Lecture 6 Current and Resistance Ch. 26

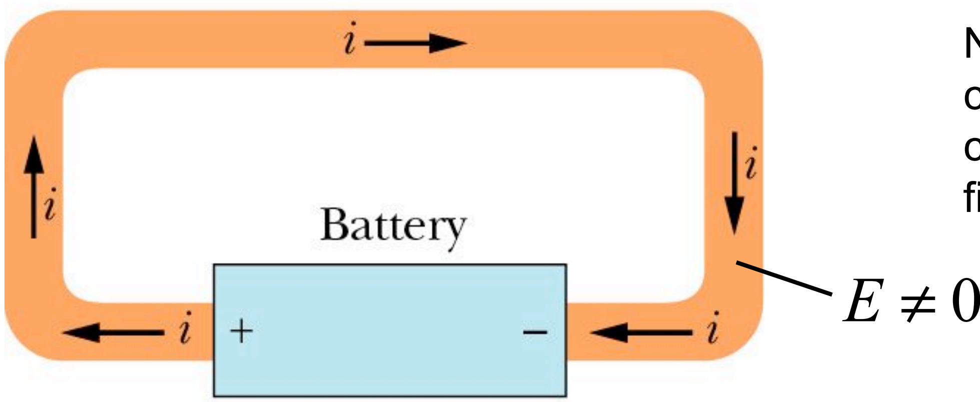
- Cartoon -Invention of the battery and Voltaic Cell
- Warm-up problem
- Topics
  - What is current?
  - Current density
  - Conservation of Current
  - Resistance
  - Temperature dependence
  - Ohms Law
  - Bateries, terminal voltage, imdedance matching
  - Power dissipation
  - Combination of resistiors
- Demos
  - Ohms Law demo on overhead projector
  - T dependence of resistance
  - Three 100 Watt light bulbs
- Puzzles
  - Resistor network figure out equivalent resistance

Loop of copper wire

(a)

Nothing moving;

electrostatic equilibrium



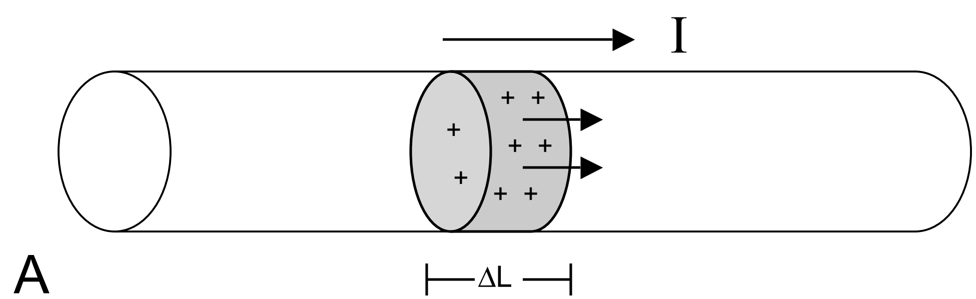
(b)

Now battery voltage forces charge through the conductor and we have a field in the wire.

What is Current?

It is the amount of positive charge that moves past a certain point per unit time.

$$I = \frac{\Delta Q}{\Delta t} = \frac{Coulomb}{\sec ond} = Amp$$



Copper wire with voltage across it

$$v\Delta t = \Delta L$$

Drift velocity of charge

$$\begin{aligned}\Delta Q &= \text{charge per unit volume} \times \text{volume}\\ &= \text{nq} \times \text{Av}\Delta\text{t}\end{aligned}$$

$$\Delta \text{Q} = \text{nqAv}\Delta\text{t}$$

Density of electrons

1.6 x 10^{-19} C

Divide both sides by Δt.

$$I = \frac{\Delta Q}{\Delta t} = nqAv$$

Question What causes charges to move in the wire?

How many charges are available to move?

Example What is the drift velocity for 1 Amp of current flowing through a 14 gauge copper wire of radius 0.815 mm?

Drift velocity $$v_d = \frac{I}{nqA}$$

$$n = \rho \frac{N_o}{\mathrm{M}} = 8.4\text{x}10^{22} \text{ atoms/cm}^3$$

$$v_d = \frac{1 amp}{8.4 \times 10^{22} \frac{atoms}{cm^3} \times 1.6 \times 10^{-19} C \times \pi (.0815 cm)^2}$$

