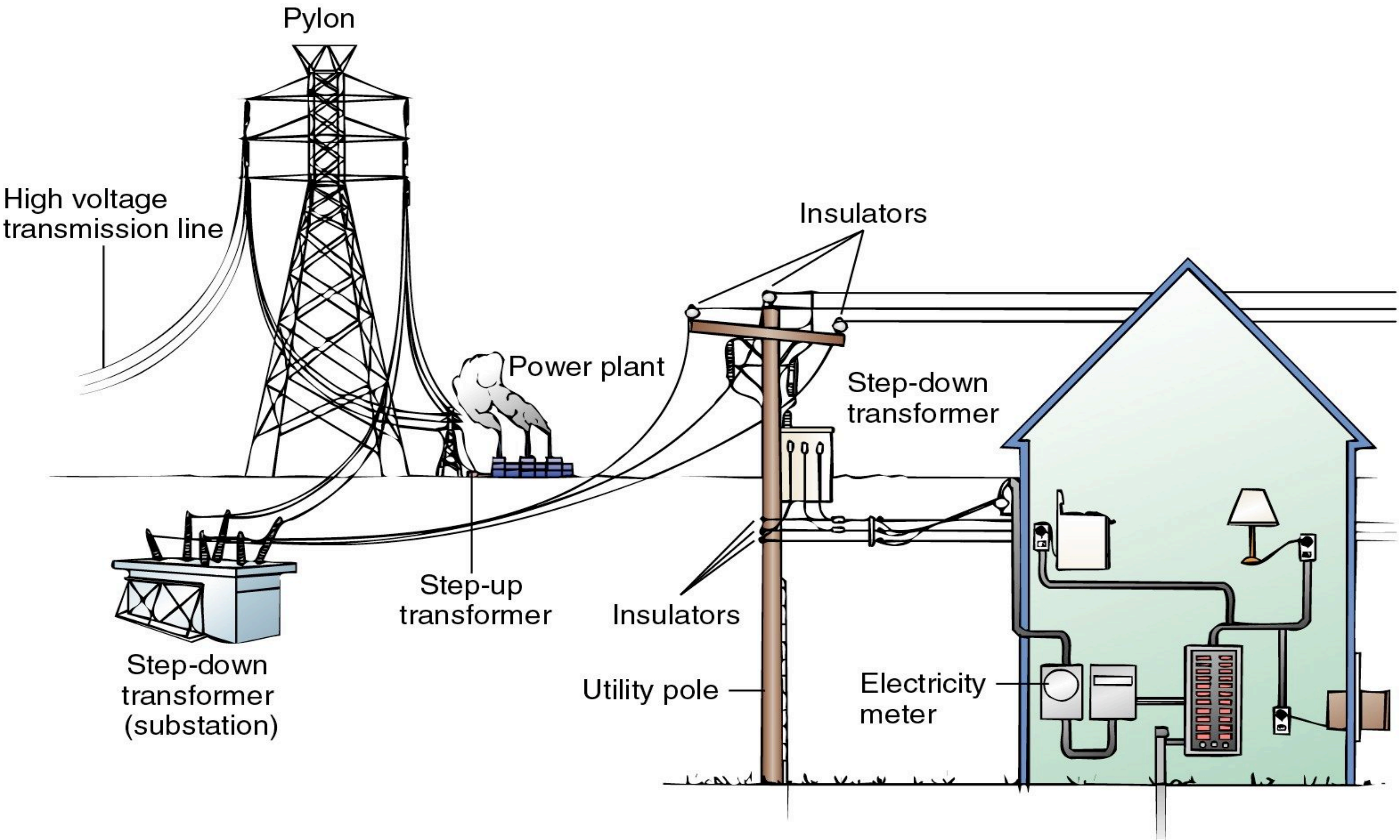
Lecture 7 Circuits Ch. 27

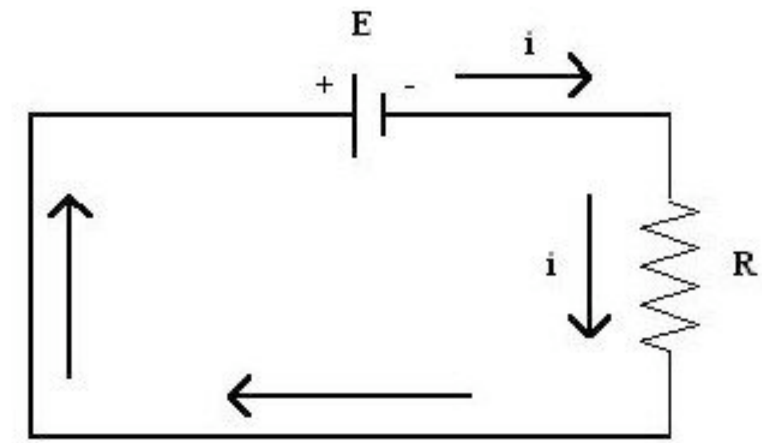
- Cartoon -Kirchhoff's Laws
- Warm-up problems
- Topics
  - Direct Current Circuits
  - Kirchhoff's Two Rules
  - Analysis of Circuits Examples
  - Ammeter and voltmeter
  - RC circuits
- Demos
  - Three bulbs in a circuit
  - Power loss in transmission lines
  - Resistivity of a pencil
  - Blowing a fuse

Transmission line demo



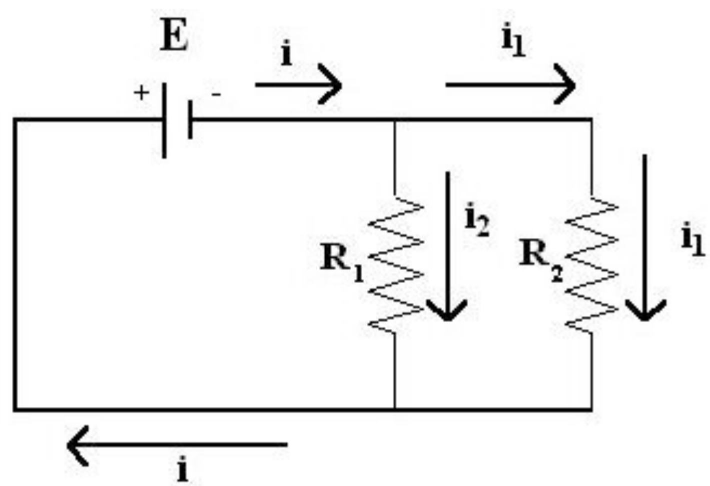
Direct Current Circuits

1. The sum of the potential drops around a closed loop is zero. This follows from energy conservation and the fact that the electric field is a conservative force.



