Lecture 5 Capacitance Ch. 25

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Capacitance

Definition of capacitance

A capacitor is a useful device in electrical circuits that allows us to store charge and electrical energy in a controllable way. The simplest to understand consists of two parallel conducting plates of area A separated by a narrow air gap d. If charge +*Q* is placed on one plate, and -*Q* on the other, the potential difference between them is V, and then the capacitance is defined as $C = \frac{Q}{V}$.

The SI unit is $\frac{Q}{V}$, which is called the Farad, named after the famous and creative scientist Michael Faraday from the early 1800's.

Applications

Radio tuner circuit uses variable capacitor

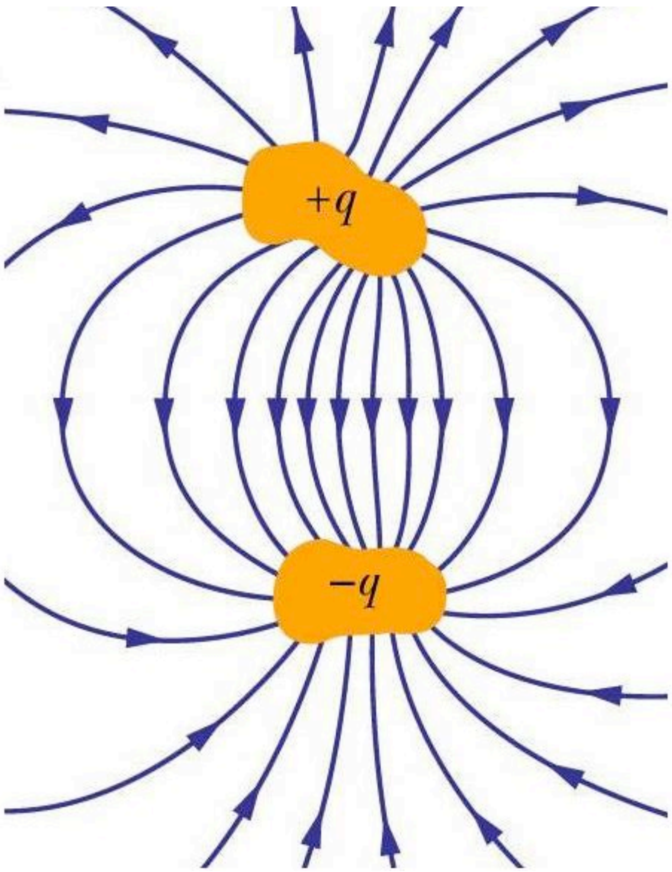
Blocks DC voltages in ac circuits

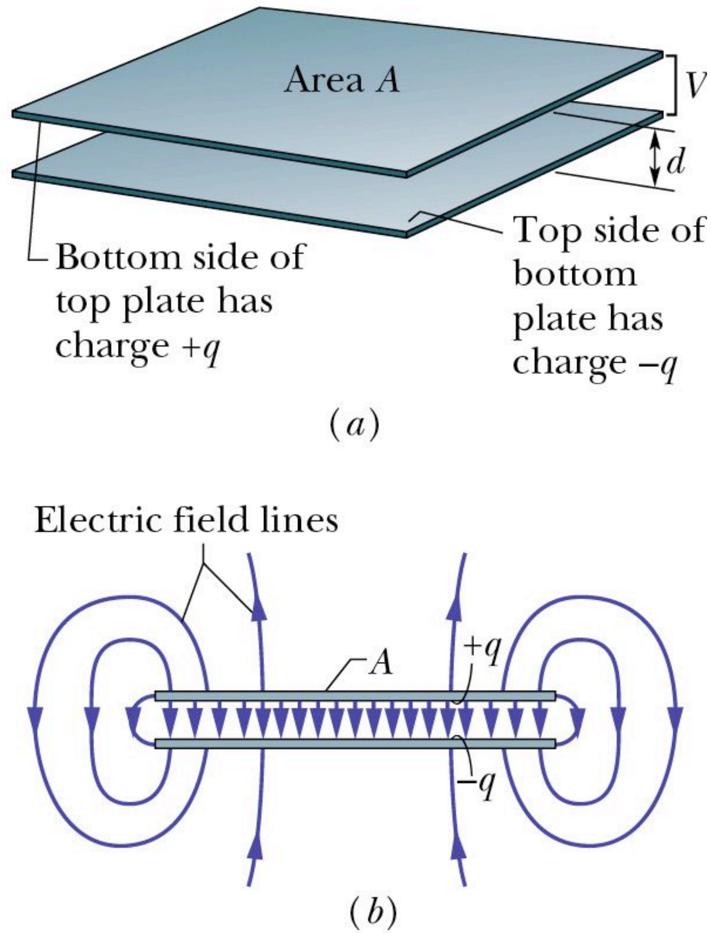
Act as switches in computer circuits

Triggers the flash bulb in a camera

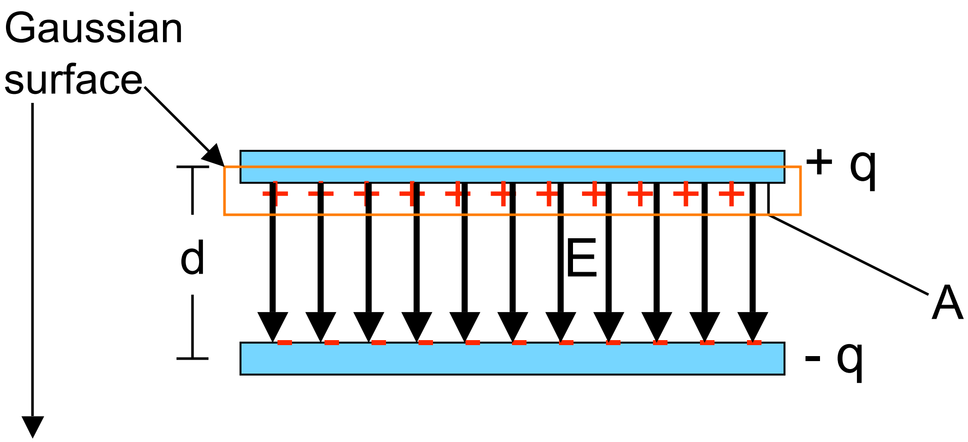
Converts AC to DC in a filter circuit

Parallel Plate Capacitor





Electric Field of Parallel Plate Capacitor



Area of plate =A

$$EA = \frac{q}{\varepsilon_0} \qquad E = \frac{q}{\varepsilon_0 A} \qquad q = \varepsilon_0 EA$$

