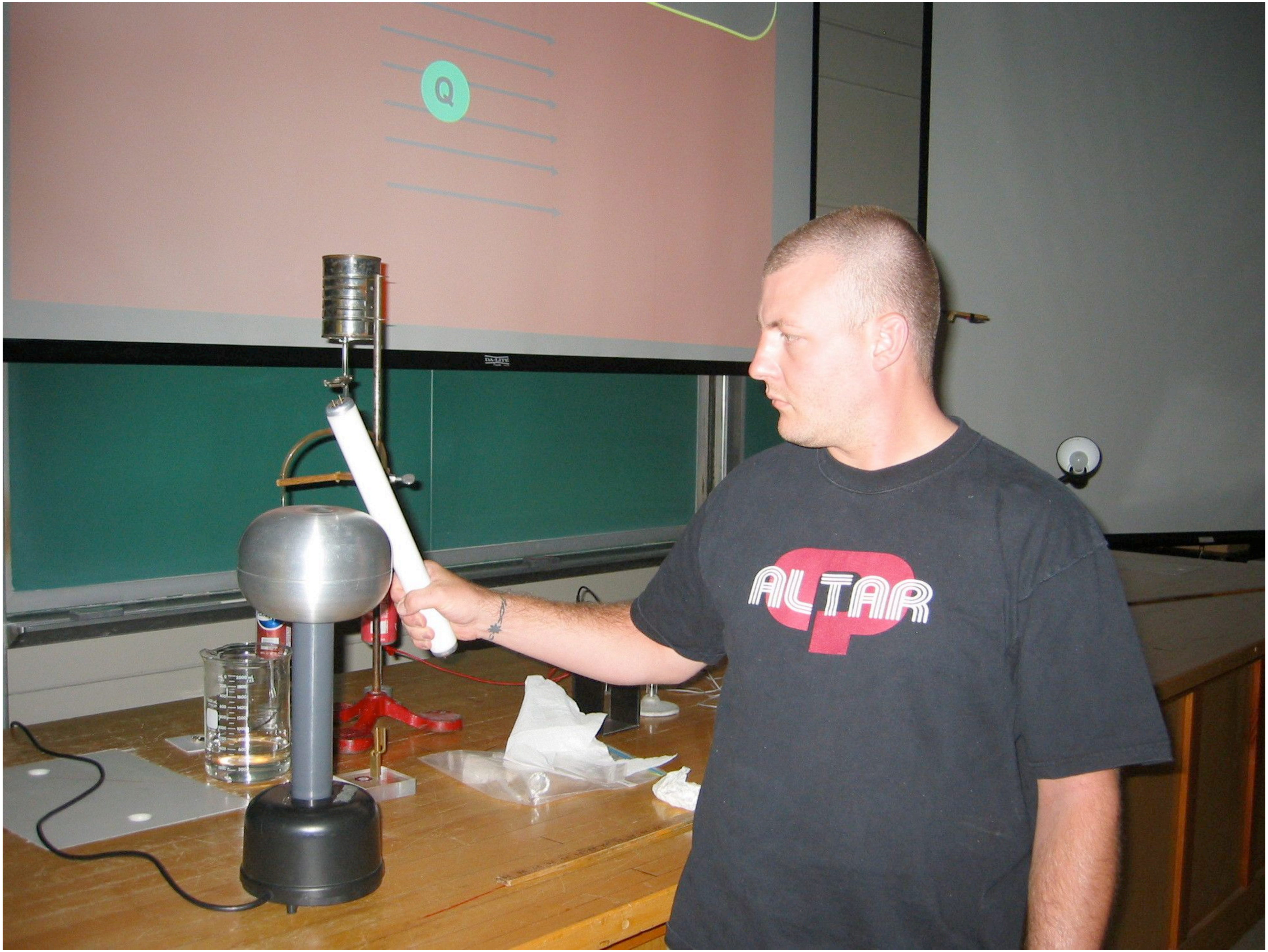
Lecture 4 Electric Potential and/ Potential Energy Ch. 25

- •Review from Lecture 3
- •Cartoon - There is an electric energy associated with the position of a charge.
- •Opening Demo -
- •Warm-up problems
- •Physlet
- •Topics
    - •Electric potential energy and electric potential
    - •Calculation of potential from field
    - •Potential from a point charge
    - •Potential due to a group of point charges, electric dipole
    - •Potential due to continuous charged distributions
    - •Calculating the filed from the potential
    - •Electric potential energy from a system of point charge
    - •Equipotential Surface
    - •Potential of a charged isolated conductor
- •Demos
    - •teflon and silk
    - •Charge Tester, non-spherical conductor, compare charge density at Radii
    - •Van de Graaff generator with pointed objects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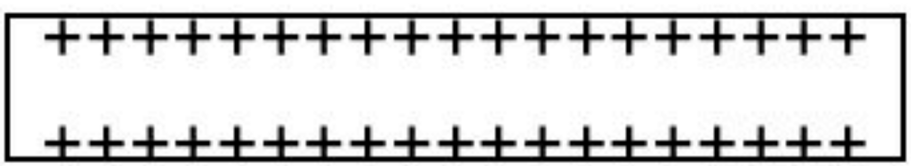
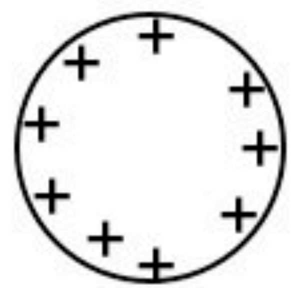
Charges on a Conductor

- Why do the charges always move to the surface of a conductor ?
- Gauss' Law tells us!!
- E = 0 inside a conductor when in equilibrium (electrostatics) !
- Why?
- If $E \neq 0$, then charges would have forces on them and they would move !
- Therefore from Gauss' Law, the charge on a conductor must only reside on the surface(s) !