$E - iR = 0$

2. The sum of currents into any junction of a closed circuit must equal the sum of currents out of the junction. This follows from charge conservation.

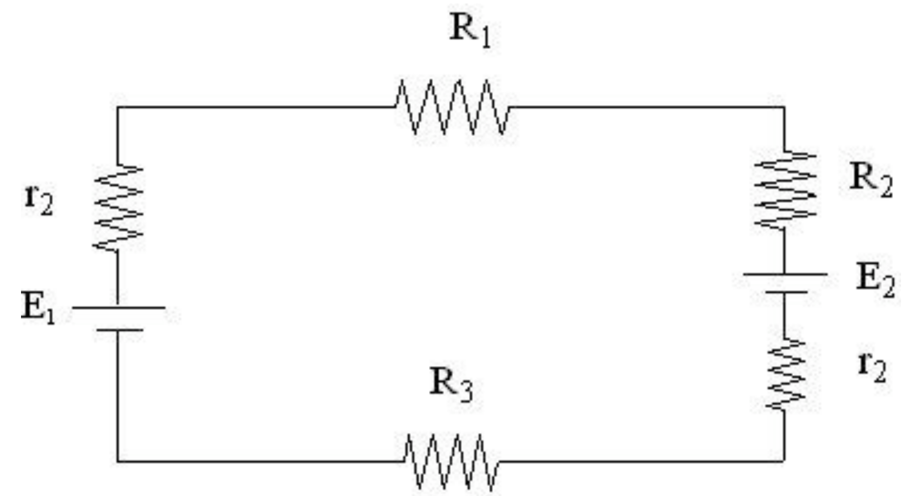


$i = i_1 + i_2$

Example (Single Loop Circuit)

No junction so we don't need that rule.

How do we apply Kirchoff's rule?



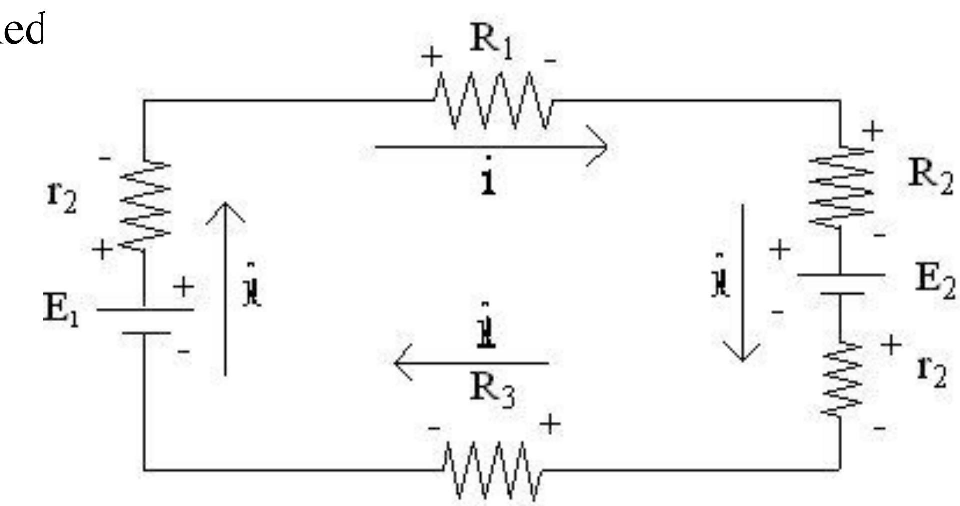
Must assume the direction of the current – assume clockwise.

Choose a starting point and apply Ohm's Law as you go around the circuit.

a. Potential across resistors is negative

b. Sign of E for a battery depends on assumed current flow

c. If you guessed wrong on the sign, your answer will be negative



Start in the upper left hand corner.

$$-iR_1 - iR_2 - E_2 - ir_2 - iR_3 + E_1 - ir_1 = 0$$

$$i = \frac{E_1 - E_2}{R_1 + R_2 + R_3 + r_1 + r_2}$$

$$i = \frac{E_1 - E_2}{R_1 + R_2 + R_3 + r_1 + r_2}$$

Now let us put in numbers.

Suppose: $R_1 = R_2 = R_3 = 10\Omega$

$r_1 = r_2 = 1\Omega$

$E_1 = 10V$

$E_2 = 5V$

$$i = \frac{10 - 5}{10 + 10 + 10 + 1 + 1} \frac{V}{\Omega} = \frac{5}{32} \text{ amp}$$

Suppose: $E_1 = 5V$

$E_2 = 10V$

$$i = \frac{(5 - 10)V}{32\Omega} = \frac{-5}{32} \text{ amp}$$

We get a minus sign. It means our assumed direction of current must be reversed.

Note that we could have simply added all resistors and get the $R_{eq.}$ and added the EMFs to get the $E_{eq.}$ And simply divided.

$$i = \frac{E_{eq.}}{R_{eq.}} = \frac{5(V)}{32(\Omega)} = \frac{5}{32} \text{ amp}$$

Sign of EMF

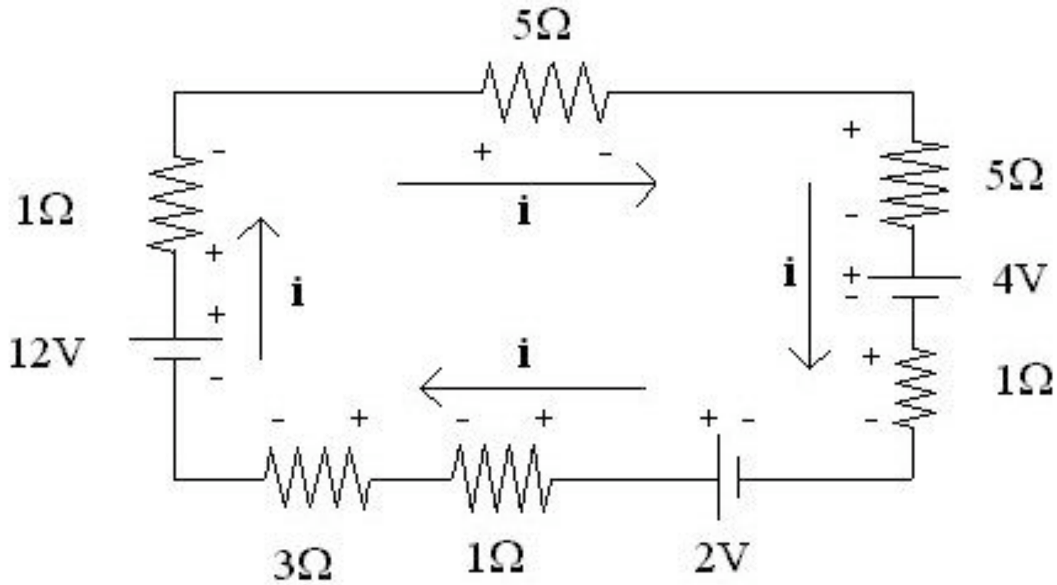
Battery **1** current flows from **- to +** in battery **$+E_1$**

