any number of point charges inside and outside the surface as long as we evaluate the net flux through it.

Applications of Gauss's Law

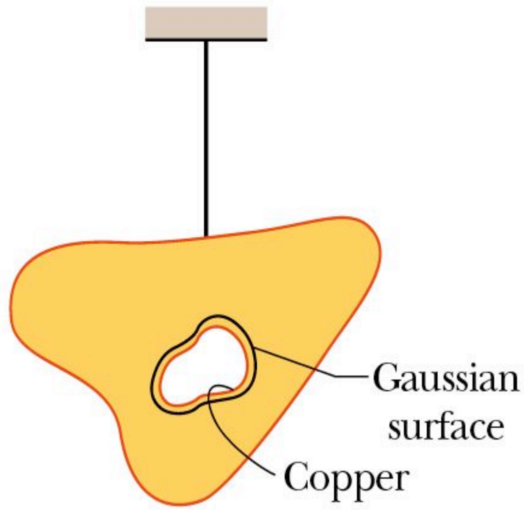
- Find electric filed of an infinite long uniformly charged wire of negligible radius.
- Find electric field of a large thin flat plane or sheet of charge.
- Find electric field around two parallel flat planes.
- Find E inside and outside of a long solid cylinder of charge density ρ and radius r.
- Find E for a thin cylindrical shell of surface charge density σ.
- Find E inside and outside a solid charged sphere of charge density ρ.

Electric field in and around conductors

- Inside a conductor in electrostatic equilibrium the electric field is zero (averaged over many atomic volumes) The electrons in a conductor move around so that they cancel out any electric field inside the conductor resulting from free charges anywhere including outside the conductor. This results in a net force of $F = eE = 0$ inside the conductor.



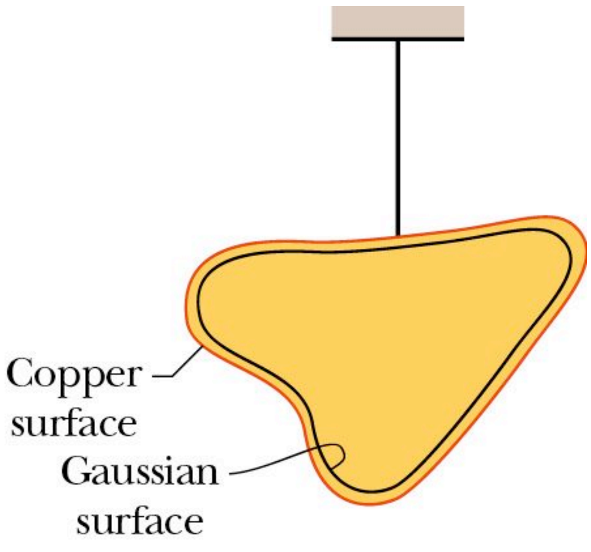
(a)



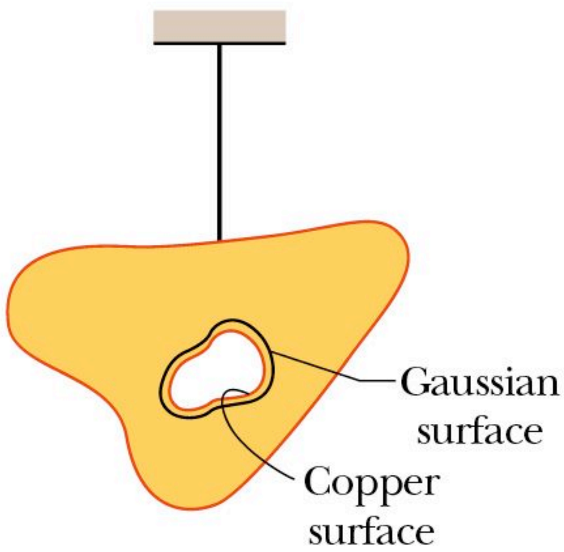
(b)

Electric field in and around conductors

- Any net electric charge resides on the surface of the conductor within a few angstroms (10^{-10} m). Draw a Gaussian surface just inside
the conductor. We know $E = 0$ everywhere on this surface. Hence, the net flux is zero. Hence, the net charge inside is zero. Show Faraday ice pail demo.