$$V = Ed = \frac{qd}{\varepsilon_0 A}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\hat{r}$$

Integrate from - charge to + charge so that $\vec{E} \cdot d\hat{r} = -Edr$

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A}$$

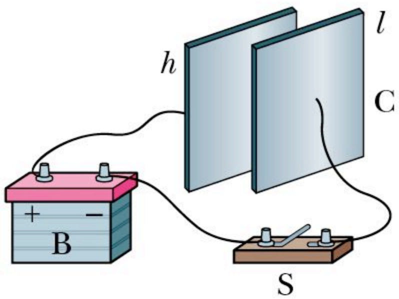
$$V_f - V_i = +\int_-^+ Edr = Ed$$

$$C = \frac{\varepsilon_0 A}{d}$$

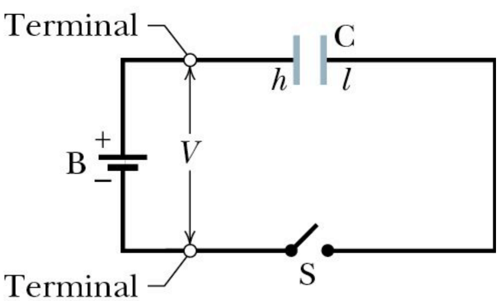
Coulomb/Volt = Farad

$$V = Ed$$

Show Demo Model, calculate its capacitance, and show how to charge it up with a battery.

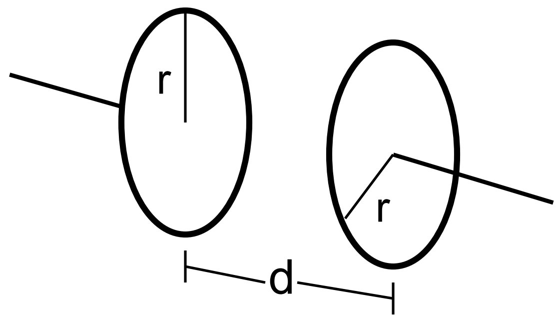


(a)



(b)

Circular parallel plate capacitor



r = 10 cm = 0.1m
$A = \pi r^2 = \pi(.1\text{m})^2$
$A = .03\ \text{m}^2$
d = 1 mm = .001 m

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = (10^{-11}\,\frac{C^2}{Nm^2})\frac{.03m^2}{.001m}$$

$$\left.\frac{\text{Coulomb}}{\text{Volt}}\right\}\text{Farad}$$

$$C = 3 \times 10^{-10}\,\text{F}$$

$$\boxed{C = 300\text{pF}}$$

p = pico = 10^{-12}

Demo Continued

Demonstrate

1. As d increases, voltage increases.

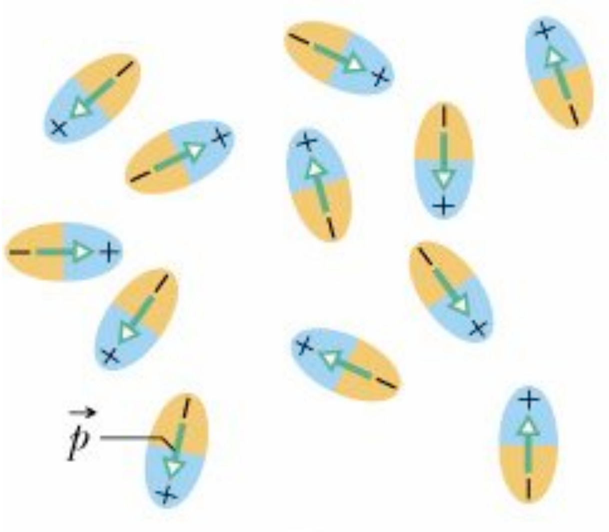
2. As d increases, capacitance decreases.