Battery **2** current flows from **+ to -** in battery **$-E_2$**

In **1** the electrical potential energy **increases**

In **2** the electrical potential energy **decreases**

Example with numbers



Quick solution:

$$\sum_{i=1}^{3} E_i = 12V - 4V + 2V = 10V$$

$$\sum_{i=1}^{6} R_i = 16\Omega$$

$$I = \frac{E_{eq.}}{R_{eq.}} = \frac{10}{16} A$$

Question: What is the current in the circuit?

Write down Kirchoff's loop equation.

Loop equation

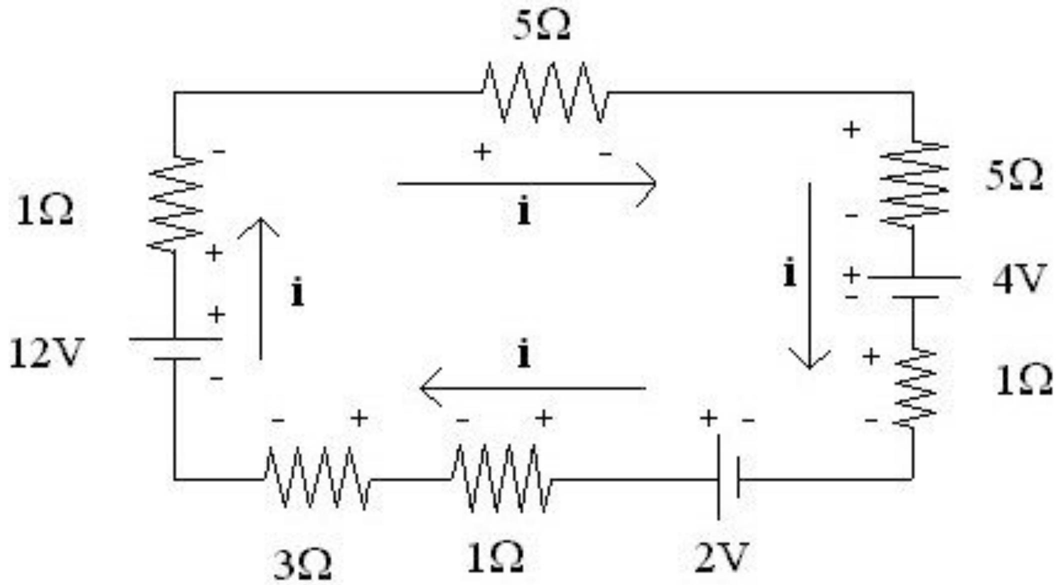
$$(+12 - 4 + 2)V - i(1 + 5 + 5 + 1 + 1 + 3)\Omega = 0$$

Assume current flow is clockwise.

$$i = \frac{10}{16}\frac{V}{\Omega} = 0.625 amps = 0.625A$$

Do the batteries first – Then the current.

Example with numbers (continued)



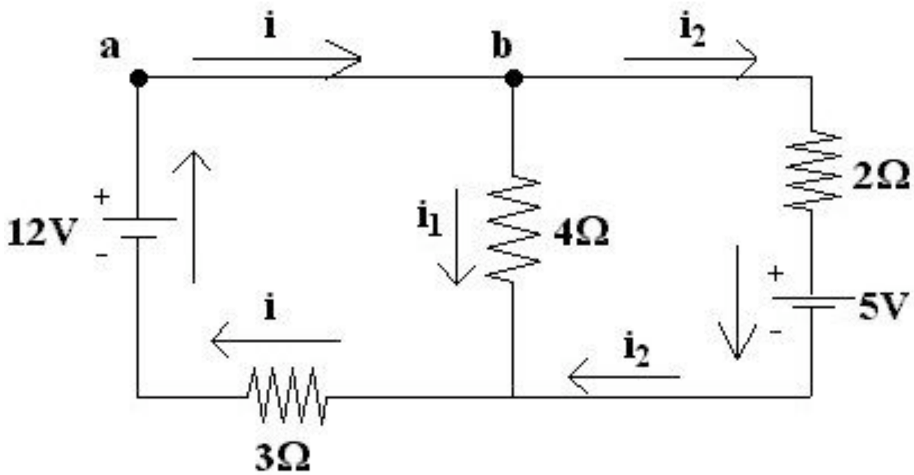
Question: What are the terminal voltages of each battery?

12V: $V = \mathcal{E} - ir = 12V - 0.625A \cdot 1\Omega = 11.375V$

2V: $V = \mathcal{E} - ir = 2V - 0.625A \cdot 1\Omega = 1.375V$

4V: $V = \mathcal{E} - ir = 4V + 0.625A \cdot 1\Omega = 4.625V$

Multiloop Circuits



Kirchoff's Rules

1. $\sum_i V_i = 0$ in any loop

2. $\sum i_{in} = \sum i_{out}$ at any junction

Rule 1 – Apply to 2 loops (2 inner loops)

a. $12 - 4i_1 - 3i = 0$

b. $-2i_2 - 5 + 4i_1 = 0$

Rule 2

a. $i = i_1 + i_2$

Find i, i_1, and i_2

We now have 3 equations with 3 unknowns.

$$12 - 4i_1 - 3(i_1 + i_2) = 0$$

$12 - 7i_1 - 3i_2 = 0$ multiply by 2

$-5 + 4i_1 - 2i_2 = 0$ multiply by 3

$$24 - 14i_1 - 6i_2 = 0$$
$$-15 + 12i_1 - 6i_2 = 0$$

subtract them

$$39 - 26i_1 = 0$$
$$i_1 = \frac{39}{26} = 1.5A$$
$$i_2 = 0.5A$$
$$i = 2.0A$$

Find the Joule heating in each resistor $P = i^2R$.

Is the 5V battery being charged?