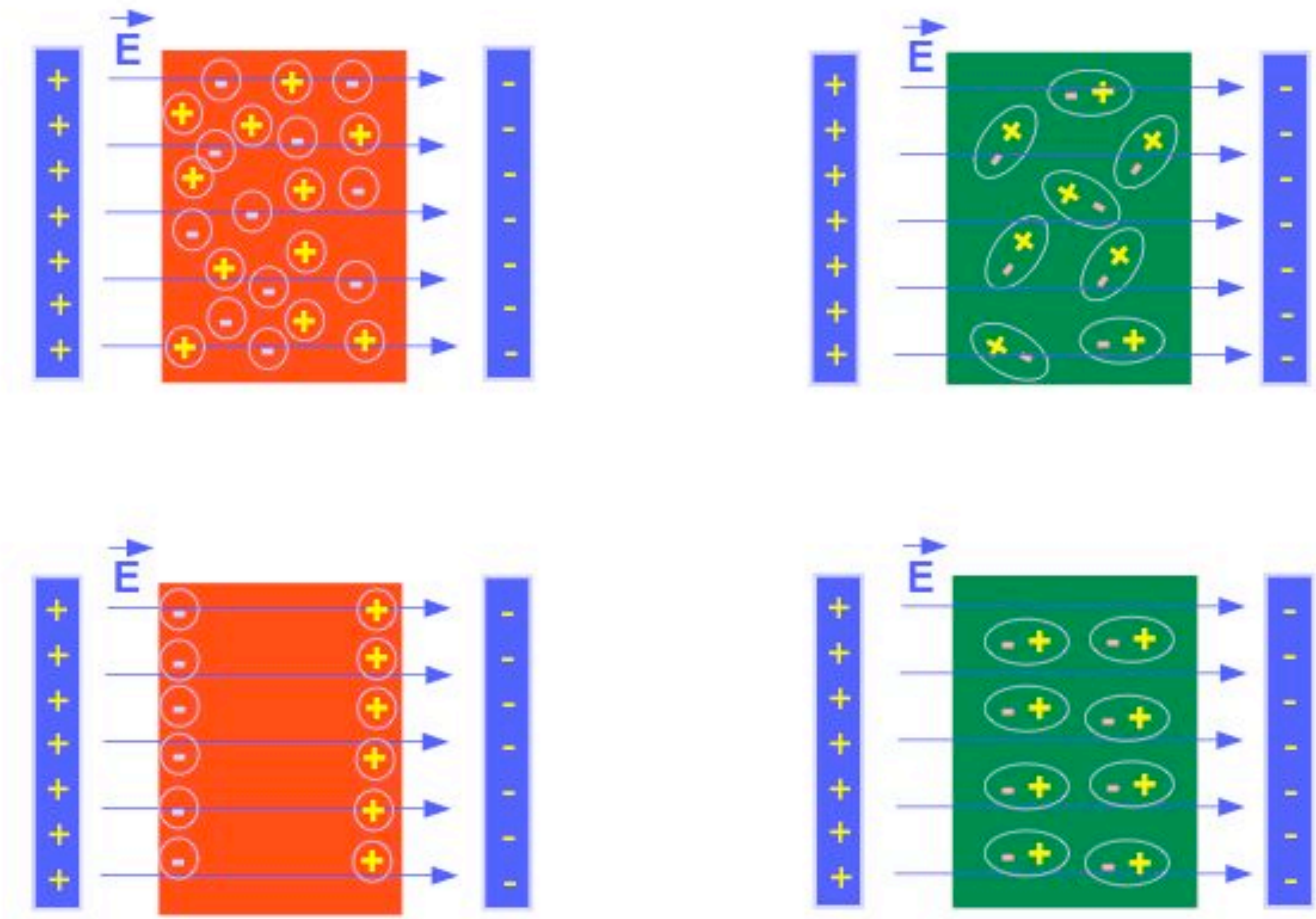


Infinite Conducting Plane



Conducting Sphere

Conductors vs. Insulators



$E_{inside} = 0$

$E_{inside} < E$

Potential Energy and Electric potential

- The electric force is mathematically the same as gravity so it too must be a conservative force. We will find it useful to define a potential energy as is the case for gravity. Recall that the change in the potential energy in moving from one point a to point b is the negative of the work done by the electric force.
- $\Delta U = U_b - U_a$ = - W = -Work done by the electric force = $-\int_a^b F \cdot ds$
- Since $F = q_0 E$, $\Delta U = -q_0 \int_a^b E \cdot ds$ and
- Electric Potential difference = Potential energy change/ unit charge

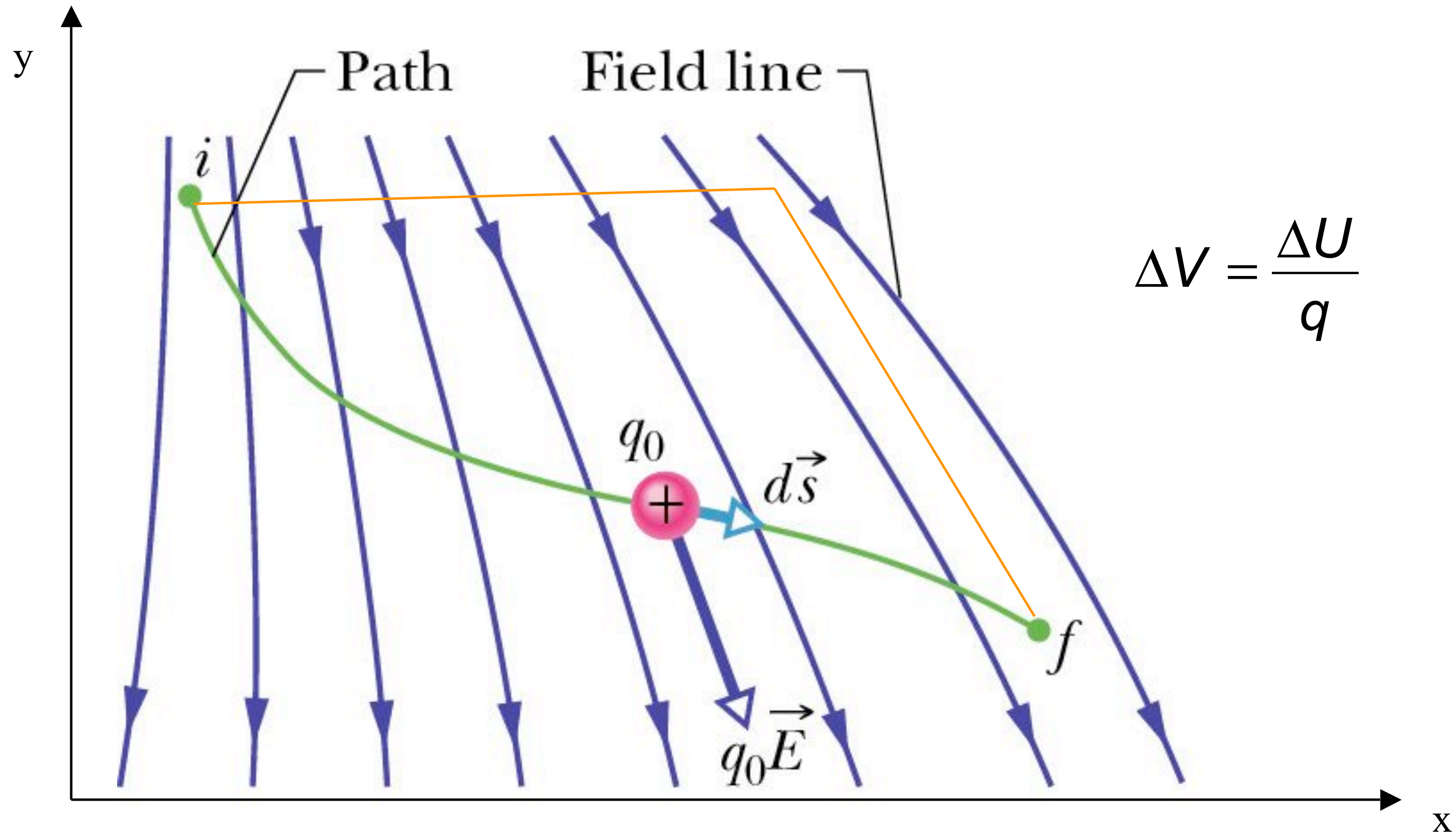
$$\Delta V = \frac{\Delta U}{q_0}$$

SI unit of electric potential is volt (V):
1 Volt = 1 Joule/Coulomb (1 V = 1 J/C)

- $\Delta V = V_b - V_a = -\int E \cdot ds$ (independent of path, ds)

- Joule is too large a unit of energy when working at the atomic or molecular level, so use the electron-volt (eV), the energy obtained when an electron moves through a potential difference of 1 V. 1 eV = 1.6 x 10-19 J

$$\Delta U = U_f - U_i = \text{- Work done by the electric force} = -\int_i^f F \cdot ds$$



$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta V = V_f - V_i = -\int E \cdot ds$$ (independent of path, ds)

Therefore, electric force is a conservative force.