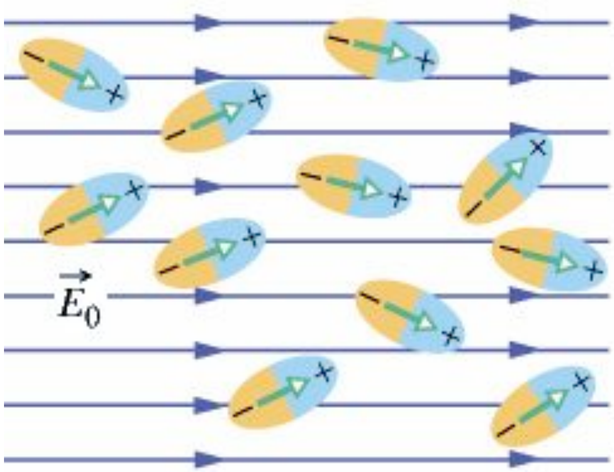
3. As d increases, E_0 and q are constant.

Dielectrics

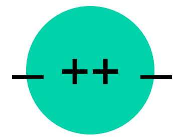
- A dielectric is any material that is not a conductor, but polarizes well. Even though they don't conduct they are electrically active
  - Examples. Stressed plastic or piezo-electric crystal will produce a spark.
  - When you put a dielectric in a uniform electric field (like in between the plates of a capacitor), a dipole moment is induced on the molecules throughout the volume. This produces a volume polarization that is just the sum of the effects of all the dipole moments. If we put it in between the plates of a capacitor, the surface charge densities due to the dipoles act to reduce the electric field in the capacitor.

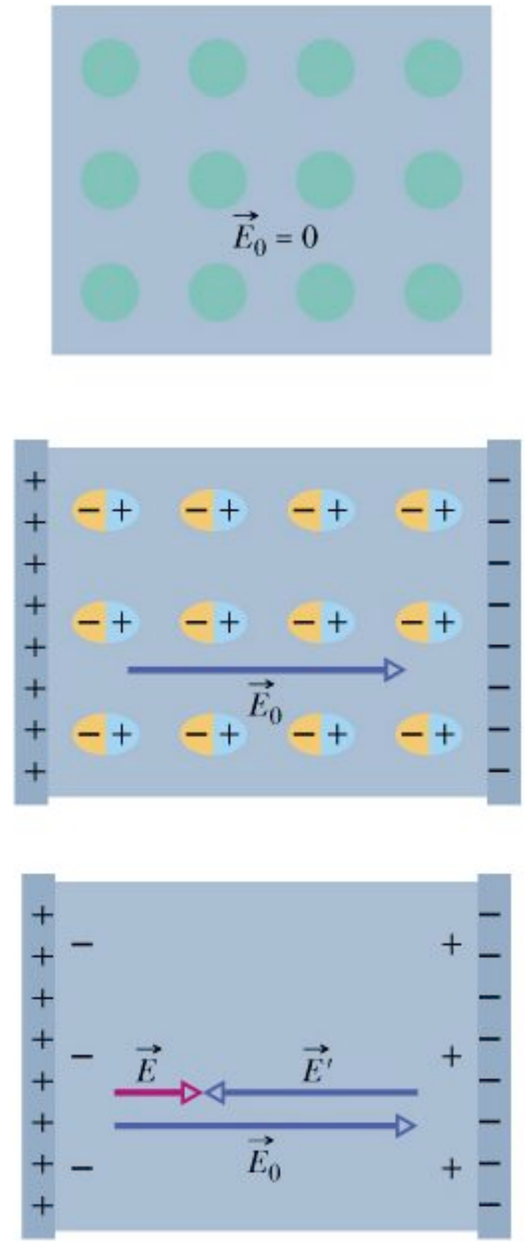
Permanent dipoles





Induced dipoles





E_0 = the applied field

E' = the field due to induced dipoles

$E = E_0 - E'$

Dielectrics

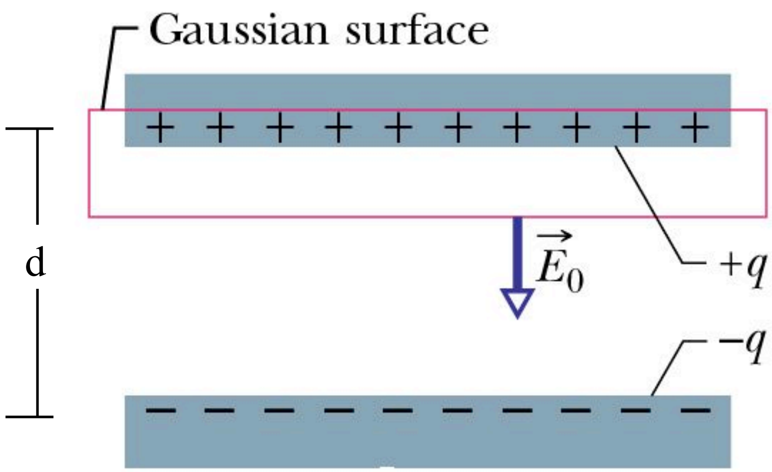
The amount that the field is reduced defines the dielectric constant κ from the formula $E = \dfrac{E_0}{\kappa}$, where E is the new field and E_0 is the old field without he dielectric.

Since the electric field is reduced and hence the voltage difference is reduced (since $E = Vd$), the capacitance is increased.

$$C = \frac{Q}{V} = \frac{Q}{\left(\dfrac{V_0}{\kappa}\right)} = \kappa C_0$$

where κ is typically between 2 – 6 with water equal to 80.