(a)



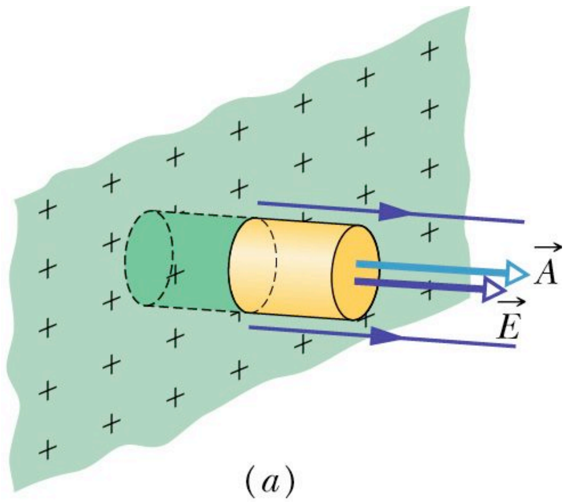
(b)

Electric field in and around conductors

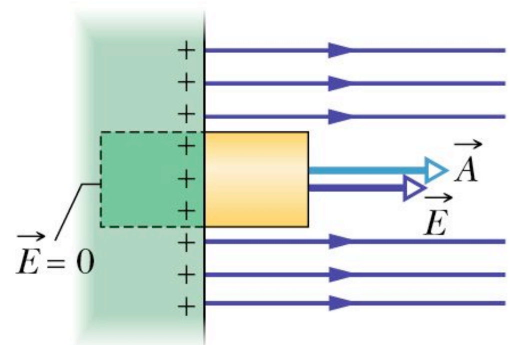
- The electric field just outside a conductor has magnitude $\frac{\sigma}{\varepsilon_0}$ and is directed perpendicular to the surface.
  - Draw a small pill box that extends into the conductor. Since there is no field inside, all the flux comes out through the top.