Method of determinants for solving simultaneous equations

$$
\begin{aligned}
i - i_1 - i_2 &= 0 \\
-3i - 4i_1 + 0 &= -12 \\
0 + 4i_1 - 2i_2 &= 5
\end{aligned}
$$

Cramer's Rule says if :

$$
\begin{aligned}
a_1 i_1 + b_1 i_2 + c_1 i_3 &= d_1 \\
a_2 i_1 + b_2 i_2 + c_2 i_3 &= d_2 \\
a_3 i_1 + b_3 i_2 + c_3 i_3 &= d_3
\end{aligned}
$$

Then,

$$
i_1 = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad
i_2 = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \qquad
i_3 = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}
$$

Method of determinants using Cramers Rule and cofactors Also use this to remember how to evaluate cross products of two vectors.

For example solve for i

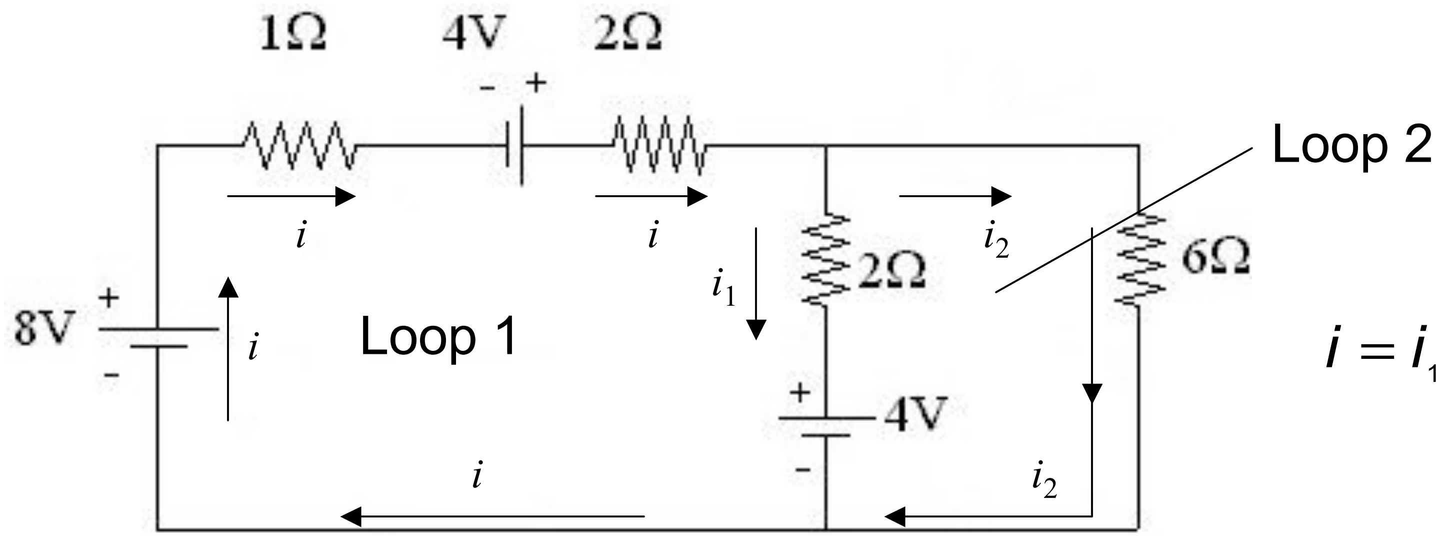
$$i = \frac{\begin{vmatrix} 0 & -1 & -1 \\ -12 & -4 & 0 \\ 5 & +4 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -1 \\ -3 & -4 & 0 \\ 0 & +4 & -2 \end{vmatrix}} = \frac{0\begin{pmatrix} -4 & 0 \\ 4 & -2 \end{pmatrix} - 1\begin{pmatrix} 0 & -12 \\ -2 & 5 \end{pmatrix} - 1\begin{pmatrix} -12 & -4 \\ 5 & 4 \end{pmatrix}}{1\begin{pmatrix} -4 & 0 \\ 4 & -2 \end{pmatrix} - 1\begin{pmatrix} 0 & -3 \\ -2 & 0 \end{pmatrix} - 1\begin{pmatrix} -3 & -4 \\ 0 & 4 \end{pmatrix}} = \frac{24 + 48 - 20}{8 + 6 + 12} = \frac{52}{26} = 2A$$

You try it for i_1 and i_2.

See inside of front cover in your book on how to use Cramer's Rule.

Another example

Find all the currents including directions.



$$i = i_1 + i_2$$

Loop 1

$$0 = +8V + 4V - 4V - 3i - 2i_1$$
$$0 = 8 - 3i_1 - 3i_2 - 2i_1$$
$$0 = 8 - 5i_1 - 3i_2$$

Loop 2

$$-6i_2 + 4 + 2i_1 = 0$$

Multiply eqn of loop 1 by 2 and subtract from the eqn of loop 2

$$-6i_2 + 4 + 2i_1 = 0$$
$$-6i_2 + 16 - 10i_1 = 0$$
$$0 - 12 + 12i_1 = 0$$

$$\boxed{i_1 = 1A}$$

$$-6i_2 + 4 + 2(1A) = 0$$

$$\boxed{i_2 = 1A}$$

$$\boxed{i = 2A}$$

Rules for solving multiloop circuits

1. Replace series resistors or batteries with their equivalent values.

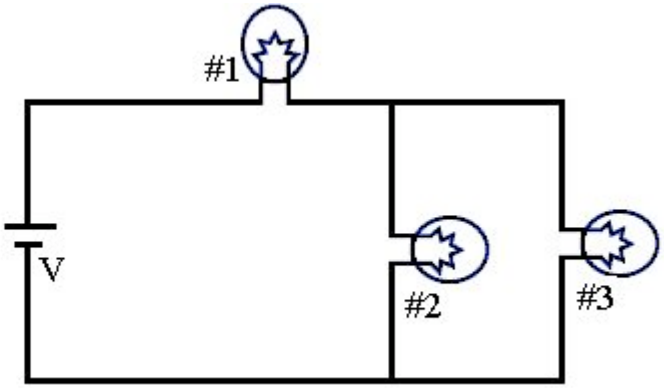
2. Choose a direction for *i* in each loop and label diagram.

