Lecture 8 Magnetic Fields Chp. 29

- Cartoon Magnesia, Bar Magnet with N/S Poles, Right Hand Rule
- Topics
  - Magnetism is likable, Compass and diclinometer, Permanent magnets
  - Magnetic field lines, Force on a moving charge, Right hand rule,
  - Non-uniform magnetic field
  - Force on a current carrying wire, Torque on a current loop
- Demos
  - Globe
  - Natural magnetic rock
  - Compass and diclinometer
  - Iron fillings and bar magnets
  - Compass needle array
  - Pair of gray magnets
  - CRT illustrating electron beam bent bent by a bar magnet
  - Gimbal mounted bar magnet
  - Wire jumping out of a horsehoe magnet.
  - Coil in a magnet

Magnetic Fields

- Magnetism has been around as long as there has been an Earth with an iron magnetic core.
- Thousands of years ago the Chinese built compasses for navigation in the shape of a spoon with rounded bottoms on which they balanced (Rather curious shape for people who eat with chopsticks).
- Certain natural rocks are ferromagnetic – having been magnetized by cooling of the Earth's core.
- **Show a sample of natural magnetic rock. Put it next to many compasses.**

Magnetism's Sociabilities

- Magnetism has always has something of a mystic aura about it. It is usually spoken of in a favorable light.
- Animal magnetism, magnetic personality, and now you can wear magnetic collars, bracelets, magnetic beds all designed to make you healthier – even grow hair.
- We do not have the same feeling about electricity. If you live near electric power lines, the first thing you want to do is to sue the electric company.

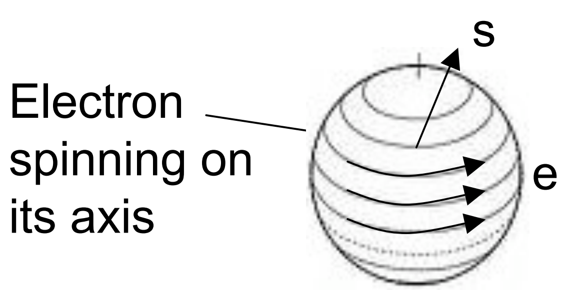
Compass and Declinometer

- In 1600 William Gilbert used a compass needle to show how it oriented itself in the direction of the north geographic pole of the Earth, which happens to be the south magnetic pole of the Earth's permanent magnetic field.
- **Show compass and declinometer.** Each has a slightly magnetized needle that is free to rotate. The compass lines up with the component of the magnetic field line parallel to the surface of the Earth. The declinometer lines up with the actual magnetic field line itself. It says that the angle between the field lines and the surface is 71 degrees as measured from the south.
- **Show model of Earth field lines assuming a uniformly magnetized sphere**
- Basically there are two types of magnets: permanent magnets and electromagnets
- **Show field lines for a bar magnet. Show bar magnet surrounded by compass needle array.**

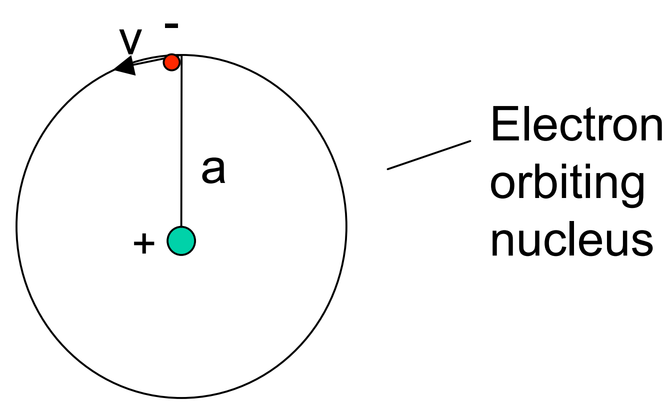
Permanent Magnets

- Bar magnet is a model of a ferromagnetic material that can be permanently magnetized. Other ferromagnetic materials are cobalt and nickel.
- The origin of magnetism in materials is due mostly to the spinning motion of the charged electron on its own axis. There is a small contribution from the orbital motion of the electron.