$$EA = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

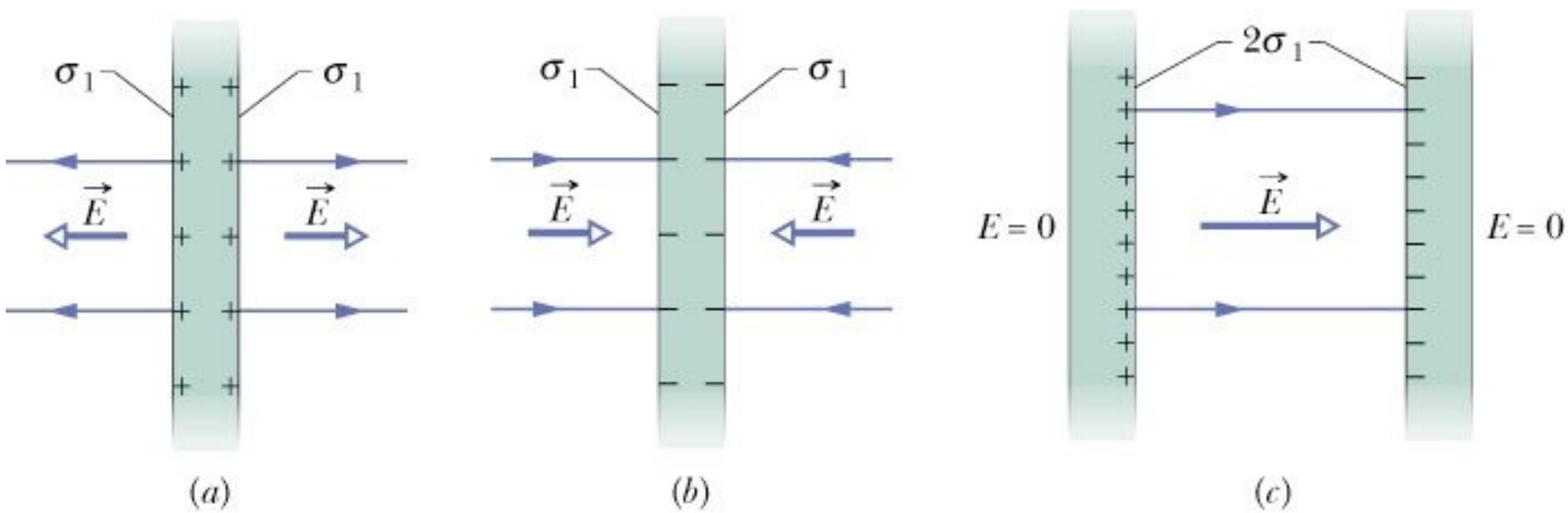


(a)