3. Write the junction rule equation for each junction.

4. Apply the loop rule n times for n interior loops.

5. Solve the equations for the unknowns. Use Cramer’s Rule if necessary.

6. Check your results by evaluating potential differences.

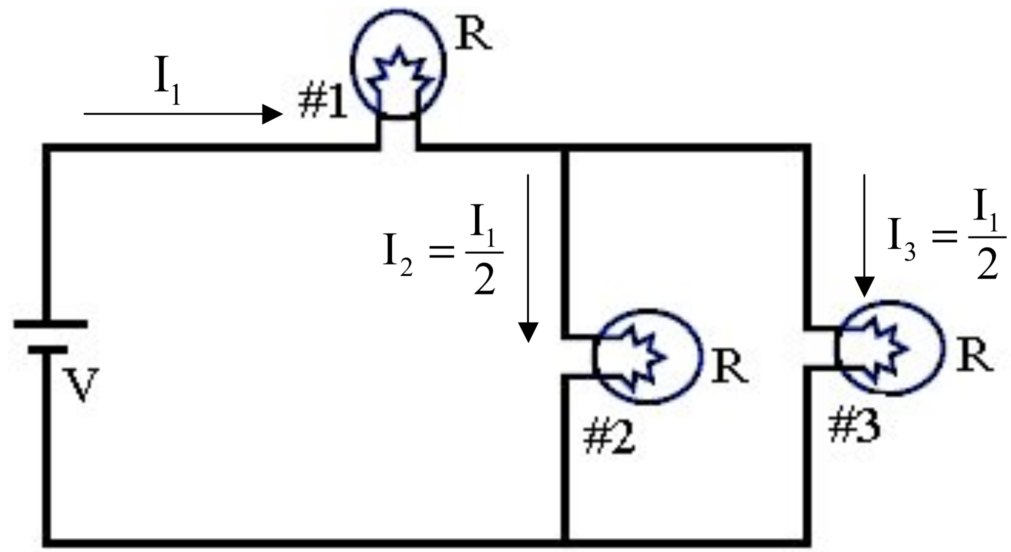


3 bulb question

The circuit above shows three identical light bulbs attached to an ideal battery. If the bulb#2 burns out, which of the following will occur?

a) Bulbs 1 and 3 are unaffected. The total light emitted by the circuit decreases.
b) Bulbs 1 and 3 get brighter. The total light emitted by the circuit is unchanged.
c) Bulbs 1 and 3 get dimmer. The total light emitted by the circuit decreases.
d) Bulb 1 gets dimmer, but bulb 3 gets brighter. The total light emitted by the circuit is unchanged.
e) Bulb 1 gets brighter, but bulb 3 gets dimmer. The total light emitted by the circuit is unchanged.
f) Bulb 1 gets dimmer, but bulb 3 gets brighter. The total light emitted by the circuit decreases.
g) Bulb 1 gets brighter, but bulb 3 gets dimmer. The total light emitted by the circuit decreases.
h) Bulb 1 is unaffected, but bulb 3 gets brighter. The total light emitted by the circuit increases.
i) None of the above.

When the bulb #2 is not burnt out:



$$R_{eq} = R + \frac{R}{2} = \frac{3}{2}R$$

$$\text{Power}, P = I^2 R \qquad I = \frac{V}{R}$$

For Bulb #1

$$I_1 = \frac{V}{\frac{3}{2}R} = \frac{2V}{3R} \qquad P_1 = I_1^2 R = \frac{4V^2}{9R} = .44\frac{V^2}{R}$$

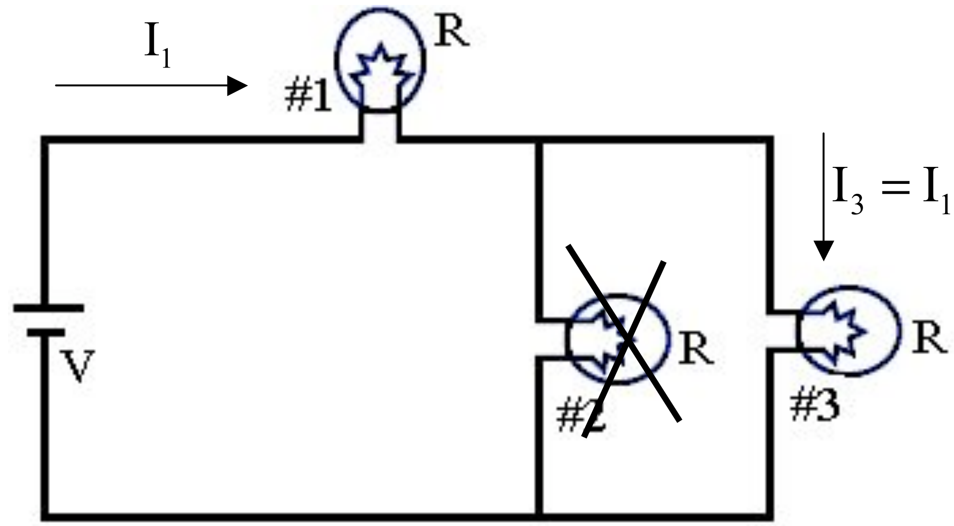
For Bulb #2

$$I_2 = \frac{I_1}{2} = \frac{V}{3R} \qquad P_2 = I_2^2 R = \frac{V^2}{9R} = .11\frac{V^2}{R}$$

For Bulb #3

$$I_3 = \frac{I_1}{2} = \frac{V}{3R} \qquad P_3 = I_3^2 R = \frac{V^2}{9R} = .11\frac{V^2}{R}$$

When the bulb #2 is burnt out:



$$R_{eq} = R + R = 2R$$

Power, $P = I^2R$ $\quad I = \frac{V}{R}$

For Bulb #1

$$I_1 = \frac{V}{2R} \quad P_1 = I_1^2 R = \frac{V^2}{4R} = .25\frac{V^2}{R}$$

For Bulb #2

$$I_2 = 0 \quad P_2 = I_2^2 R = 0$$

For Bulb #3

$$I_3 = I_1 = \frac{V}{2R} \quad P_3 = I_3^2 R = \frac{V^2}{4R} = .25\frac{V^2}{R}$$

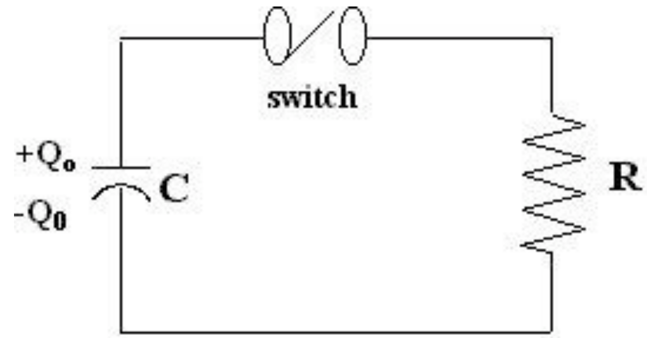
Before total power was $P_b = \frac{V^2}{R_{eq}} = \frac{V^2}{\frac{3}{2}R} = .66\frac{V^2}{R}$