$$\Delta V = \frac{-W}{q_0} = -\int \frac{\vec{F}}{q_0} \cdot d\vec{s} = -\int \vec{E} \cdot d\vec{s}$$

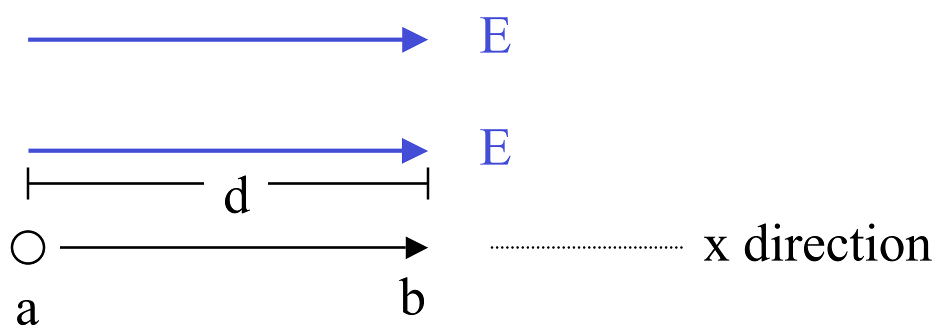
•The potential difference is the negative of the work done
per unit charge by an electric field on a positive unit
charge
when it moves from one point *to another.*

• V is a scalar not a vector. Simplifies solving problems.

•We are free to choose V to be 0 at any location. Normally
V is chosen to be 0 at the negative terminal of a battery or
0 at infinity for a point charge.

Example of finding the potential difference in a Uniform Field

What is the electric potential difference for a unit positive charge moving in an uniform electric field from a to b?



$$\Delta V = -\int_a^b E \cdot ds = -E\int_a^b dx = -E\underbrace{(x_b - x_a)}_{d}$$

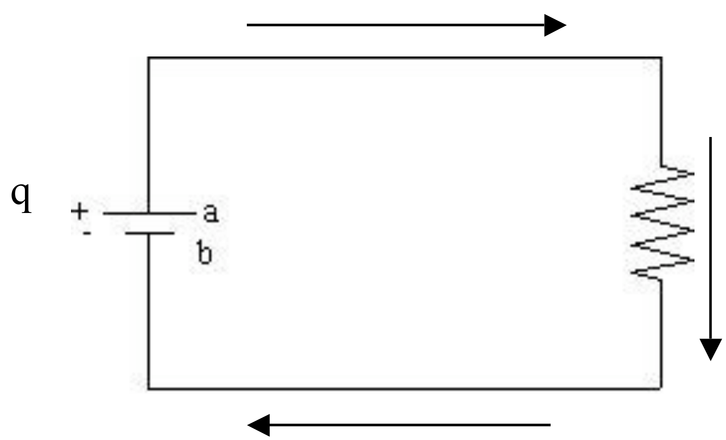
$\Delta V = -Ed$ $\qquad$ $dV = -Edx$

$\Delta U = q\Delta V$ $\qquad$ $E = -dV / dx$

$\Delta U = -qEd$

Example for a battery in a circuit

- In a 9 volt battery, typically used in IC circuits, the positive terminal has a potential 9 v higher than the negative terminal. If one micro-Coulomb of positive charge flows through an external circuit from the positive to negative terminal, how much has its potential energy been changed?



$$\Delta V = \frac{\Delta U}{q} = V_b - V_a = (0-9)V$$

$$\Delta U = -9q$$

$$= (-9V) \times 1 \times 10^{-6} C$$

$$\Delta U = -9 \times 10^{-6} \, Joules$$

$$\Delta U = -\ 9 \ microJoules$$

$$= \ -9 \ \mu\text{J}$$

Potential energy is lower by $9 \ \mu J$

We also assumed that the potential at b was 0

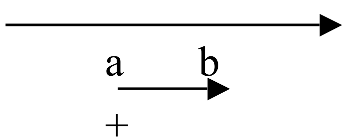
Example of a proton accelerated in a uniform field

A proton is placed in an electric field of E=10^5 V/m and released. After going 10 cm, what is its speed?

Use conservation of energy.

E = 10^5 V/m

d = 10 cm



$$\Delta V = V_b - V_a = -Ed$$

$$\Delta U = q\Delta V = -qEd$$

$$\Delta U + \Delta K = 0$$

$$\Delta K = -\Delta U$$

$$\Delta K = qEd$$

$$\frac{1}{2}mv^2 = qEd$$

$$v = \sqrt{\frac{2qEd}{m}}$$

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 10^5 \frac{V}{m} \times 0.1 m}{1.67 \times 10^{-27} kg}}$$

$$v = 1.4 \times 10^8 \frac{m}{s}$$

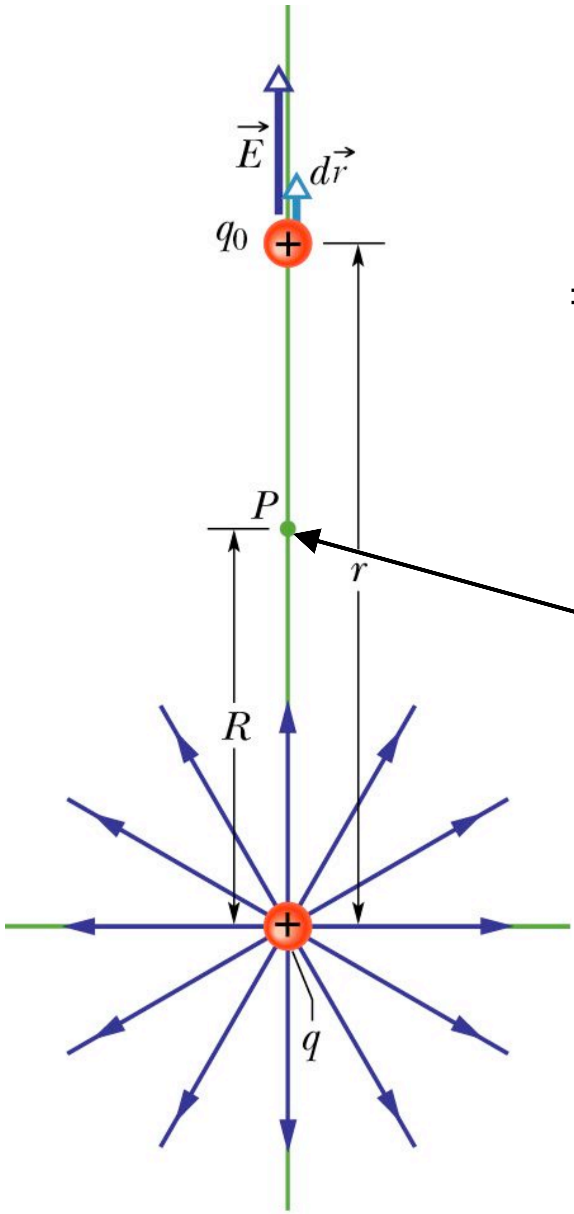
What is the electric potential when moving from one point to another in a field due to a point charge?