Atomic origin of magnetic field



Magnetic dipole



Magnetic dipole

Permanent Magnets (continued)

- In ferromagnetic materials there are whole sections of the iron called domains where the magnetism does add up from individual electrons. Then there are other sections or domains where contributions from different domains can cancel. However, by putting the iron in a weak magnetic field you can align the domains more or less permanently and produce a permanent bar magnet as you see here.

- In nonmagnetic materials the contributions from all

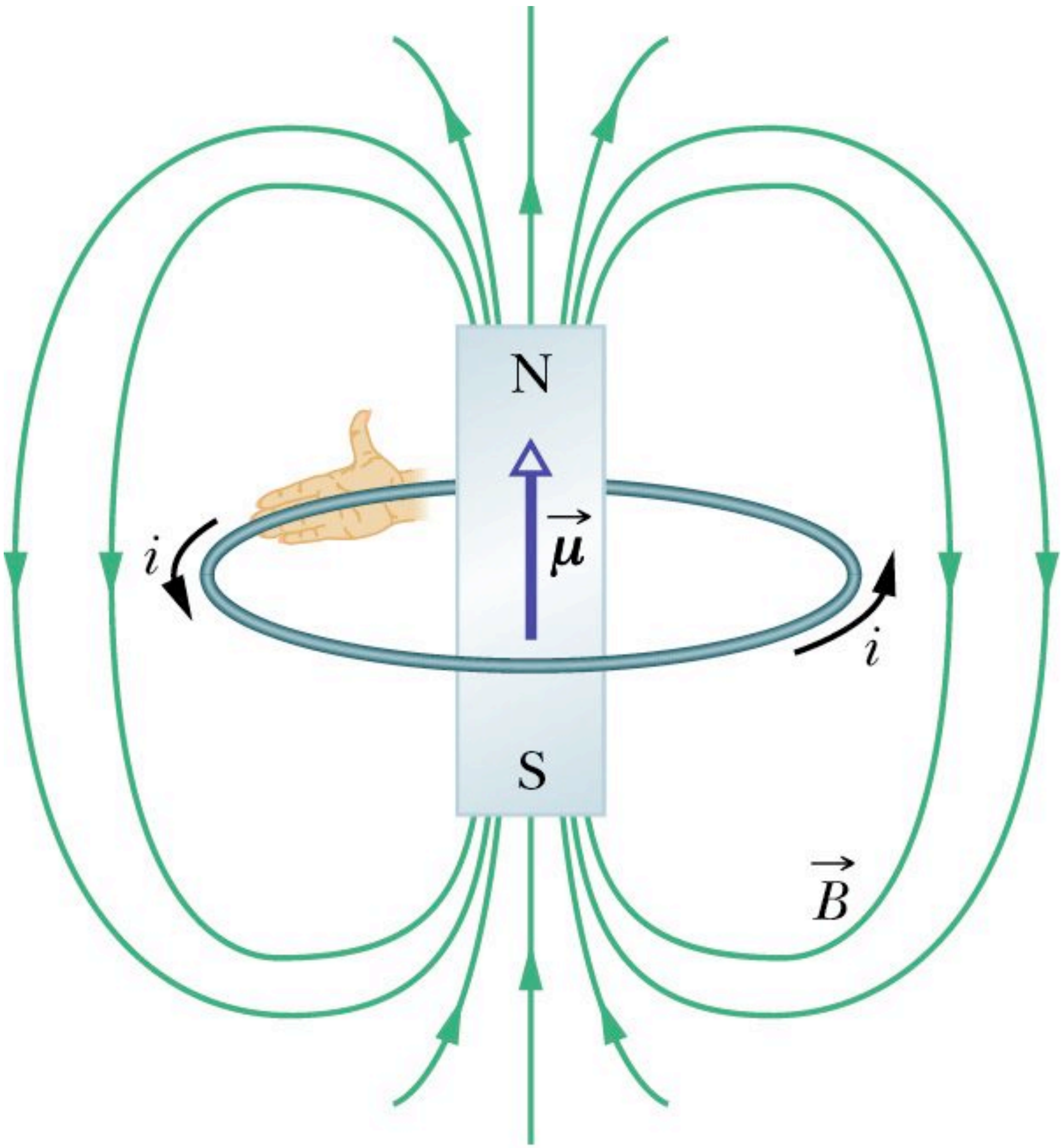
  The electrons cancel out. Domains are not even formed.