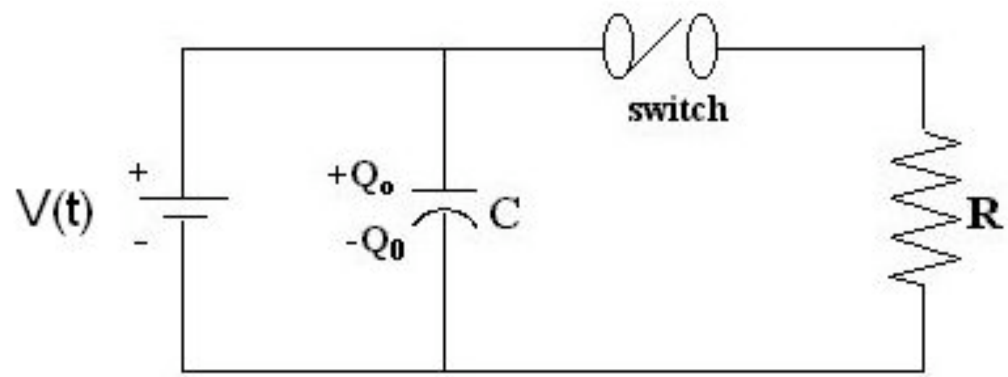
After total power is $P_a = \frac{V^2}{R_{eq}} = \frac{V^2}{2R} = .50\frac{V^2}{R}$

So, Bulb #1 gets dimmer and bulb #3 gets brighter. And the total power decreases.

f) is the answer.

How does a capacitor behave in a circuit with a resistor?





Charge capacitor with 9V battery with switch open, then remove battery.

Now close the switch. What happens?

Discharging a capacitor through a resistor

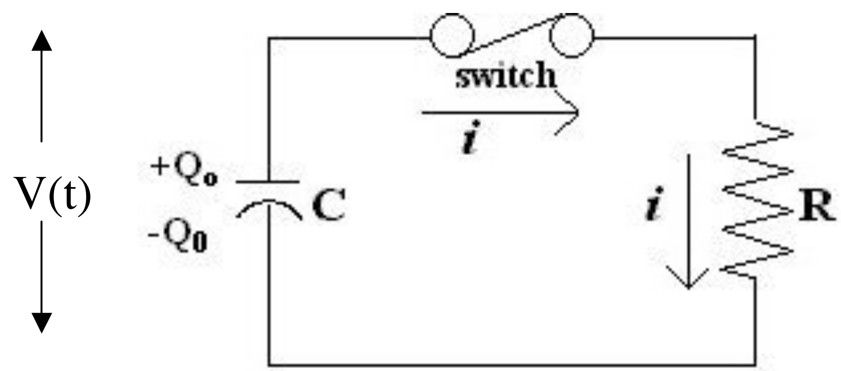
Potential across capacitor = V = $\frac{Q_o}{C}$

just before you throw switch at time t = 0.

Potential across Resistor = iR

$$\frac{Q_o}{C} = i_o R \Rightarrow i_o = \frac{Q_o}{RC}$$

at t > 0.



What is the current I at time t?

$$i(t) = \frac{Q(t)}{RC}$$

or $i = \frac{Q}{RC}$

What is the current I at time t?