$$v_d = 3.5 \times 10^{-5} \ {}^{m}\!/\!{}_{s}$$

I = 1 Amp

$q = 1.6\text{x}10^{-19}$ C

$A = \pi(.0815 \text{ cm})^2$

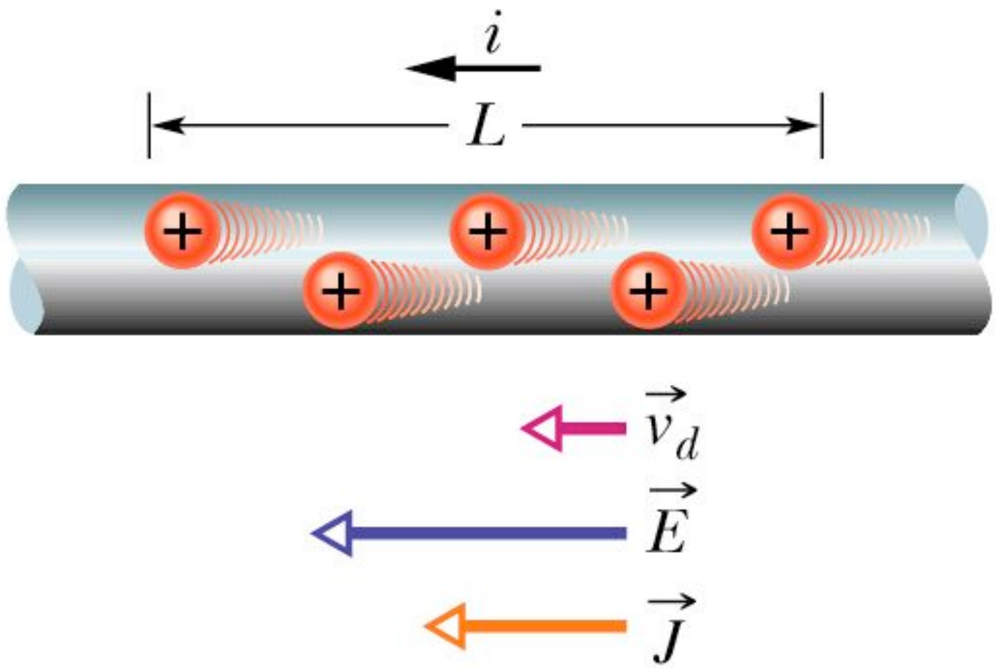
$\rho = 8.9$ grams/cm ^{3}

$N_o = 6\text{x}10^{23}$ atoms/mole

M = 63.5 grams/mole

The higher the density the smaller the drift velocity

Drift speed of electrons and current density



Directions of current i is defined as the direction of positive charge.

$$i = nAqv_d$$

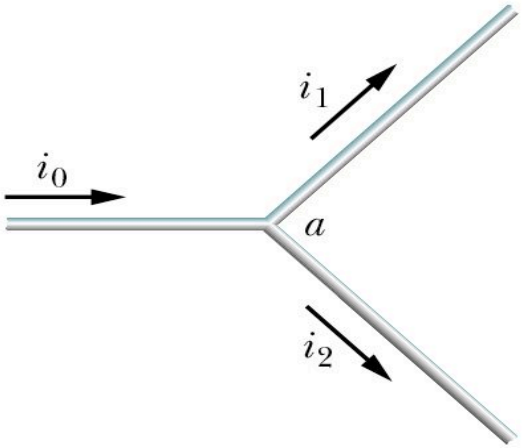
$$J = \frac{i}{A}$$

$$J = nqv_d$$

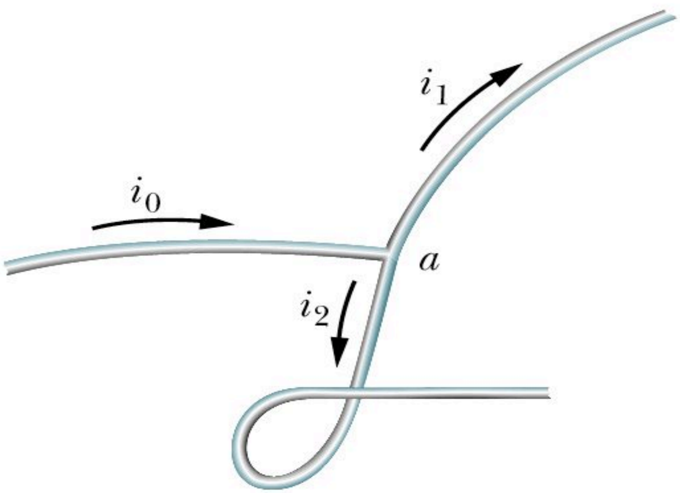
(Note positive charge moves in direction of $\vec{E}$) electron flow is opposite $\vec{E}$.

Currents: Steady motion of charge and conservation of current

$i = i_1 = i_2$ (Kirchoff's 2nd Rule)



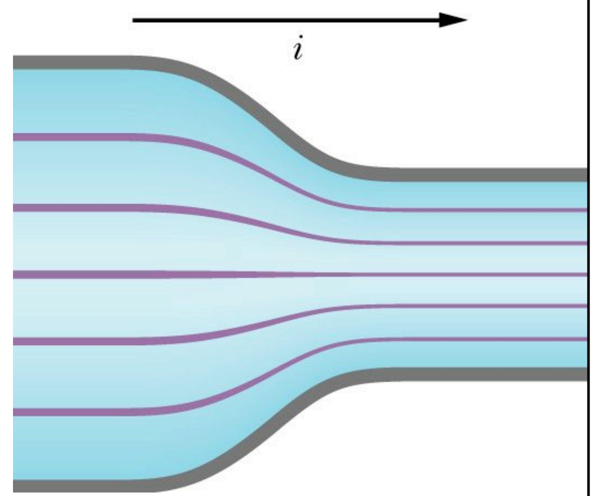
(a)



(b)

Current is the same throughout all sections in the diagram below; it is continuous.

Current density **J** does vary.



Question: How does the drift speed compare to the instantaneous speed?

Instantaneous speed ~ 10^6 m/s

$$v_d \approx 3.5\times10^{-11} v_{instant}$$

(This tiny ratio is why Ohm's Law works so well for metals.)

At this drift speed $3.5\text{x}10^{-5}$ m/s, it would take an electron 8 hours to go 1 meter.

Question: So why does the light come on immediately when you turn on the light switch?

It's like when the hose is full of water and you turn the faucet on, it immediately comes out the ends. The charge in the wire is like the water. A wave of electric field travels very rapidly down the wire, causing the free charges to begin drifting.