Magnetic field lines do not stop at surface.

They are continuous.

They make complete loops.

Field lines for a bar magnet are the same as for a current loop



Magnetic field lines

Similarities to electric lines

- A line drawn tangent to a field line is the direction of the field at that point.
- The density of field lines still represent the strength of the field.

Differences

- The magnetic field lines do not terminate on anything. They form complete loops. There is no magnetic charge on as there was electric charge in the electric case. This means if you cut a bar magnet in half you get two smaller bar magnets ad infinitum all the way down to the atomic level – Magnetic atoms have an atomic dipole – not a monopole as is the case for electric charge.
- They are not necessarily perpendicular to the surface of the ferromagnetic material.

$$\phi_B = \text{Magnetic flux} = \int \vec{B} \cdot d\vec{A}$$

$$\phi_E = \text{Electric flux} = \int \vec{E} \cdot d\vec{A}$$

Definition of magnetic Field

- $B = \frac{F}{qv}$ definition of a magnetic field
- The units of B are $\frac{N}{(\frac{C.m}{s})}$ or $\frac{N}{(A.m)}$ in SI units(MKS).

  This is called a Tesla (T). One Tesla is a very strong field.

- A commonly used smaller unit is the Gauss. 1 T = 10^4 G
  (Have to convert Gauss to Tesla in formulas in MKS)
- In general the force depends on angle $\vec{F} = q\vec{v} \times \vec{B}$. This is called the Lorentz Force

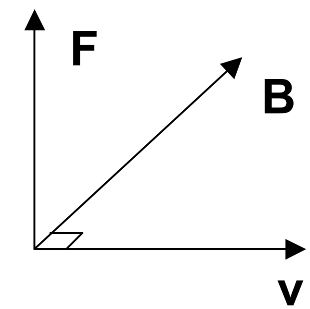
In analogy with the electric force on a point charge, the corresponding equation for a force on a moving point charge in a magnetic field is:

$$\vec{F}_m = q\vec{v} \times \vec{B} \qquad\qquad \vec{F}_e = q\vec{E}$$

Magnitude of $F_m = qvB\sin\theta$

– Direction of F is given by the right hand rule (see next slide).

- Consider a uniform B field for simplicity.



If the angle between v and B is $\theta = 0$, then the force = 0.