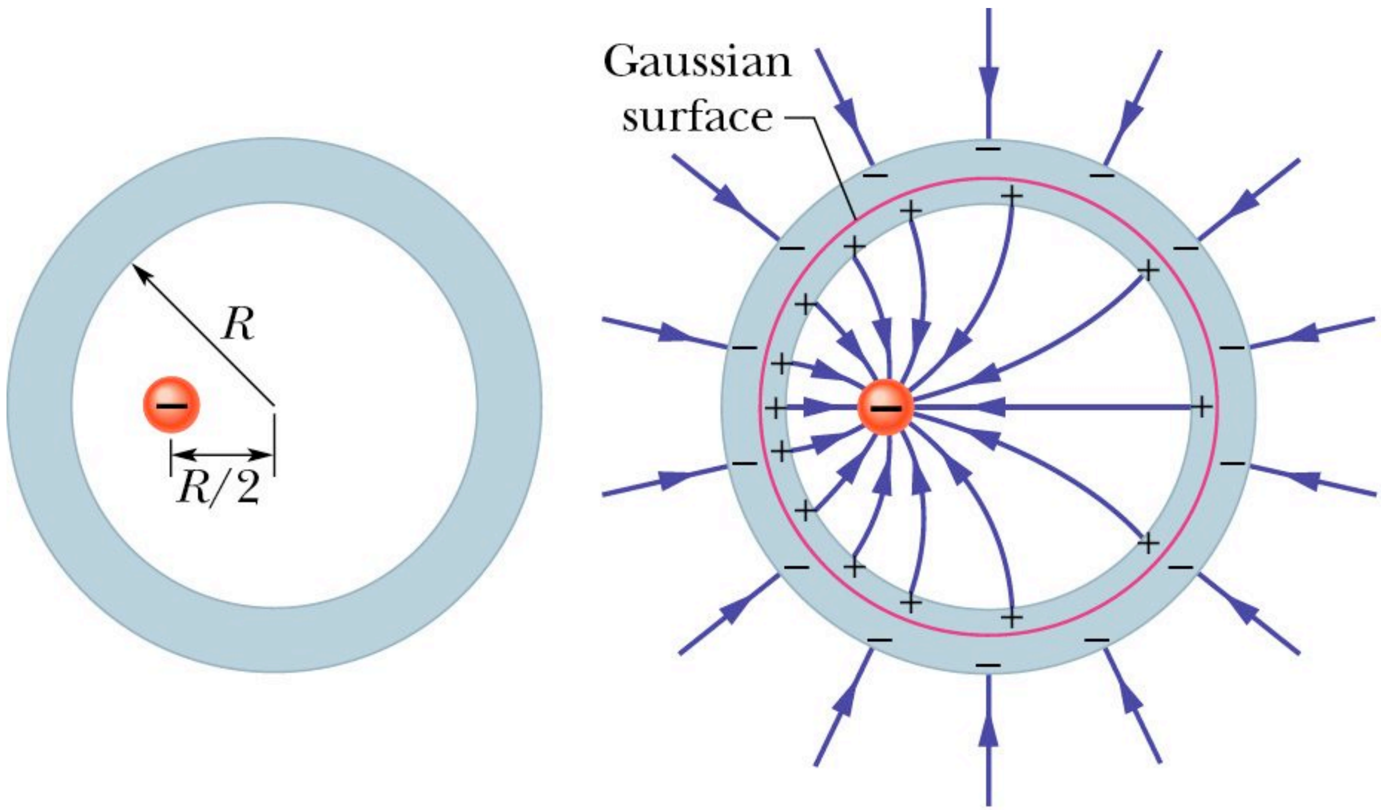


(b)

Two Conducting Plates

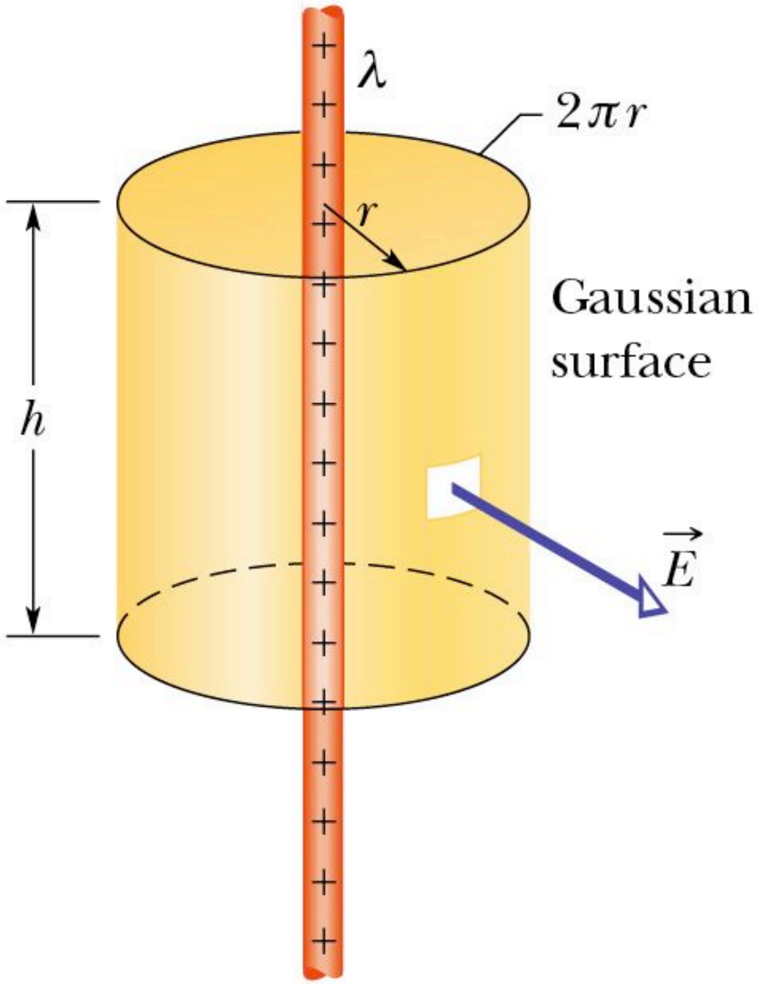


Negative charge in a neutral conducting metal shell



(a) (b)

Find the electric field for an infinite long wire



Charge per unit length $= \frac{Q}{L} = \lambda$

$$E_n = \vec{E} \cdot \hat{n}$$

$$E \parallel \hat{n} \quad \Rightarrow \theta = 0°$$

$$\oint E_n dA = E \oint dA = E \cdot 2\pi r h$$

$$\phi = \oint E_n dA = \frac{q}{\varepsilon_0}$$

$$\phi = \text{endcaps} + \text{side}$$

$$= \quad 0 \quad + E \cdot 2\pi r h \quad = \frac{\lambda h}{\varepsilon_0}$$

$\vec{E} \perp \hat{n}$

$\theta = 90°$

$\text{Cos}90° = 0$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Application of Gauss's Law