Example: Recall typical TV tube, CRT, or PC monitor. The electron beam has a speed 5×10^7 m/s. If the current is I = 100 microamps, what is n?

$$n = \frac{I}{qAv} = \frac{10^{-4}\,A}{1.6\times10^{-19}\,C\cdot10^{-6}\,m^2\cdot5\times10^{7}\,{}^{m}\!/\!_{s}}$$

Take A $= 1mm^2$

$$= (10^{-3}m)^2$$

$$= 10^{-6}m^2$$

<u>For CRT</u>

$$n = 1.2\times10^{13}\,\frac{\text{electrons}}{m^3} = 1.2\times10^{7}\,\frac{\text{electrons}}{cm^3}$$

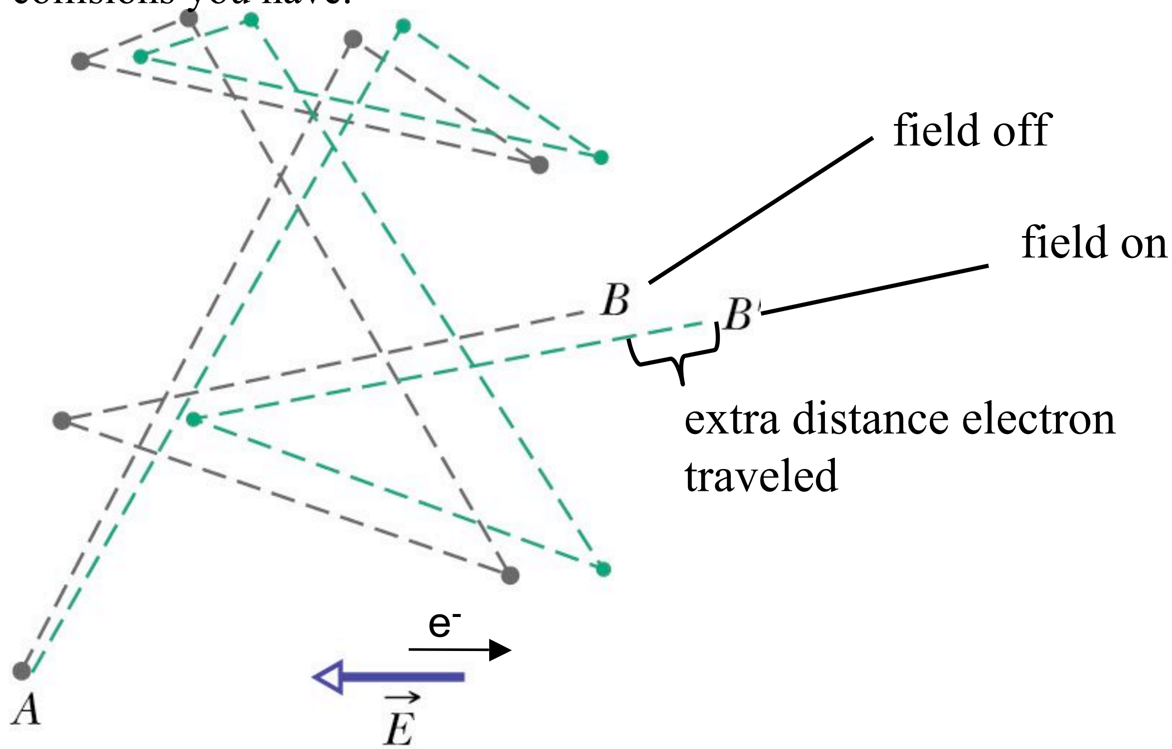
<u>For Copper</u>

$$n = 8.5\times10^{22}\,\frac{\text{electrons}}{cm^3}$$

The lower the density the higher the speed.

What is Resistance?

The collisions between the electrons and the atoms is the cause of resistance and the cause fo a very slow drift velocity of the electrons. The higher the density, the more collisions you have.



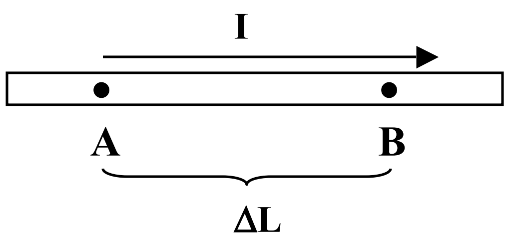
The dashed lines represent the straight line tracks of electrons in between collisions

•Electric field is off.

•Electric field is on. When the field is on, the electron traveled drifted further to B'.

Ohm's Law

Want to emphasize here that as long as we have current (charge moving) due to an applied potential, the electric field is no longer zero inside the conductor.



Potential difference

$$V_B - V_A = E\Delta L, \text{ where E is constant.}$$

$$I = \text{current} \propto E\Delta L \quad \text{(Ohm's law)}$$

True for many materials – not all. Note that Ohms Law is an experimental observation and is not a true law.