So, $i = \frac{Q}{RC}$, but $i = -\frac{dQ}{dt}$

$$-\frac{dQ}{dt} = \frac{Q}{RC}$$

$$-\frac{dQ}{Q} = \frac{dt}{RC}$$

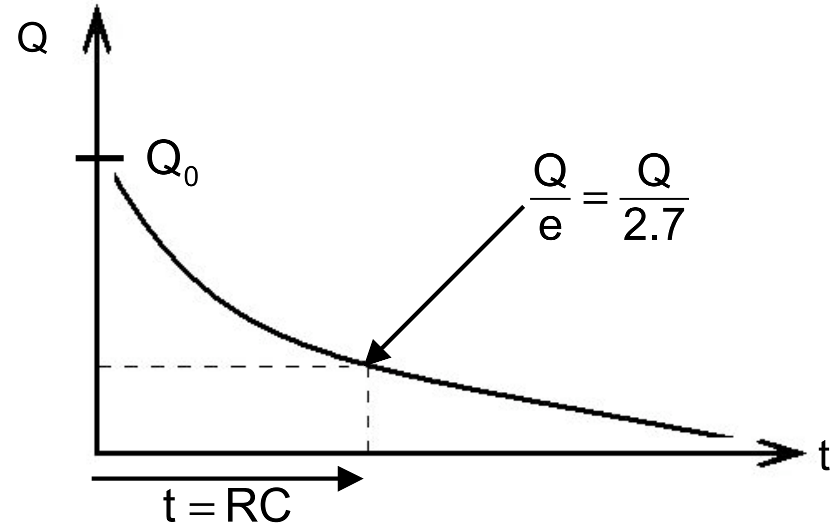
Time constant =RC

Integrating both the sides

$$-\int \frac{dQ}{Q} = \int \frac{dt}{RC}$$

$$-\ln Q = \frac{t}{RC} + A$$

$$\ln Q = -\frac{t}{RC} - A$$



So, $Q = e^{-\frac{t}{RC} - A} = e^{-\frac{t}{RC}} e^{-A}$

At t=0, $Q=Q_0$

So, $Q_0 = e^{-\frac{0}{RC} - A} = e^{-A}$

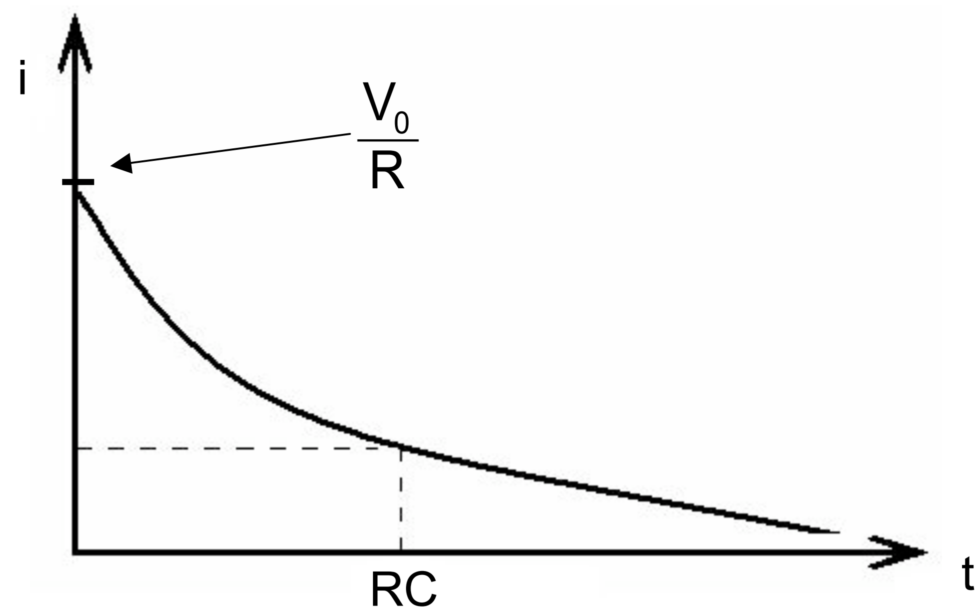
$$\Rightarrow \boxed{Q = Q_0 e^{-\frac{t}{RC}}}$$

What is the current?

$$Q = Q_0 e^{-\frac{t}{RC}}$$

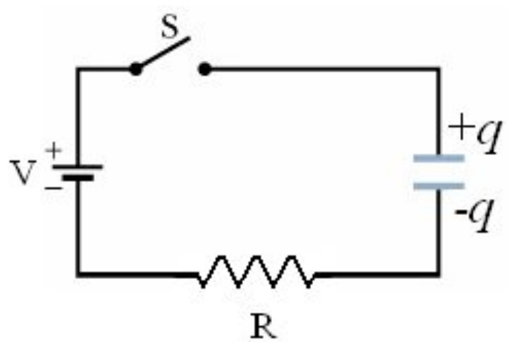
$$i = \frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = -\frac{V_0}{R} e^{-\frac{t}{RC}}$$

Ignore - sign



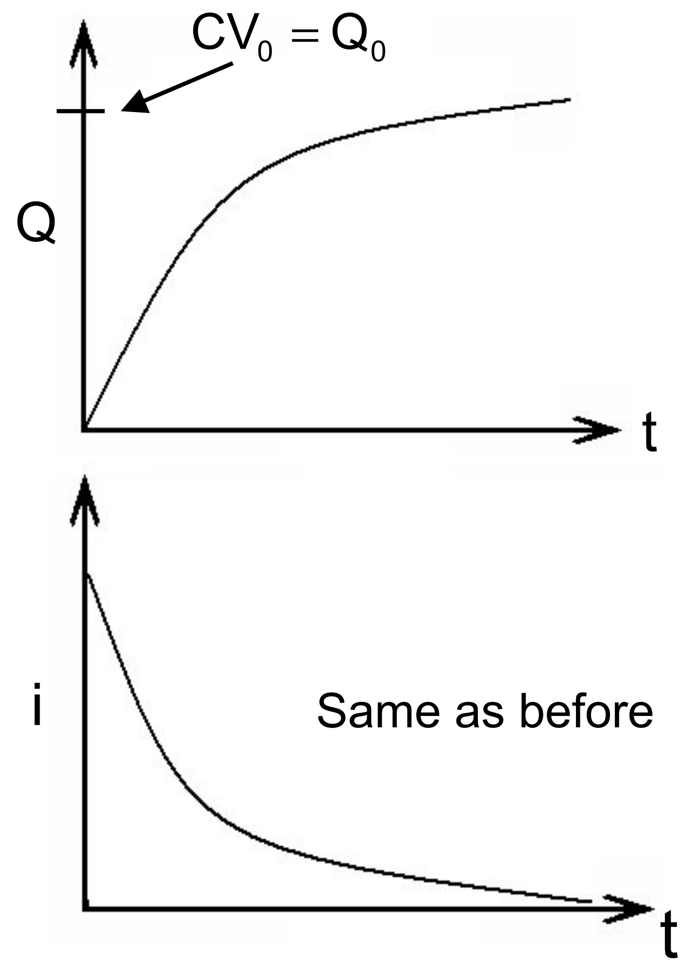
How the charge on a capacitor varies with time as it is being charged

What about charging the capacitor?



$$Q = CV_0(1 - e^{-\frac{t}{RC}})$$

$$i = \frac{V_0}{R} e^{-\frac{t}{\gamma}}$$



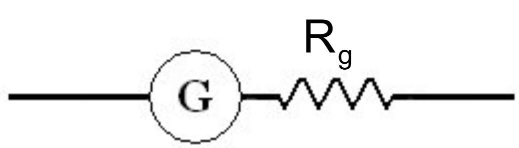
Note that the current is zero when either the capacitor is fully charged or uncharged. But the second you start to charge it or discharge it, the current is maximum.

Instruments

Galvanometers:	a coil in a magnetic field that senses current.
Ammeters:	measures current.
Voltmeter:	measures voltage.
Ohmmeters:	measures resistance.
Multimeters:	one device that does all the above.

Galvanometer is a needle mounted to a coil that rotates in a magnetic field. The amount of rotation is proportional to the current that flows through the coil.