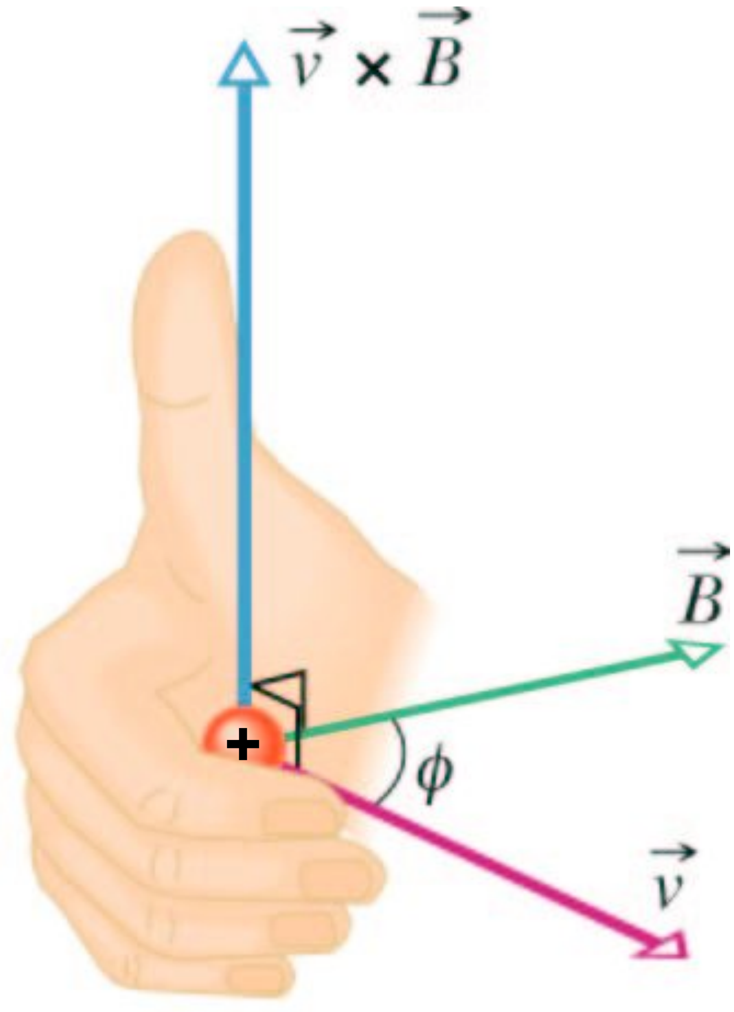
v ⟶ ⟶ B $\quad \vec{v} \parallel \vec{B} \quad \sin(0^o) = 0 \quad F = 0$

- If $\theta = 90$, then he force = qvB and the particle moves in a circle.

Use right hand rule to find the direction of **F**

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

Positive Charge



Rotate **v** into **B** through the smaller angle ϕ and the force **F** will be in the direction a right handed screw will move.

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

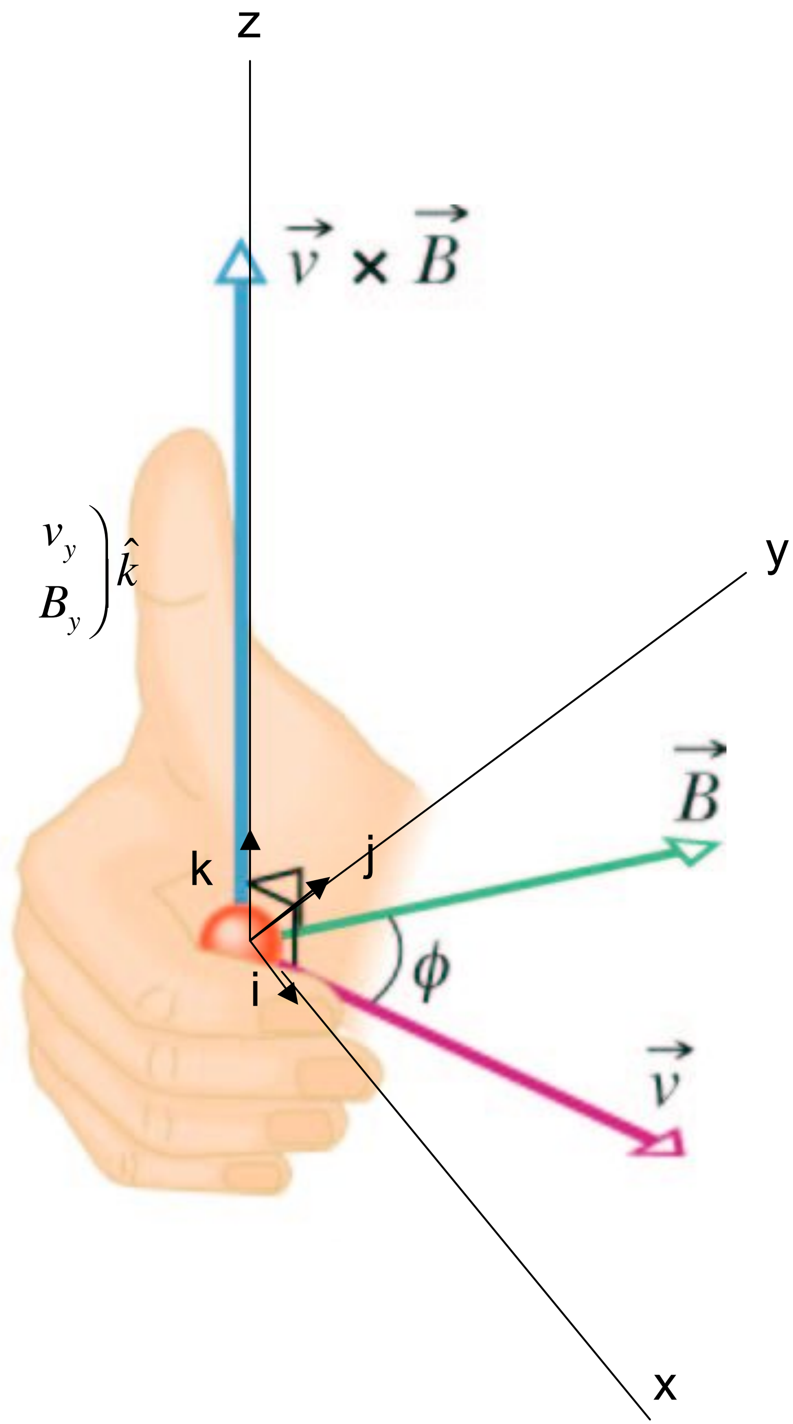
$$\hat{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