Constant of proportionality between V and I is known as the resistance. The SI unit for resistance is called the ohm.

$$V = RI \qquad \mathrm{R} = \frac{\mathrm{V}}{\mathrm{I}} \qquad Ohm = \frac{Volt}{amp}$$

Demo: Show Ohm's Law

<u>Best conductors</u>

Silver

Copper – oxidizes

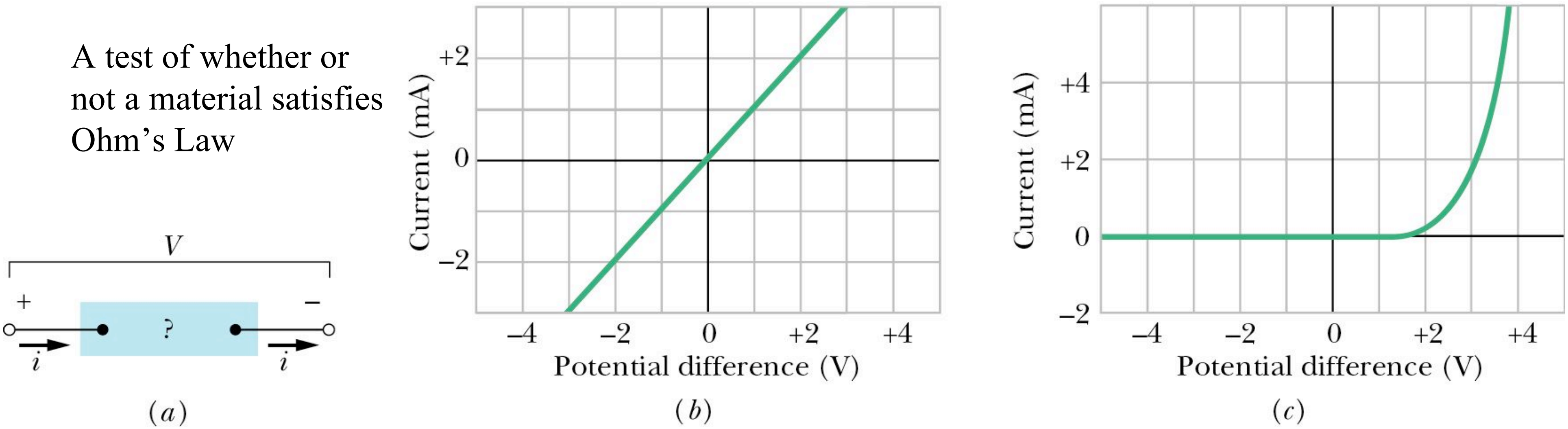
Gold – pretty inert

<u>Non-ohmic materials</u>

Diodes

Superconductors

A test of whether or not a material satisfies Ohm's Law



$$V = IR$$

$$I = \frac{V}{R}$$

$$\text{Slope} = \frac{1}{R} = \text{constant}$$

Ohm's law is satisfied

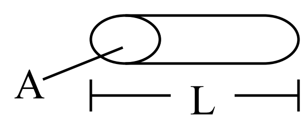
Here the slope depends on the potential difference. Ohm's Law is violated for a pn junction diode.

Resistance: What is it? Denote it by R

- Depends on shape, material, temperature.
- Most metals: R increases with increasing T
- Semi-conductors: R decreases with increasing T

Define a new constant which characterizes materials.

Resistivity $\rho = R\dfrac{A}{L}$



$$R = \frac{L}{A}\rho$$

Demo: Show temperature dependence of resistance

For materials $\rho = 10^{-8}$ to 10^{15} ohms-meters

Example: What is the resistance of a 14 gauge Cu wire? Find the resistance per unit length.

$$\frac{R}{L} = \frac{\rho_{cu}}{A} = \frac{1.7\times 10^{-8}\,\Omega m}{3.14(.815\times 10^{-3})^2} \cong 8\times 10^{-3}\ \Omega/m$$

Build circuits with copper wire. We can neglect the resistance of the wire. For short wires 1-2 m, this is a good approximation.

Note Conductivity = 1/Resistivity $\sigma = 1/\rho$

Example Temperature variation of resistivity.

$$\rho = \rho_{20}\left[1+\alpha(T-20)\right]$$

can be positive or negative

$$R = \frac{L}{A}\rho$$

Consider two examples of materials at T = 20^{o}C.

	ρ_{20} (Ω-m)	α(C^{-1})	L	Area	R (20^{o}C)
Fe	10^{-7}	0.005	$6x10^{6}$ m	1mm^{2}(10^{-6}m^{2})	60,000 Ω
Si	640	- 0.075	1 m	1 m^{2}	640 Ω

Fe – conductor - a long $6x10^{6}$ m wire.

Si – insulator - a cube of Si 1 m on each side

Question: You might ask is there a temperature where a conductor and insulator are one and the same?

Condition: $R_{Fe} = R_{Si}$ at what temperature?

Use $R = \frac{L}{A}\rho$ $R = \rho_{20}\left[1+\alpha(T-20\ C)\right]\frac{L}{A}$

$R_{Fe} = 10^{-7}\ \Omega\text{-m}\ [\ 1 + .005\ (T\text{-}20)]\ \frac{6\times10^{6}m}{10^{-6}m^{2}}$

$R_{Si} = 640\ \Omega\text{-m}\ [\ 1 + .075\ (T\text{-}20)]\ \frac{1m}{1m^{2}}$

Now, set $R_{Fe} = R_{Si}$ and solve for T

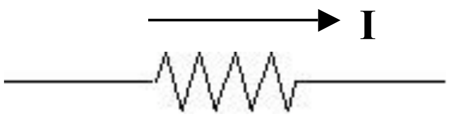
T – 20 C = – 196 C

$\boxed{\text{T = – 176 C or 97 K (pretty low temperature)}}$

Resistance at Different Temperatures

	T =293K	T = 77K (Liquid Nitrogen)	
Cu	.1194 Ω	.0152 Ω	conductor
Nb	.0235 Ω	.0209 Ω	impure
C	.0553 Ω	.069 Ω	semiconductor

Power dissipation resistors



Potential energy decrease

$$\Delta U = \Delta Q(-V)$$

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}(-V)$$

$$P = IV$$ (drop the minus sign)

Rate of potential energy decreases equals rate of thermal energy increases in resistor.