Show demo dielectric slab sliding in between plates. Watch how capacitance and voltage change. Also show aluminum slab.



$$C = q/V$$

$$V = E_0 d$$

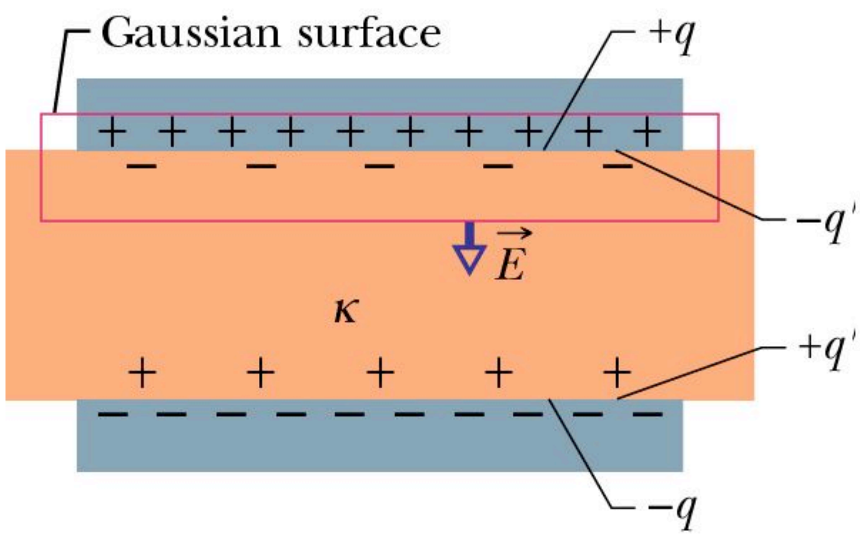
$$E_0 = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = q/A$$

$$E_0 = \frac{q}{\varepsilon_0 A}$$

$$V = \frac{qd}{\varepsilon_0 A}$$

$$C = \frac{\varepsilon_0 A}{d}$$



$$E = \frac{E_0}{\kappa}$$

$$V = \frac{E_0}{\kappa} d$$

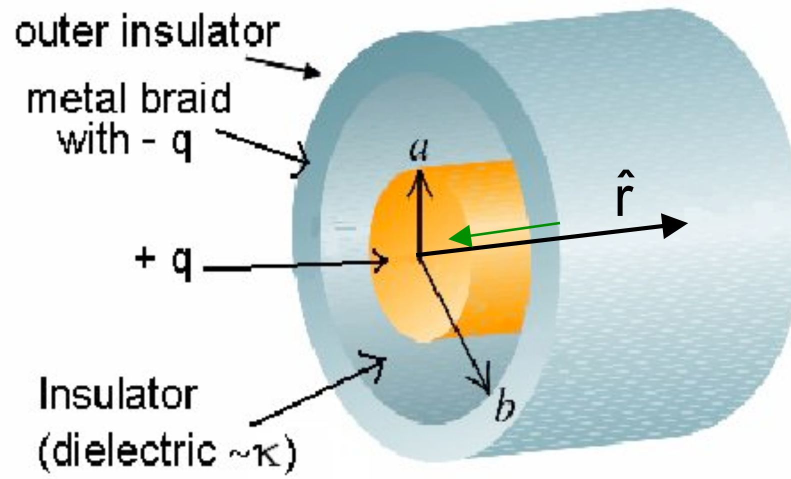
$$V = \frac{V_0}{\kappa}$$

$$C = q/V$$

$$C = \frac{\kappa q}{V_0}$$

$$C = \kappa C_0$$

Find the capacitance of a ordinary piece of coaxial cable (TV cable)



$$V_f - V_i = -\int_i^f \vec{E} \cdot d\hat{r}$$

$$\vec{E}.\, d\hat{r} = Edr\cos 180 = -Edr \qquad E_r = \frac{2k\lambda}{r}$$

Integrate from b to a   or - to +

$$V_a - V_b = -\int_b^a \vec{E}.\, d\hat{r} = \int_b^a Edr = 2k\lambda \int_b^a \frac{dr}{r} = +2k\lambda \ln r \Big|_a^b$$

$$V_a - V_b = 2k\lambda \ln\frac{b}{a} \qquad \lambda = Q/L \qquad k = \frac{1}{4\pi\varepsilon_0} \quad \text{~ air}$$

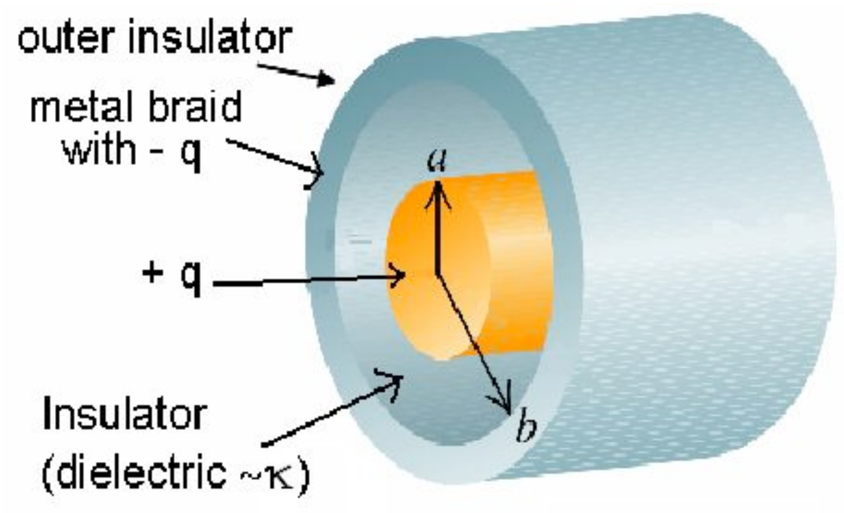
V_a is higher than V_b

capacitance of a coaxial cable cont.