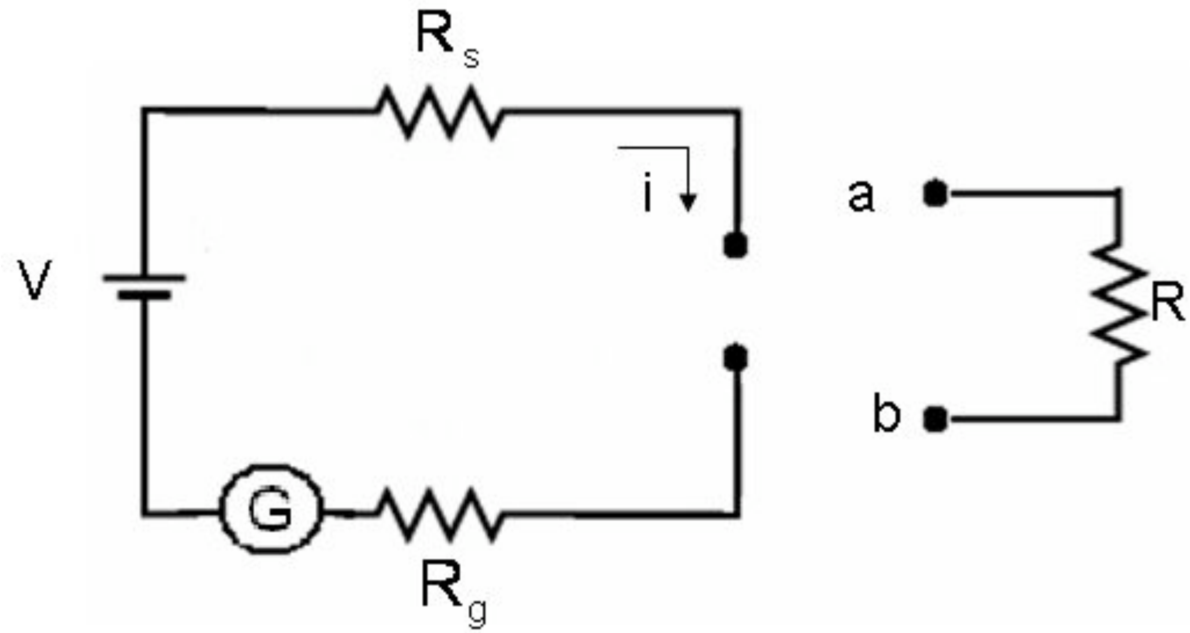
Symbolically we write



Usually when $R_g = 20\Omega$

$I_g = 0 \rightarrow 0.5\text{milliAmp}$

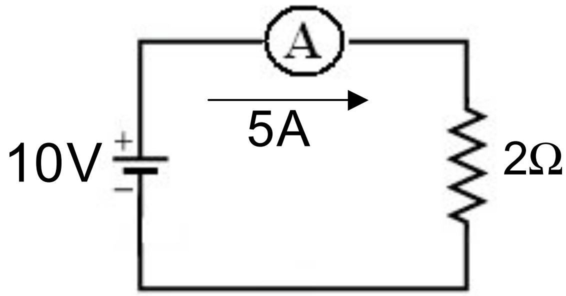
Ohmmeter

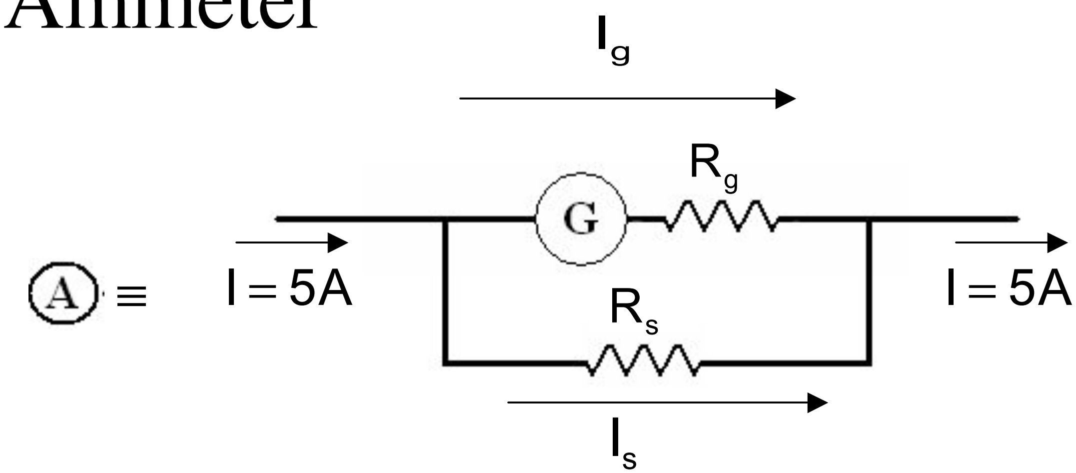


$$i = \frac{V}{R + R_s + R_g}$$

Adjust R_s so when R=0 the galvanometer read full scale.

Ammeter





$$I = I_g + I_s = 5A$$

$$I_g R_g = I_s R_s$$

The idea is to find the value of R_S that will give a full scale reading in the galvanometer for 5A

$$R_g = 20\Omega \quad \text{and} \quad I_g = 0.5\times10^{-3}\,A, \quad \text{So,} \quad I_s = 5A - .0005A \approx 5A$$

$$\text{So,} \quad R_s = \frac{I_g}{I_s} R_g = \frac{0.5\times10^{-3}\,A}{5A}(20\Omega) = 0.002\Omega$$

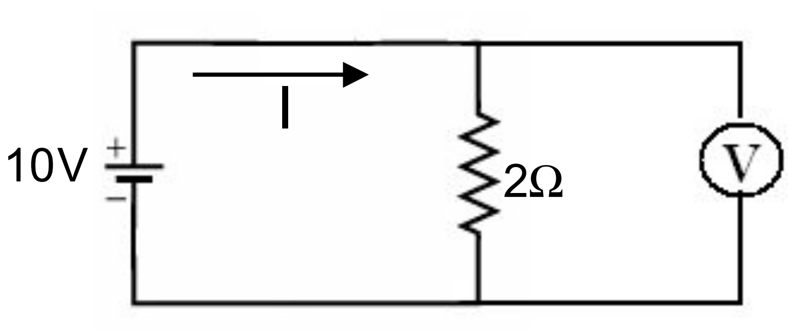
Very small

Ammeters have very low resistance when put in series in a circuit.

You need a very stable shunt resistor.

Voltmeter

Use the same galvanometer to construct a voltmeter for which full scale reading in 10 Volts.



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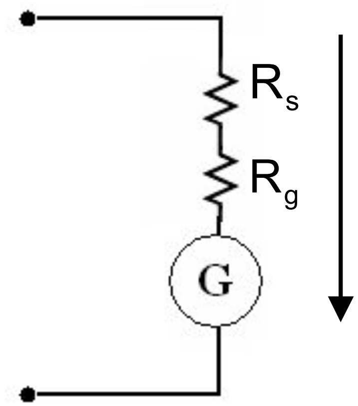

$$I_g = 0.5 \times 10^{-3}\,A$$
$$R_g = 20\Omega$$

What is the value of R_S now?

We need
$$10V = I_g(R_s + R_g)$$
$$R_s + R_g = \frac{10V}{I_g} = \frac{10V}{5 \times 10^{-4}\,A}$$
$$R_s + R_g = 20{,}000\Omega$$
$$R_s = 19{,}980\Omega$$

So, the shunt resistor needs to be about 20KΩ.

Note: the voltmeter is in parallel with the battery.