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{r}$$

Potential of a point charge at a distance R



$$V_f - V_i = -\int_i^f \vec{E} \cdot d\hat{r}$$

$$= -\int_R^\infty \vec{E} \cdot d\hat{r} = -kq\cos 0^\circ \int_R^\infty \frac{1}{r^2} dr = kq \frac{1}{r}\Big|_R^\infty = kq\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$$V_f - V_i = 0 - V_i = -k\frac{q}{R}$$

$$V = \frac{kq}{R}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

Replace R with r

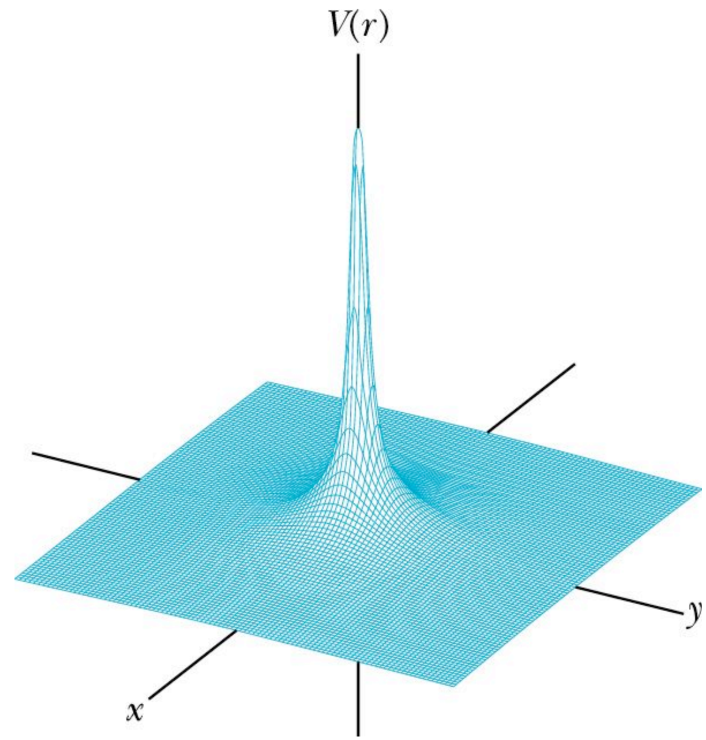
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

eqn 25-26

Electric potential for a positive point charge

$$V(r) = \frac{kq}{r}$$

$$r = \sqrt{x^2 + y^2}$$



- V is a scalar
- V is positive for positive charges, negative for negative charges.
- r is always positive.
- For many point charges, the potential at a point in space is the simple algebraic sum (Not a vector sum)

Electric potential due to a positive point charge

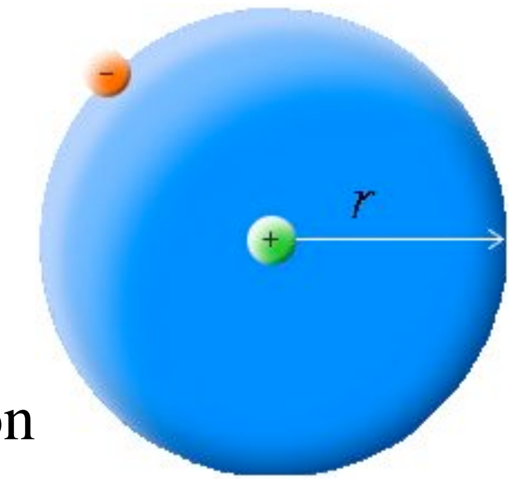
Hydrogen atom.

- What is the electric potential at a distance of 0.529 A from the proton? 1A= 10^{-10} m

$$V = \frac{kq}{R} = \frac{\left(8.99\times10^{9}\, Nm^2/C^2\right)\times1.6\times10^{-19}C}{.529\times10^{-10}m}$$

$$V = 27.2\frac{J}{C} = 27.2 Volts$$

r = 0.529 A



What is the electric potential energy of the electron at that point?

U = qV= (-1.6 x 10^{-19} C) (27.2 V)= - 43.52 x 10^{-19} J or - 27.2 eV where eV stands for electron volts.

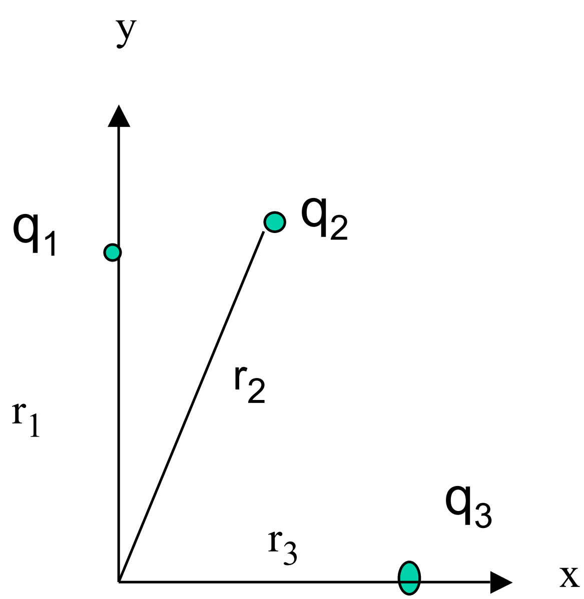
Total energy of the electron in the ground state of hydrogen is - 13.6 eV

Also U= 2E = -27.2 eV. This agrees with above formula.

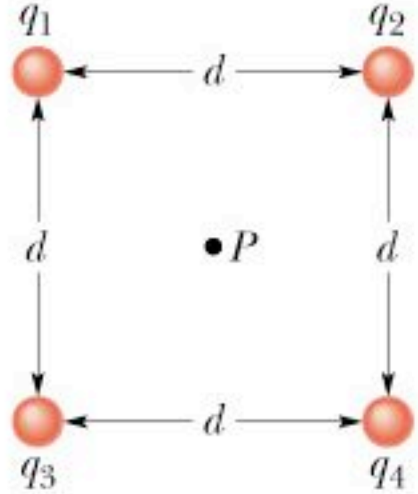
What is the electric potential due to several point charges?

- For many point charges, the potential at a point in space is the simple algebraic sum (Not a vector sum)

$$V = \sum_i \frac{kq_i}{r_i}$$



$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$



Problem

q_1 = +12 nC q_2 = -24 nC
q_3 = +31 nC q_4 = +17nC
d = 1.3 m.
What is the electric potential at P?

Key Idea: Electric potential is algebraic sum of the electric potentials of the 4 charges. The relative orientations of the charges does not matter.

$$V(r) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right) \qquad r = \frac{d}{\sqrt{2}}$$

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}C$$
$$= 36 \times 10^{-9}C$$

$$V = \frac{(8.99 \times 10^9 Nm^2/C^2)(36 \times 10^{-9}C)}{0.919m} = \simeq 350V$$

Potential due to a dipole

For two point charges, the total potential is the sum of the potentials of each point charge.

So, $V_{dipole} = V_{total} = V_a + V_b$

$$V_{dipole} = V_a + V_b = k\left(\frac{q}{r_a} + \frac{(-q)}{r_b}\right)$$

$$= kq\left(\frac{r_b - r_a}{r_a r_b}\right)$$

We are interested in the regime where r>>d.

As in fig 2, r_a and r_b are nearly parallel. And the difference in their length is $d\cos\theta$. Also because r>>d, $r_a r_b$ is approximately r^2.

$$V_{dipole} \simeq kq\frac{d\cos\theta}{r^2} \simeq \frac{kp\cos\theta}{r^2}$$

where **p** is the dipole moment.