So, $V = \dfrac{Q}{2\pi\varepsilon_0 L} \ln\dfrac{b}{a}$

$$C = \frac{Q}{V} = \frac{\cancel{Q}2\pi\varepsilon_0 L}{\cancel{Q}\ln\frac{b}{a}}$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$$



a = 0.5 mm

b = 2.0 mm

$\kappa \cong 2$

Now if a=0.5mm and b=2.0mm, then

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}$$

$$\frac{C}{L} = \frac{6\times10^{-11}}{\ln 4} = \frac{6\times10^{-11}}{1.38}$$

$$\frac{C}{L} = 43\frac{pF}{m}$$

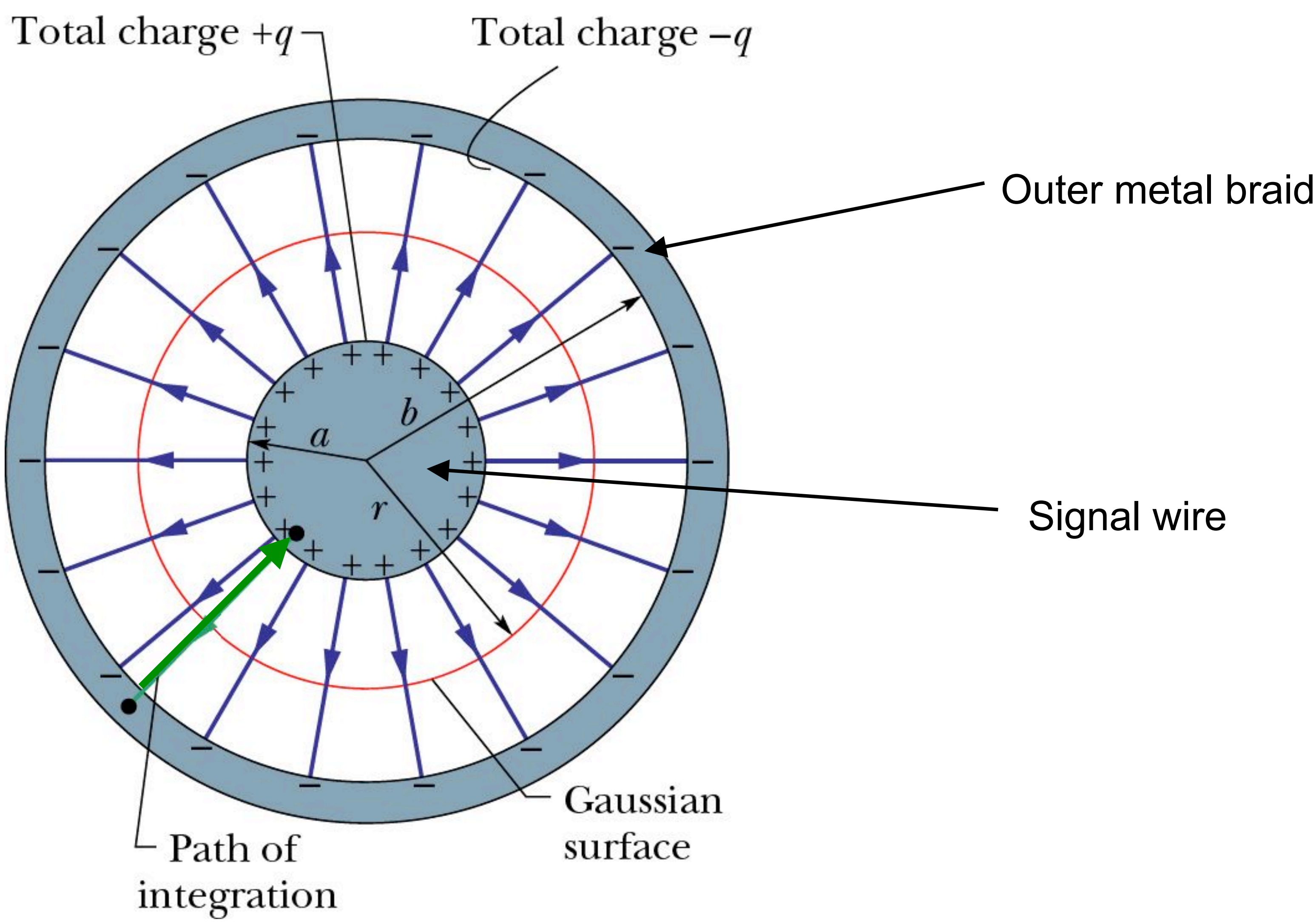
ε_0 (for air)