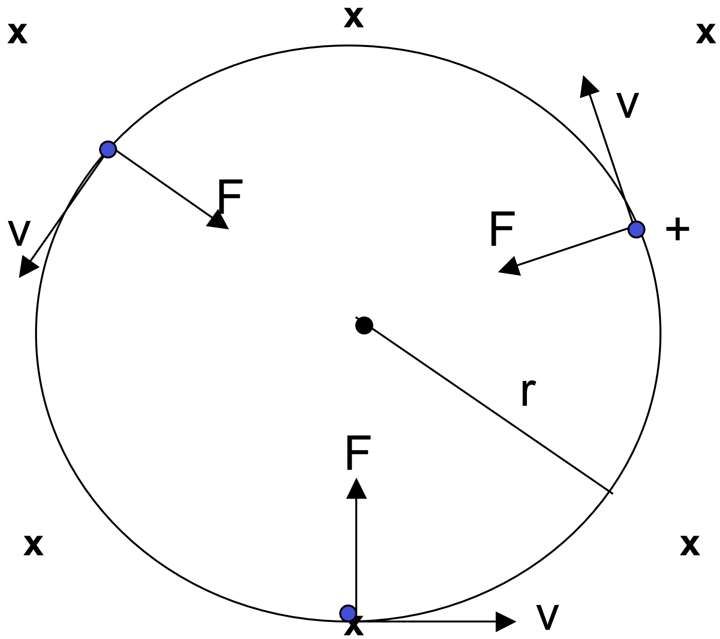
$$\vec{F} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ B_x & B_y & 0 \end{pmatrix} = \begin{pmatrix} v_y & 0 \\ B_y & 0 \end{pmatrix}\hat{i} + \begin{pmatrix} 0 & v_x \\ 0 & B_x \end{pmatrix}\hat{j} + \begin{pmatrix} v_x & v_y \\ B_x & B_y \end{pmatrix}\hat{k}$$

$$\vec{F} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 0 \\ B_x & B_y & 0 \end{pmatrix} = (v_x B_y - B_x v_y)\hat{k}$$

Note $\vec{F} \perp xy\ plane$



Motion of a point positive charge "•" in a magnetic field.