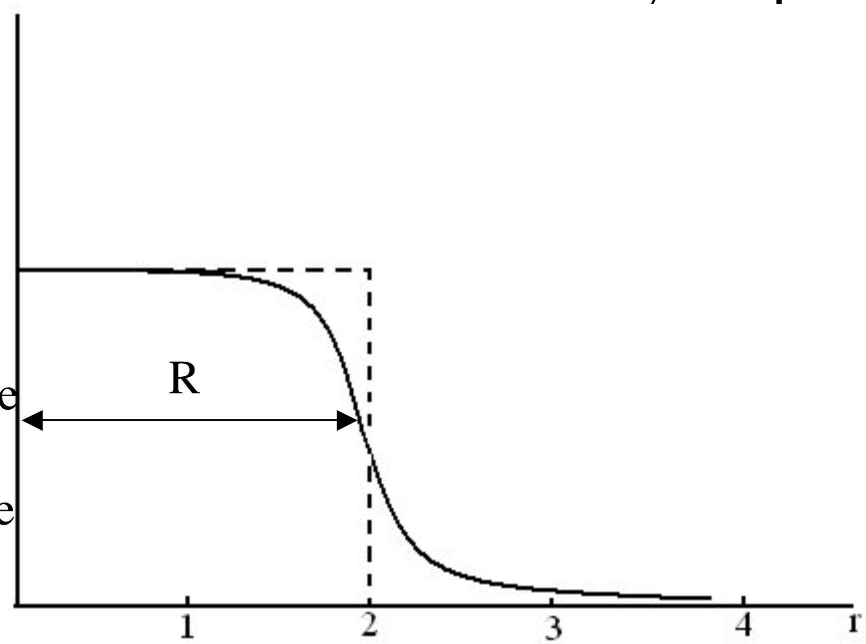
Electric field inside and outside a solid uniformly charged sphere

- Often used as a model of the nucleus.
- Electron scattering experiments have shown that the charge density is constant for some radius and then suddenly drops off at about $2-3\times10^{-14}$ m.

For the nucleus,

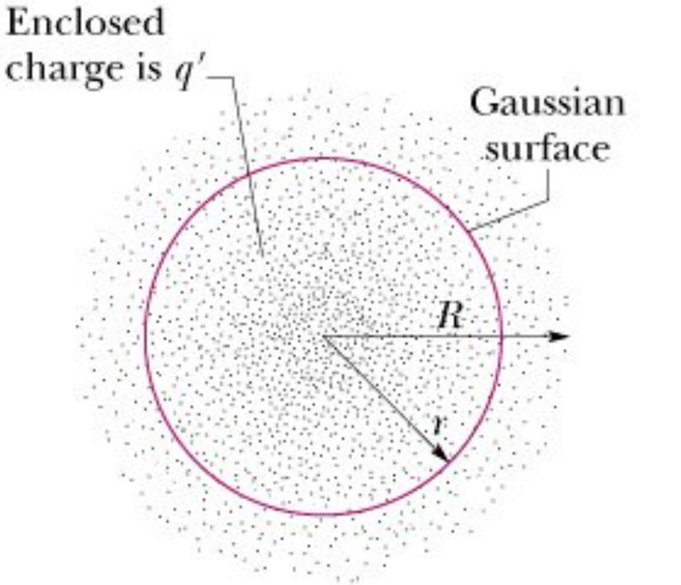
$$\rho = 10^{-26}\,\frac{\text{C}}{\text{m}}$$