And if **κ** = 2, then

$$\frac{C}{L} = 86\frac{pF}{m}$$

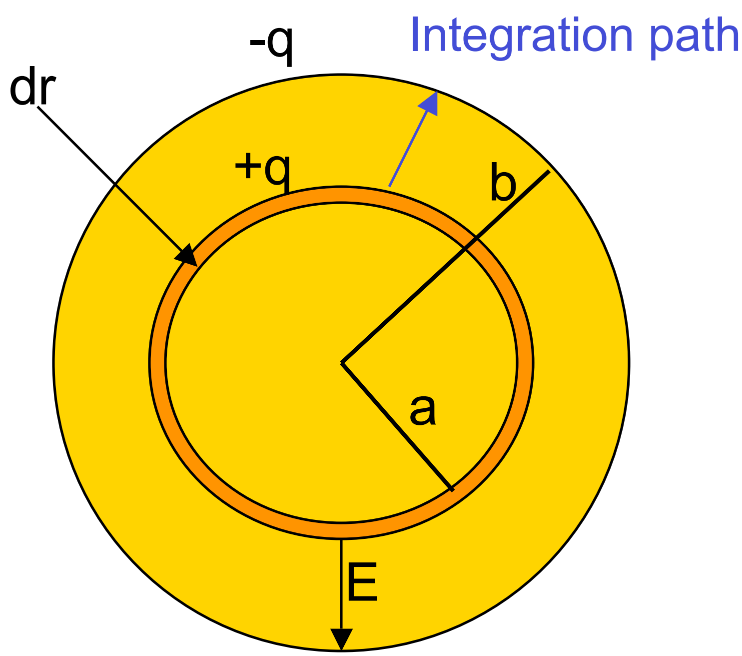
For κ = 2

Model of coaxial cable for calculation of capacitance



- to +

Capacitance of two concentric spherical shells



$$V = V_a - V_b = -\int_b^a \vec{E}\cdot d\hat{r} = +\int_b^a E dr$$

as $\vec{E}.\, d\hat{r} = E ds \cos 180 = -E dr$

$$V_a - V_b = +\int_b^a E dr = +\int_b^a \frac{kq}{r^2} dr = +kq\int_b^a \frac{dr}{r^2}$$

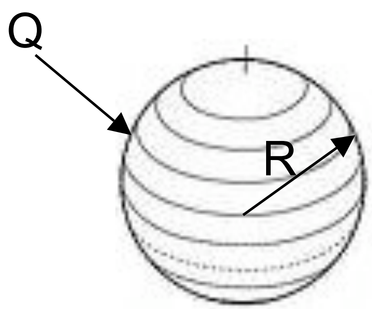
$$V = kq\frac{1}{r}\bigg|_b^a = kq\left(\frac{1}{a} - \frac{1}{b}\right) = kq\left(\frac{b-a}{ab}\right)$$

$$C = q/V = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Let b get very large. Then $C = 4\pi\varepsilon_0 a$ for an isolated sphere

Spherical capacitor or sphere

Recall our favorite example for E and V is spherical symmetry



The potential of a charged sphere is $V = \frac{kQ}{R}$ with V = 0 at r = ∞ .

The capacitance is

$$C = \frac{Q}{V} = \frac{Q}{kQ/R} = \frac{R}{k} = 4\pi\varepsilon_0 R$$

Where is the other plate (conducting shell)?

It's at infinity where it belongs, since that's where the electric lines of flux terminate.

k = 10^{10} and R in meters we have

$$C = \frac{R}{10^{10}} = 10^{-10} R(m) = 10^{-12} R(cm)$$

$$\boxed{C = R(cm)pF}$$

Earth: C = (6×10^8 cm)pF = 600 μF

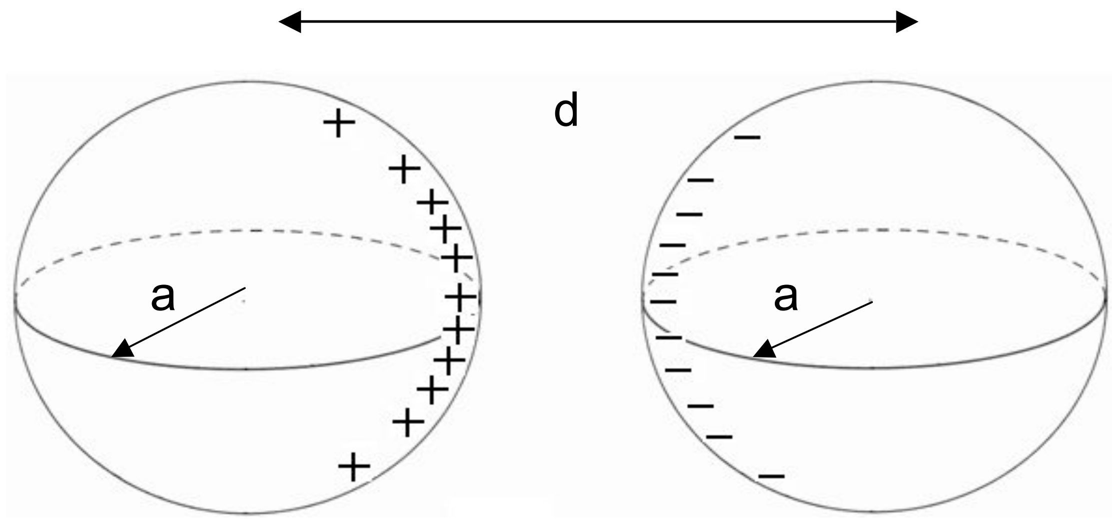
Marble: 1 pF

Basketball: 15 pF

You: 30 pF

Demo: Show how you measured capacitance of electroscope

Capacitance of one charged conducting sphere of radius a relative to another oppositely charged sphere of radius a



$C = 4\pi\varepsilon_0 a\ (1+m+m^2+m^3+m^4+\ldots..)$

$m = a/d$

d >>I←a

If d gets very large, then C= 10 pF

d =20 cm

a =10 cm

m =0.5

$C=10^{-10}(.1)\ (1+.5\ +.25\ +.125\ldots.)$

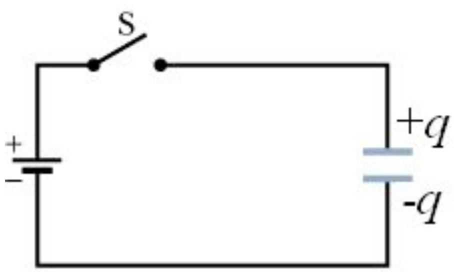
$C=10^{-10}(.1)(1/(1-m))$

$C= 0.2 \times 10^{-10}$ F

C= 0.02 nF =20 pF

Electric Potential Energy of Capacitor

As we begin charging a capacitor, there is initially no potential difference between the plates. As we remove charge from one plate and put it on the other, there is almost no energy cost. As it charges up, this changes.