B is directed into the paper

$$\vec{F} \perp \vec{v} \perp \vec{B}$$

$$\vec{F}_m = q\vec{v} \times \vec{B} = qvB\sin 90^o$$

Magnitude of F = qvB

Direction of the RHR (right hand rule)

For a "+" charge, the particle rotates counter clockwise.

For a "-" charge, the particle rotates counter clockwise.

•Since **F** $\perp$ **v**, the magnetic force does no work on the particle.

W = **F** • **d** = 0 ; F $\perp$ d

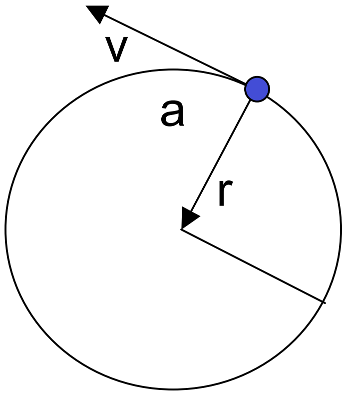
•This means kinetic energy remains constant.

•The magnitude of velocity doesn't change.

•Then the particle will move in a circle forever.

•The B field provides the centripetal force needed for circular motion.

Apply Newton's 2nd Law to circular motion



$$F = ma = \frac{mv^2}{r} = qvB \qquad \boxed{r = \frac{mv}{qB}}$$

Radius of the orbit

Important formula in Physics

$$a = \frac{v^2}{r}$$

$$v = qBr / m$$

What is the period of revolution of the motion?

$$T = \frac{2\pi r}{v} = \boxed{\frac{2\pi m}{qB} = period = T}$$

Note the period is independent of the radius, amplitude, and velocity. Example of simple harmonic motion in 2D.

T is also the cyclotron period.

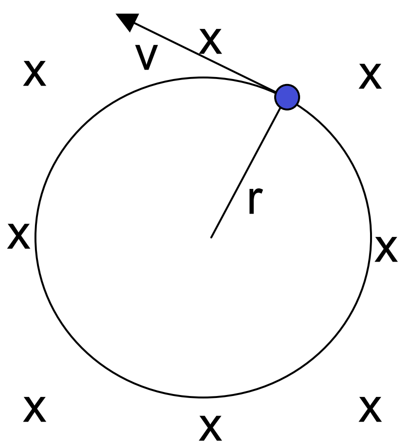
$$f = 1/t$$

$$\boxed{f = \frac{qB}{2\pi m}}$$

Cyclotron frequency

It is important in the design of the cyclotron accelerator. Of course, this is important because today it is used to make medical isotopes for radiation therapy.

Example: If a proton moves in a circle of radius 21 cm perpendicular to a B field of 0.4 T, what is the speed of the proton and the frequency of motion?



① $$v = \frac{qBr}{m}$$

$$v = \frac{1.6\times10^{-19}C\ (0.4T)\ 0.21m}{1.67\times10^{-27}kg}$$

$$v = \frac{1.6\ (0.4)\ 0.21}{1.67}\times10^{8}\ \tfrac{m}{s} = 8.1\times10^{6}\ \tfrac{m}{s}$$