$\rho =$ Charge density per unit volume



$\times 10^{-14}$ m

Electric Field inside and outside a uniformly charged sphere



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}, \quad r \le R$$

$Q =$ Total charge

$= z \times 1.6 \times 10^{-19}$C

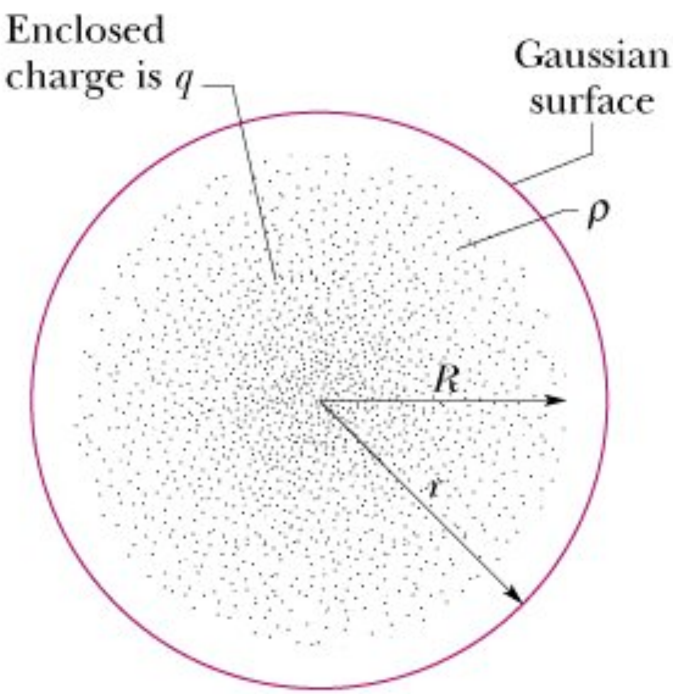
Inside the sphere:

To find the charge at a distance r<R

Draw a gaussian surface of radius r

By symmetry E is radial and parallel to normal at the surface. By Gauss's Law:

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0} \qquad E = \frac{\rho r}{3\varepsilon_0}$$

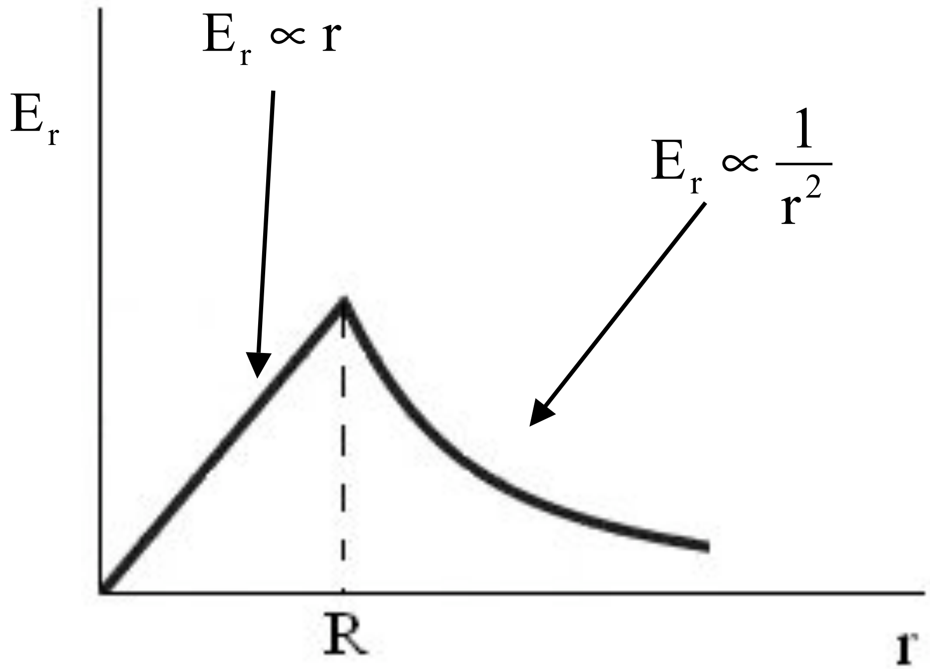


Outside the sphere:

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi R^3}{\varepsilon_0} \qquad E = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

Same as a point charge q

Electric field vs. radius for a conducting sphere



(similar to gravity)