At some point during the charging, we have a charge q on the positive plate.

The potential difference between the plates is $V = \frac{q}{C}$

As we transfer an amount *dq* of positive charge from the negative plate to the positive one, its potential energy increases by an amount *dU*.

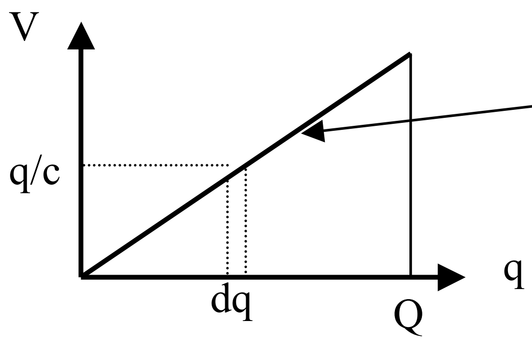
$$dU = Vdq = \frac{q}{C}dq.$$

The total potential energy increase is

$$U = \int_0^Q \frac{q}{C}dq = \frac{q^2}{2C}\Big| = \frac{Q^2}{2C}$$

Also $$\boxed{U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}}$$ using $C = \frac{Q}{V}$

Graphical interpretation of integration



$$dU = Vdq$$

$$U = \int_0^Q Vdq \quad \text{where} \quad V = \frac{q}{C}$$

$$\int_0^Q \frac{q}{C}dq = \frac{q^2}{2}\Big|_0^Q = \frac{Q^2}{2C}$$

$$U = \frac{1}{C}\sum_{i=1}^{N} q_i \Delta q_i$$ = Area under the triangle

Area under the triangle is the value of the integral $\int_0^Q \frac{q}{C}dq$

Area of the triangle is also = $\frac{1}{2} b \cdot h$

$$\text{Area } = \frac{1}{2}(b)(h) = \frac{1}{2}(Q)(\frac{Q}{C}) = \frac{1}{2}\frac{Q^2}{C}$$

Where is the energy stored in a capacitor?

- Find energy density for parallel plate capacitor. When we charge a capacitor we are creating an electric field. We can think of the work done as the energy needed to create that electric field. For the parallel plate capacitor the field is constant throughout, so we can evaluate it in terms of electric field E easily.

Use U = (1/2)QV

$$E = \frac{\sigma}{\kappa\varepsilon_0} = \frac{\sigma}{\varepsilon} = \frac{Q}{\varepsilon A} \quad \text{and} \quad V = ES$$

We are now including dielectric effects: ε

Solve for Q = εAE, V = ES and substitute in

$$U = \frac{1}{2}QV = \frac{1}{2}(\varepsilon AE)(ES) = \frac{1}{2}\varepsilon E^2 \underbrace{(AS)}_{\text{volume occupied by E}}$$

$$\frac{U}{AS} = \frac{1}{2}\varepsilon E^2 = \eta$$

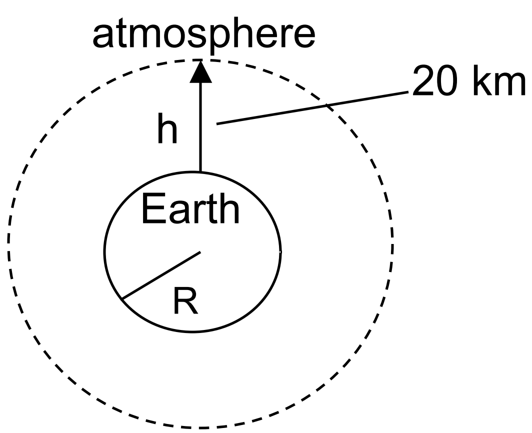
$$\boxed{\eta = \frac{1}{2}\varepsilon E^2}$$

Electrostatic energy density general result for all geometries.

To get total energy you need to integrate over volume.

How much energy is stored in the Earth's atmospheric electric field?

(Order of magnitude estimate)



R = 6x10^6 m

$$E = 100 \tfrac{V}{m} = 10^2$$

$$U = \frac{1}{2}\varepsilon_0 E^2 \cdot Volume$$

$$Volume = 4\pi R^2 h$$

$$Volume = 4\pi (6\times10^6)^2 (2\times10^4) = 8.6\times10^{18} m^3$$

$$U = \frac{1}{2}(10^{-11} \tfrac{C^2}{Nm^2})(10^2 \tfrac{V}{m})(8.6\times10^{18} m^3)$$

$$\boxed{U = 4.3\times10^{11} J}$$

This energy is renewed daily by the sun. Is this a lot?