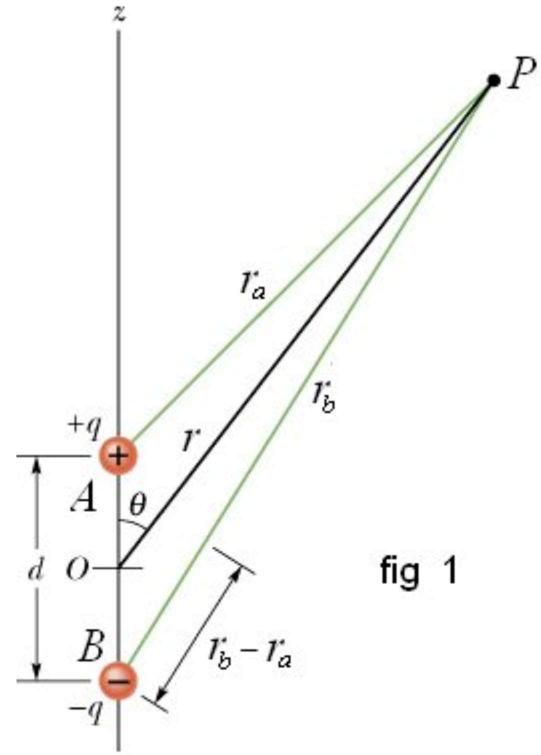


fig 1

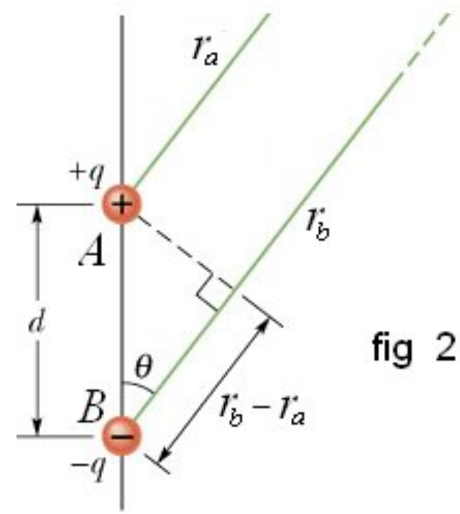


fig 2

Fields and potentials for continuous charge distributions

- Fields and potentials are closely related, the basic relation is

$$\Delta V = V_b - V_a = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{s}$$

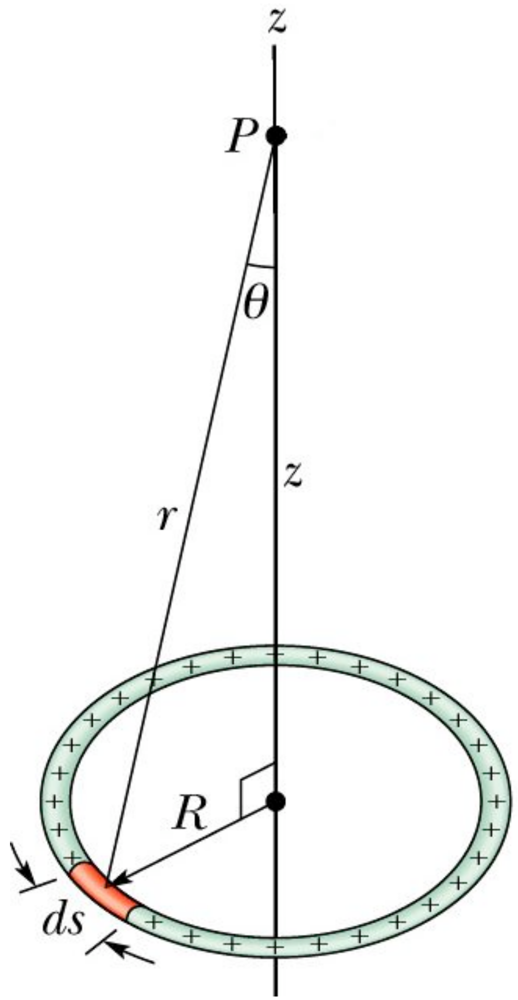
- If E is known, then one can get ΔV by performing the integral (or one can get just V by starting the integral path at infinity)

- Alternatively, if the charge distribution is known, one also has

$$V(r) = \frac{1}{4\pi\epsilon_o} \int \frac{dq}{r}$$

Potential due to a ring of charge

- Direct integration. Since V is a scalar, it is easier to evaluate V than E.
- Find V on the axis of a ring of total charge Q. Use the formula for a point charge, but replace q with elemental charge *dq* and integrate.



Point charge $V = \dfrac{kq}{r}$

For an element of charge dq, $dV = \dfrac{kdq}{r}$

r is a constant as we integrate.

$$V = \int \frac{kdq}{r}$$

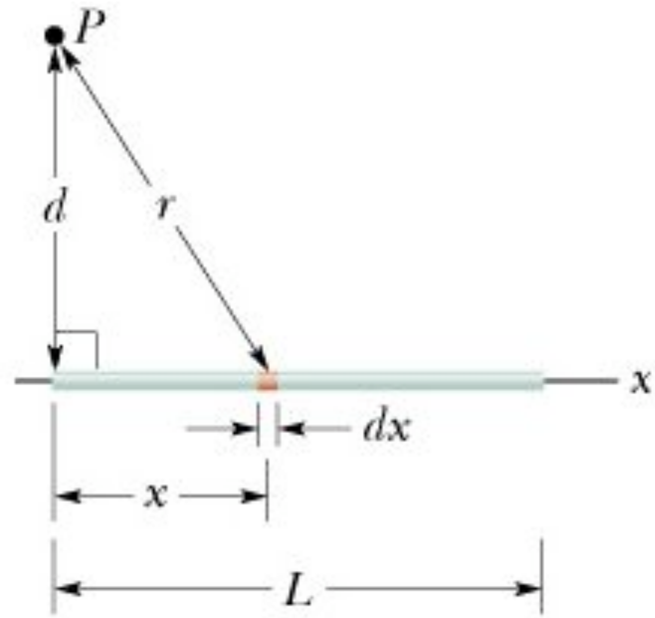
$$= \int \frac{kdq}{\sqrt{(z^2 + R^2)}}$$

$$= \frac{k}{\sqrt{(z^2 + R^2)}} \int dq \qquad \Rightarrow \mathrm{V} = \frac{\mathrm{k}}{\sqrt{(\mathrm{z}^2 + \mathrm{R}^2)}} \mathrm{Q}$$

This is simpler than finding **E** because V is not a vector.

Potential due to a line charge

We know that for an element of charge **dq** the potential is $dV = k\frac{dq}{r}$



For the line charge let the charge density be λ.
Then **dq=λdx**

So, $dV = k\frac{\lambda dx}{r}$ But, $r = \sqrt{x^2 + d^2}$

Then, $dV = k\frac{\lambda dx}{\sqrt{x^2 + d^2}}$

Now, we can find the total potential **V** produced by the rod at point **P** by integrating along the length of the rod from x=0 to x=L

$$V = \int_0^L dV = \int_0^L k\frac{\lambda dx}{\sqrt{x^2+d^2}} = k\lambda\int_0^L \frac{dx}{\sqrt{x^2+d^2}}$$

$$\Rightarrow V = k\lambda \ln(x + \sqrt{x^2+d^2})\Big|_0^L$$

So, $V = k\lambda(\ln(L + \sqrt{L^2+d^2}) - \ln d)$

Or, $V = k\lambda \ln\left(\frac{L + \sqrt{L^2+d^2}}{d}\right)$

A new method to find E if the potential is known.

If we know V, how do we find E?