Called Joule heating

• good for stove and electric oven

• nuisance in a PC – need a fan to cool computer

Also since V = IR,

$$P = I^2R \text{ or } \frac{V^2}{R}$$ All are equivalent.

Example: How much power is dissipated when I = 2A flows through the Fe resistor of

R = 10,000 Ω. $P = I^2R = 2^2 x 10^4\ \Omega = 40{,}000$ Watts

Batteries

A device that stores chemical energy and converts it to electrical energy.

Emf of a battery is the amount of increase of electrical potential of the charge when it flows from negative to positive in the battery. (Emf stands for electromotive force.)

Carbon-zinc = Emf = 1.5V

Lead-acid in car = Emf = 2V per cell

(large areas of cells give lots of current)

Car battery has 6 cells or 12 volts.

Power of a battery = P

$\boxed{P = \varepsilon I}$

ε is the Emf

Batteries are rated by their energy content. Normally they give an equivalent measure such as the charge content in $\boxed{\text{mA-Hrs}}$

milliamp-Hours

Charge = (coulomb/seconds) x seconds

Internal Resistance

As the battery runs out of chemical energy the internal resistance increases.

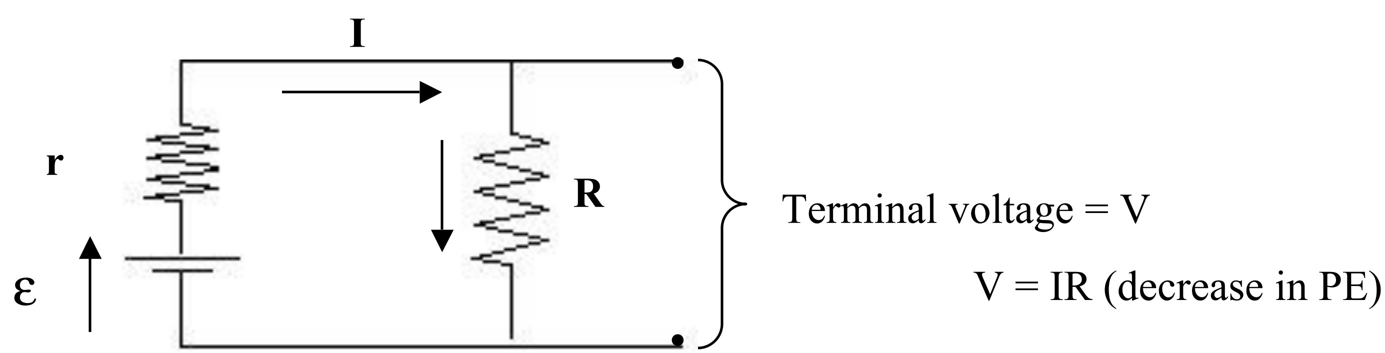
What is terminal voltage?

Terminal Voltage decreases quickly.

How do you visualize this?

What is the relationship between Emf, resistance, current, and terminal voltage?

Circuit model looks like this:



$$\varepsilon = Ir + IR \qquad \varepsilon = I(r + R)$$

$$\varepsilon - Ir = V = IR \qquad \mathrm{I} = \frac{\varepsilon}{(\mathrm{r} + \mathrm{R})}$$

The terminal voltage decrease $= \varepsilon$ - Ir as the internal resistance r increases or when I increases.

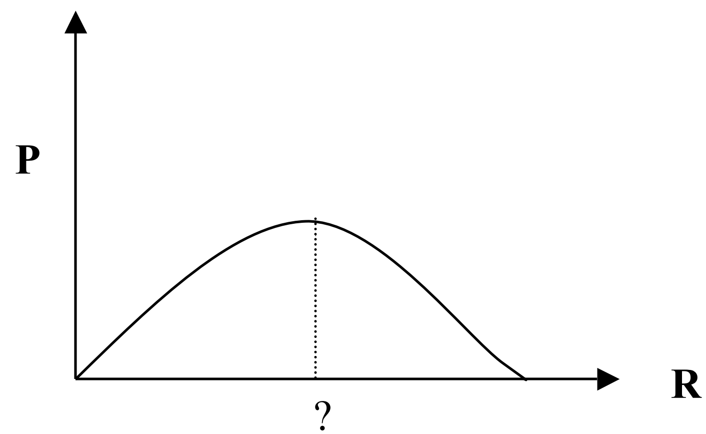
Example: This is called *impedance matching*. The question is what value of load resistor R do you want to maximize power transfer from the battery to the load.

$$I = \frac{E}{r+R}$$ =current from the battery

$P = I^2R$ = power dissipated in load

$$P = \frac{E^2}{(r+R)^2}R$$

$$\frac{dP}{dR} = 0$$



Solve for R

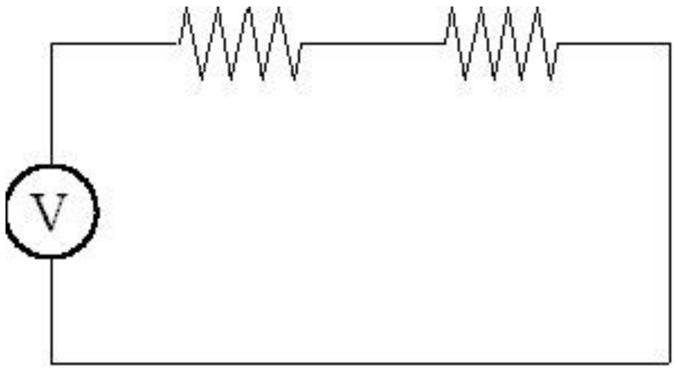
$\boxed{R = r}$

You get max. power when load resistor equals internal resistance of battery.