The total solar influx is 200 Watts/m^2

$$U_{sun} = 200 \cdot 3.14(6\times10^6)^2 \cong 2\times10^{16} J/s = 2\times10^{21} J/day$$

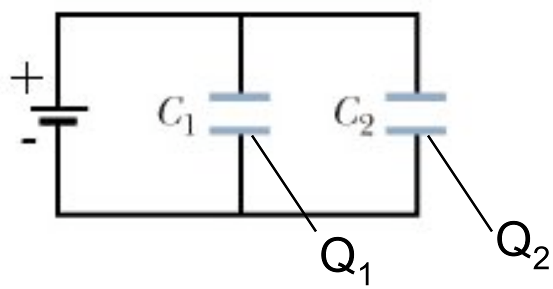
$$\boxed{U/U_{sun} \cong 2\times10^{-10}}$$

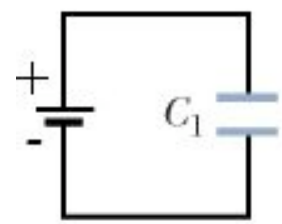
Only an infinitesimal fraction gets converted to electricity.

World consumes about 10^{18} J/day. This is 1/2000 of the solar flux.

Parallel Combination of Capacitors

Typical electric circuits have several capacitors in them. How do they combine for simple arrangements? Let us consider two in parallel.





$$C = \frac{Q}{V}$$

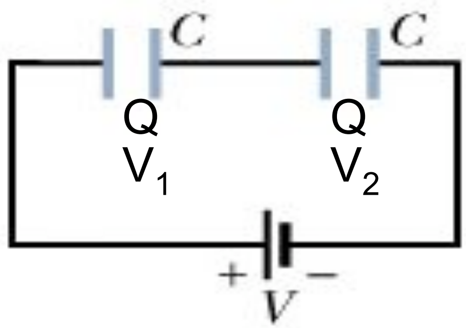
We wish to find one equivalent capacitor to replace C_1 and C_2. Let's call it C.

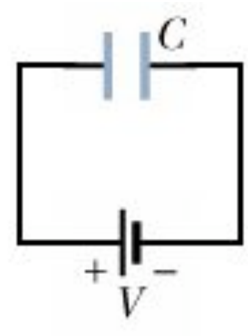
The important thing to note is that the voltage across each is the same and equivalent to V. Also note what is the total charge stored by the capacitors? Q.

$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V$$

$$\frac{Q}{V} = C_1 + C_2 \Rightarrow \boxed{C = C_1 + C_2}$$

Series Combination of Capacitors





$$C = \frac{Q}{V}$$
$$V = \frac{Q}{C}$$

What is the equivalent capacitor C?

Voltage across each capacitor does not have to be the same.

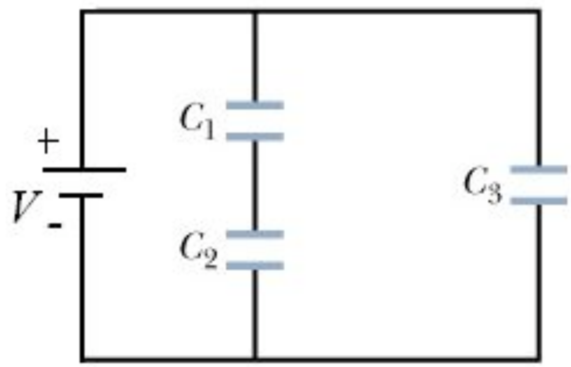
The charges on each plate have to be equal and opposite in sign by charge conservation.

The total voltage across each pair is:

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = Q\left(\frac{1}{C}\right)$$

So $\frac{1}{C_1} = \frac{1}{C_1} + \frac{1}{C_2}$ Therefore, $\boxed{C = \frac{C_1 C_2}{C_1 + C_2}}$

Sample problem



$C_1 = 10\ \mu F$

$C_2 = 5.0\ \mu F$

$C_3 = 4.0\ \mu F$

a) Find the equivalent capacitance of the entire combination.

C_1 and C_2 are in series.

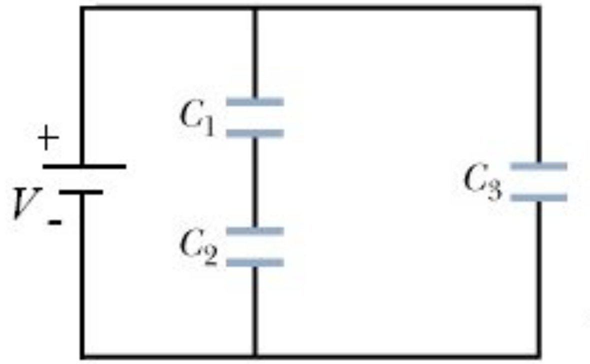
$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{12} = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = 3.3 \mu F$$

C_{12} and C_3 are in parallel.

$$C_{eq} = C_{12} + C_3 = 3.3 + 4.0 = 7.3 \mu F$$

Sample problem (continued)



C_1 = 10 μF

C_2 = 5.0 μF

C_3 = 4.0 μF

b) If V = 100 volts, what is the charge Q_3 on C_3?

C = Q/V

$$Q_3 = C_3 V = 4.0\times10^{-6} \cdot 100$$

$$\boxed{Q_3 = 4.0\times10^{-4}\,Coulombs}$$

c) What is the total energy stored in the circuit?

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \times 7.3\times10^{-6} F \times 10^4 V^2 = 3.6\times10^{-2} J$$

$$\boxed{U = 3.6\times10^{-2} J}$$