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$d\vec{s} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$dV = -\vec{E} \cdot d\vec{s}$$

$$dV = -E_x dx - E_y dy - E_z dz$$

$$E_x = -\frac{dV}{dx}$$

$$E_y = -\frac{dV}{dy}$$

$$E_z = -\frac{dV}{dz}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

So the x component of E is the derivative of V with respect to x, etc.

–If V = a constant, then E_x = 0. The lines or surfaces on which V remains constant are called equipotential lines or surfaces.

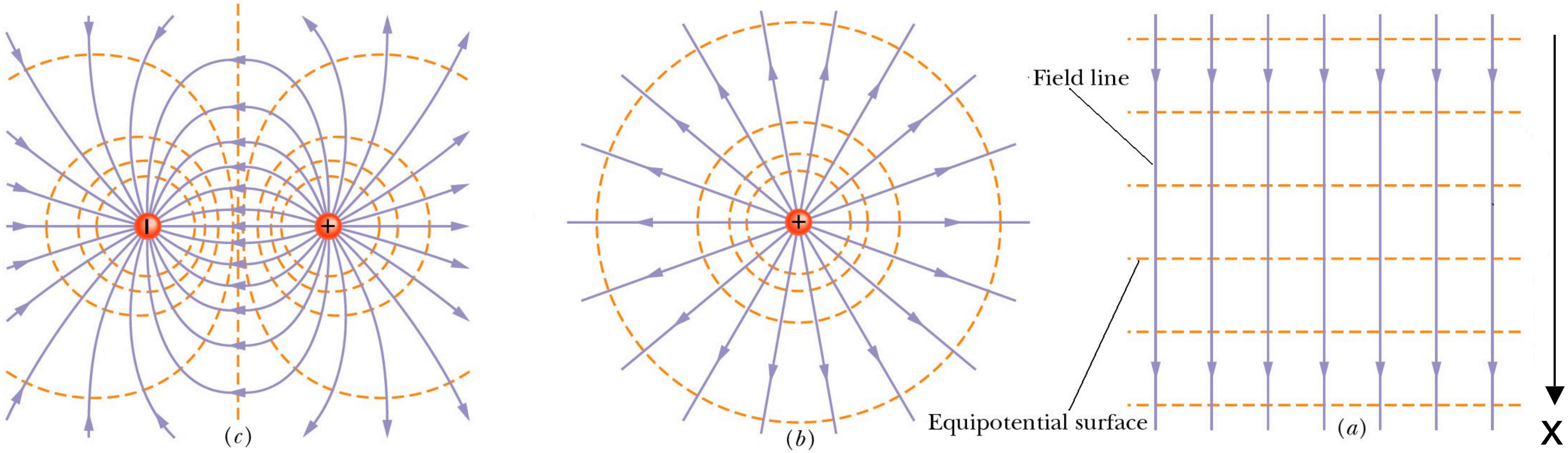
–See example on next slide

Equipotential Surfaces

- Three examples

- What is the obvious equipotential surface and equipotential volume for an arbitrary shaped charged conductor?

- See physlet 9.3.2 Which equipotential surfaces fit the field lines?



Blue lines are the electric field lines

Orange dotted lines represent the equipotential surfaces

a) Electric Dipole

(ellipsoidal concentric shells)

b) Point charge

(concentric shells)

c) Uniform E field

$$E = E_x, E_y = 0, E_z = 0$$

$$E_x = -\frac{dV}{dx}$$

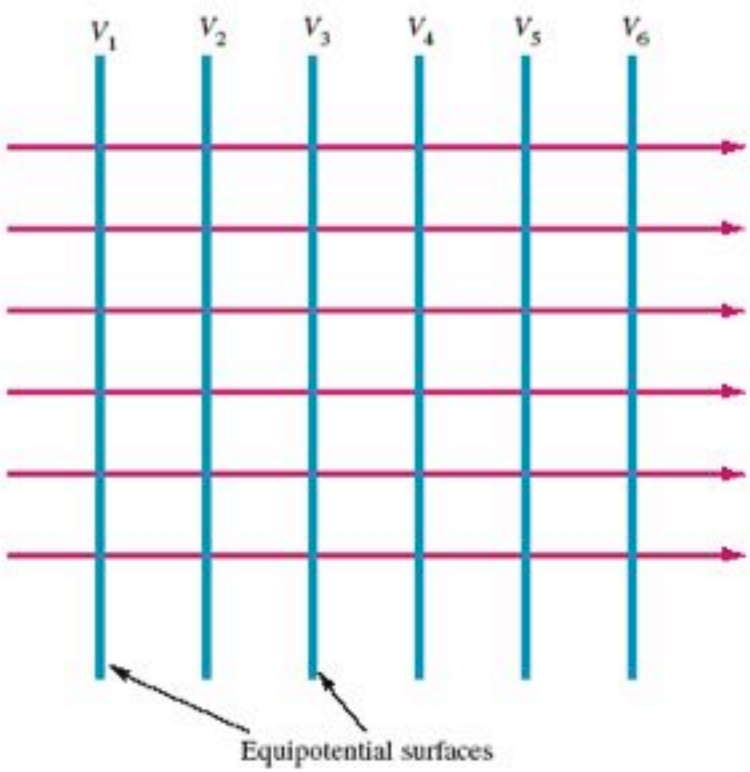
$$V = -E_x d$$

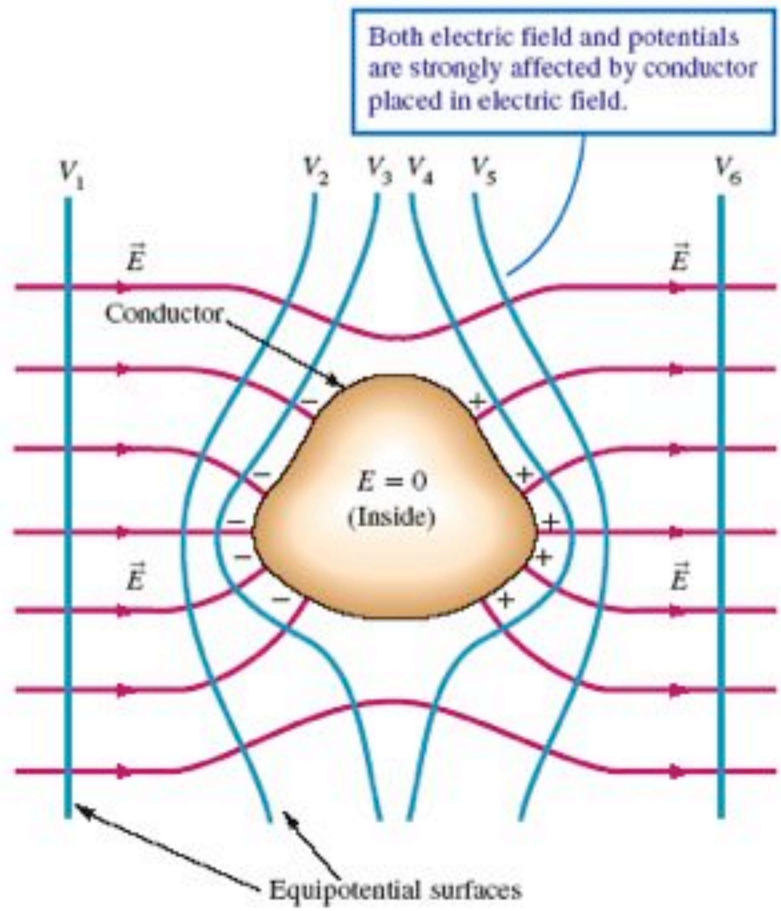
V = constant in y and z directions

Potentials and Fields Near Conductors

If a conductor is positioned within a region of electric field, it will distort the field shape.