(battery doesn't last long)

Combination of resistors

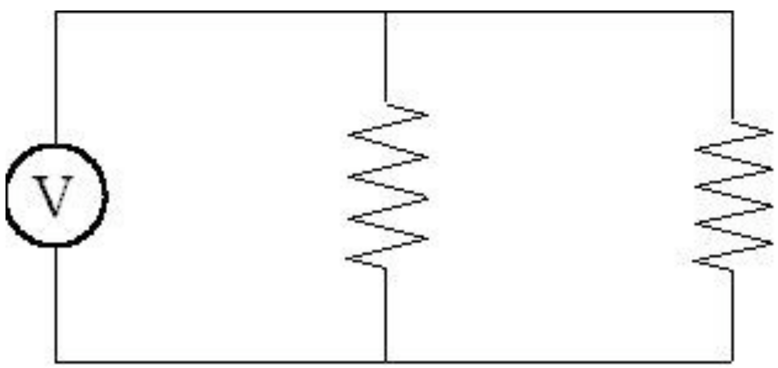
Resistors in series



Current is the same in both the resistors

$$V = R_1 I + R_2 I = (R_1 + R_2) I$$
$$R_{eqiv} = R_1 + R_2$$

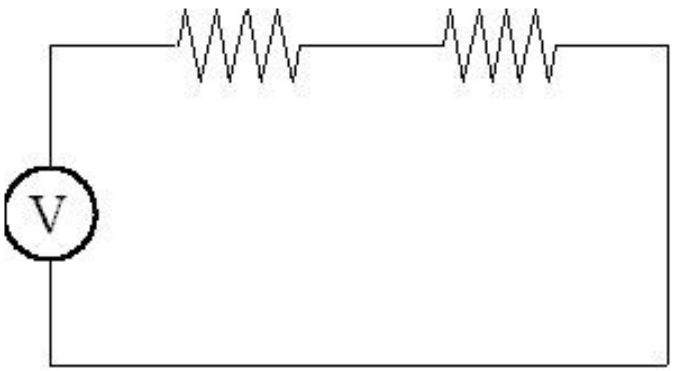
Resistors in parallel



Voltages are the same, currents add.

$$I = I_1 + I_2$$
$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$
$$So, \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R_{equiv} = \frac{R_1 R_2}{R_1 + R_2}$$

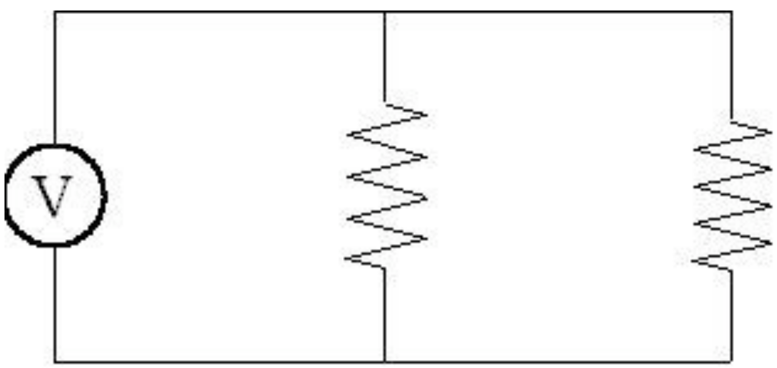
Resistors in series



$V = R_1 I + R_2 I = (R_1 + R_2)I$

$R_{equiv} = R_1 + R_2$

Resistors in parallel



Voltages are the same, currents add.