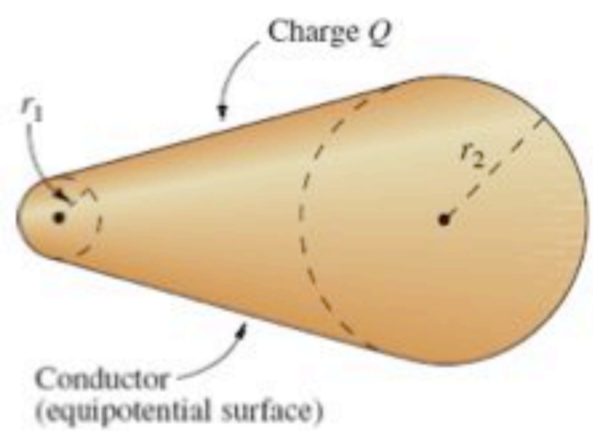
We have seen that field lines are perpendicular to the surface of a conductor (otherwise free charges would not be in equilibrium). Consequently, near the conductor, equipotential lines, which are perpendicular to field lines, are parallel to the conductor's surface





The Role of Sharp Points on Conducting Surfaces
(a semi-rigorous argument)

Thinking of the two ends of this irregular object as two spheres of radii r_1 and r_2, if q_1 and q_2 are the charges carried by each "sphere", and keeping in mind that the whole object is an equipotential, then $V_1 = V_2 \rightarrow q_1/4\pi\epsilon_0 r_1 = q_2/4\pi\epsilon_0 r_2$.



Remembering also that, in terms of local charge density, $q = \sigma A$, with $A=4\pi r^2$ and $E=\sigma/\epsilon_0$, with a few simple steps one gets

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

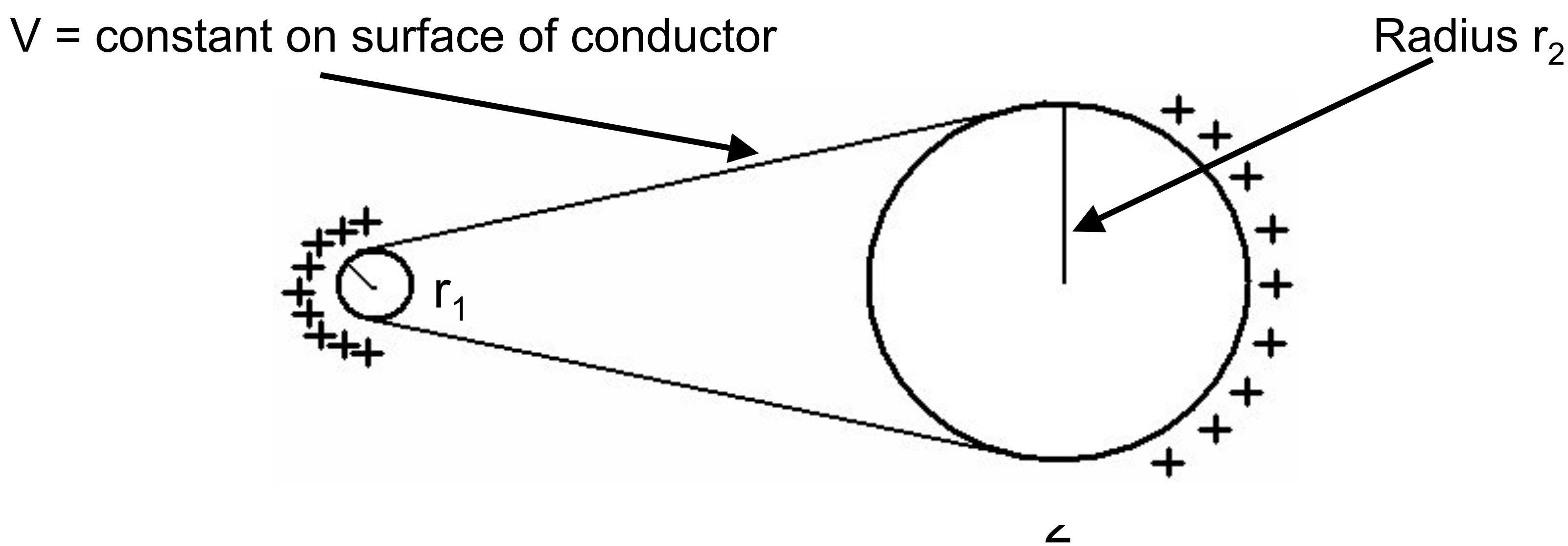
The electric field is larger in the regions of small radius of curvature. Fields can be very large around a sharp point, even though the whole conductor is at the same potential.

In air, fields of the order of 3 MV/m will cause "electric breakdown" (the air gets to be ionized, and discharges can occur).

Dielectric Breakdown: Application of Gauss's Law

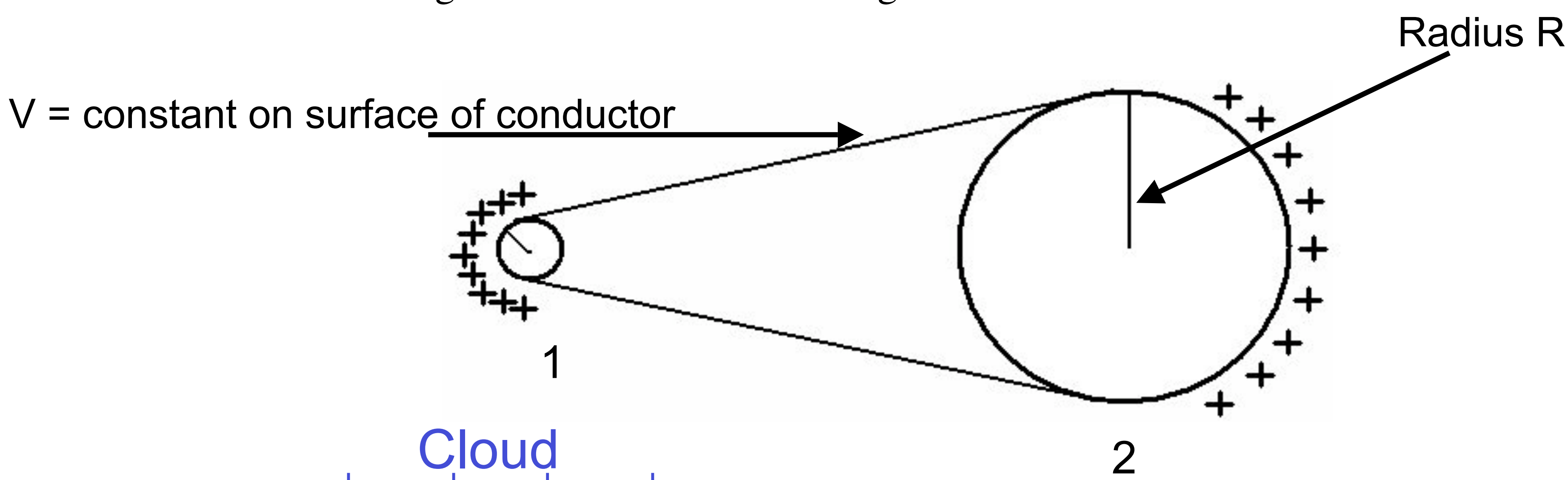
If the electric field in a gas exceeds a certain value, the gas breaks down and you get a spark or lightning bolt if the gas is air. In dry air at STP, you get a spark when