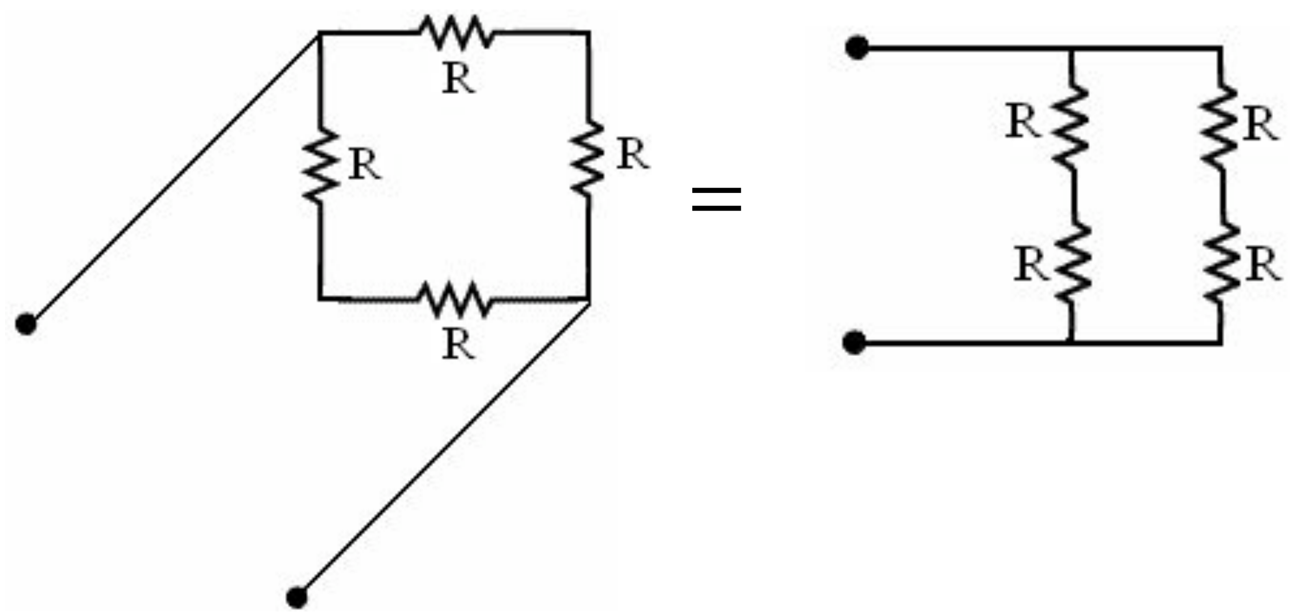
$I = I_1 + I_2$

$V/R = V/R_1 + V/R_2$

$\Rightarrow \quad 1/R = 1/R_1 + 1/R_2$

$R_{equiv} = R_1 R_2 / (R_1 + R_2)$

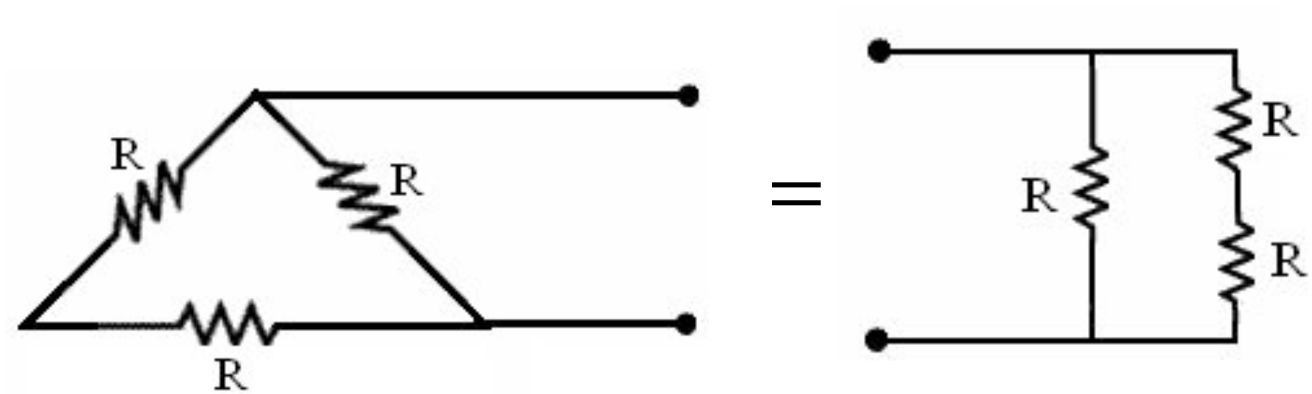
Equivalent Resistance



$$R_{eq} = (R+R)\|(R+R) = 2R\|2R$$

$$R_{eq} = \frac{4R^2}{4R}$$

$$R_{eq} = R$$

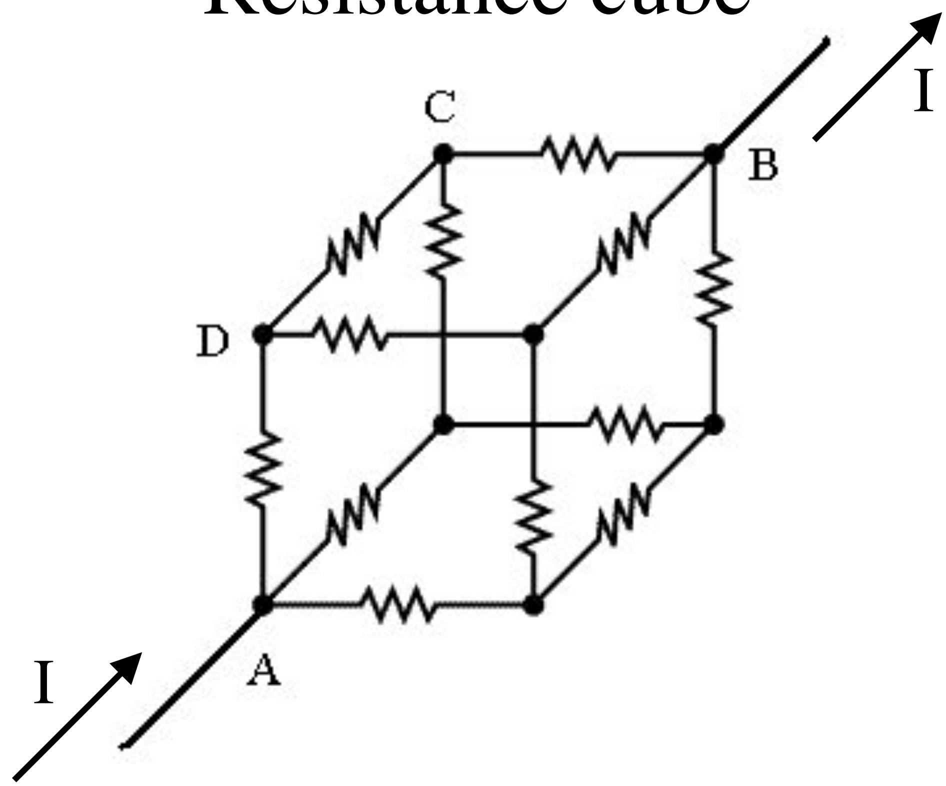


$$R_{eq} = R\|(R+R) = R\|2R$$

$$R_{eq} = \frac{2R^2}{3R}$$

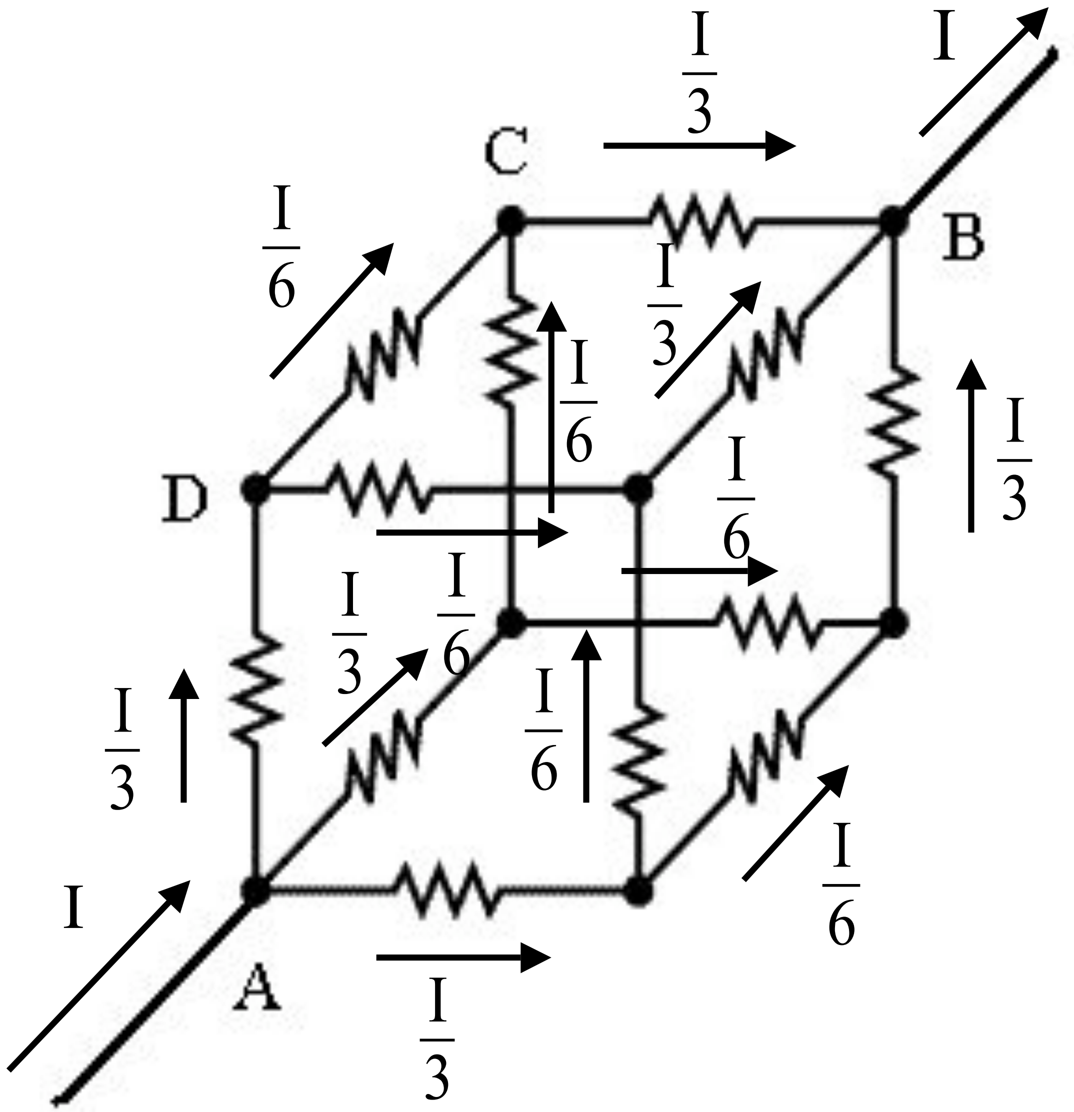
$$R_{eq} = \frac{2}{3}R$$

Resistance cube



The figure above shows 12 identical resistors of value R attached to form a cube. Find the equivalent resistance of this network as measured across the body diagonal---that is, between points A and B. (Hint: Imagine a voltage V is applied between A and B, causing a total current I to flow. Use the symmetry arguments to determine the current that would flow in branches AD, DC, and CB.)

Resistance Cube cont.



Because the resistors are identical, the current divides uniformly at each junction.

$$V = R_{eq} I$$

$$V = V_{AD} + V_{DC} + V_{CB}$$

$$R_{eq} I = R\frac{I}{3} + R\frac{I}{6} + R\frac{I}{3}$$

$$R_{eq} I = \frac{5}{6} R I$$

$$\boxed{R_{eq} = \frac{5}{6} R}$$