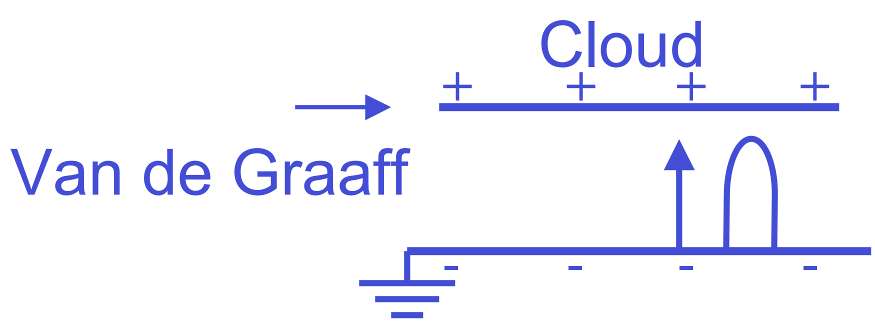
$$E \geq 3 \times 10^4 \frac{V}{cm}$$



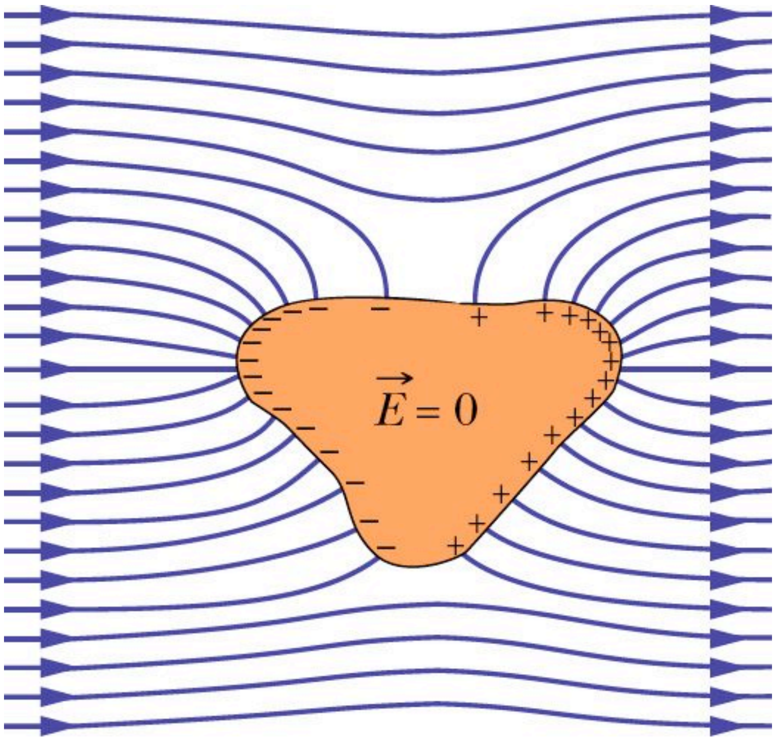
This explains why:

- Sharp points on conductors have the highest electric fields and cause corona discharge or sparks.
- Pick up the most charge with charge tester from the pointy regions of the non-spherical conductor.
- Use non-spherical metal conductor charged with teflon rod. Show variation of charge across surface with charge tester.





How does a conductor shield the interior from an exterior electric field?



• Start out with a uniform electric field with no excess charge on conductor. Electrons on surface of conductor adjust so that:

1. *E=0* inside conductor
2. Electric field lines are perpendicular to the surface. Suppose they weren't?
3. Does $E = \dfrac{\sigma}{\varepsilon_0}$ just outside the conductor
4. Is σ uniform over the surface?
5. Is the surface an equipotential?

6. If the surface had an excess charge, how would your answers change?

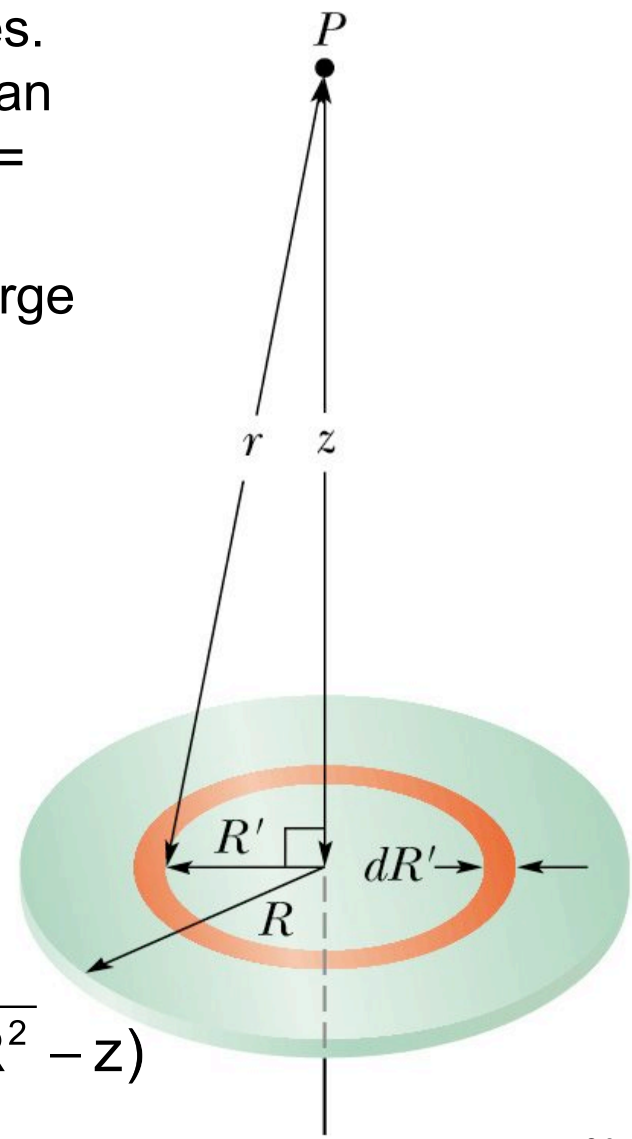
A metal slab is put in a uniform electric field of 10^6 N/C with the field perpendicular to both surfaces.

- Show how the charges are distributed on the conductor.
- Draw the appropriate pill boxes.
- What is the charge density on each face of the slab?
- Apply Gauss's Law. $\int E \cdot da = \frac{q_{in}}{\varepsilon_0}$

What is the electric potential of a uniformly charged circular disk?

We can treat the disk as a set of ring charges. The ring of radius R' and thickness dR' has an area of $2\pi R'dR'$ and it's charge is $dq = \sigma dA = \sigma(2\pi R')dR'$ where $\sigma=Q/(\pi R^2)$, the surface charge density. The potential due to the charge on this ring at point P given by

$$V = \frac{k}{\sqrt{(z^2 + (R')^2)}} Q$$



The potential dV at a point P due to the charged ring of radius R' is

$$dV = \frac{kdq}{\sqrt{(z^2 + (R')^2)}} = \frac{k\sigma 2\pi R'dR'}{\sqrt{(z^2 + (R')^2)}}$$

Integrating R' from R'=0 to R'=R

$$V = \int_0^R \frac{k\sigma 2\pi R'dR'}{\sqrt{(z^2 + (R')^2)}} \Rightarrow V = 2k\sigma\pi(\sqrt{z^2 + R^2} - z)$$