$$\boxed{v = 8.1\times10^{6}\ \tfrac{m}{s}}$$

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② $$f = \frac{qB}{2\pi m}$$

$$f = \frac{1.6\times10^{-19}C\ (0.4T)}{(2\pi)\ 1.67\times10^{-27}kg}$$

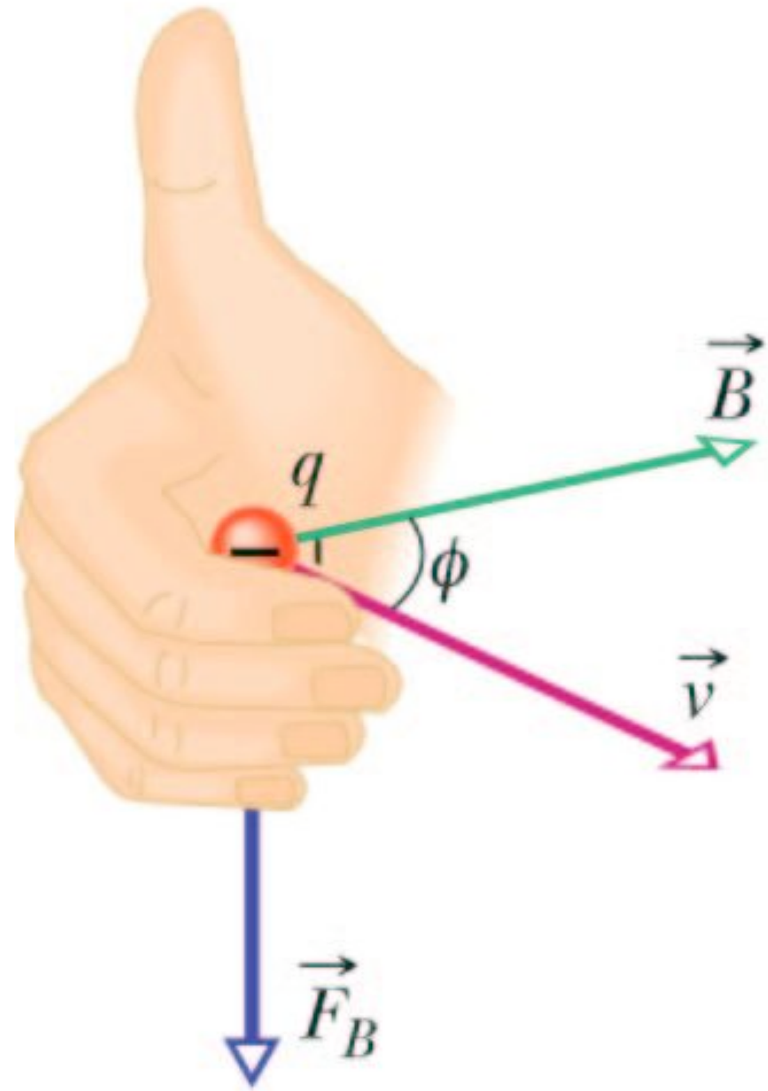
$$f = \frac{1.6\ (0.4)}{(6.28)\ 1.67}\times10^{8}\,Hz = 6.1\times10^{6}\,Hz$$

$$\boxed{f = 6.1\times10^{6}\,Hz}$$

Use right hand rule to find the direction of **F**

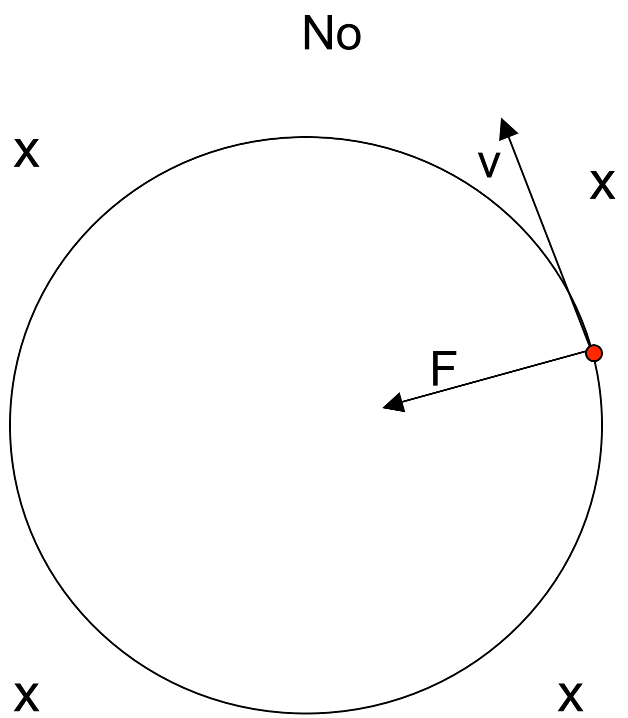
$$\vec{F}_m = q\vec{v} \times \vec{B}$$

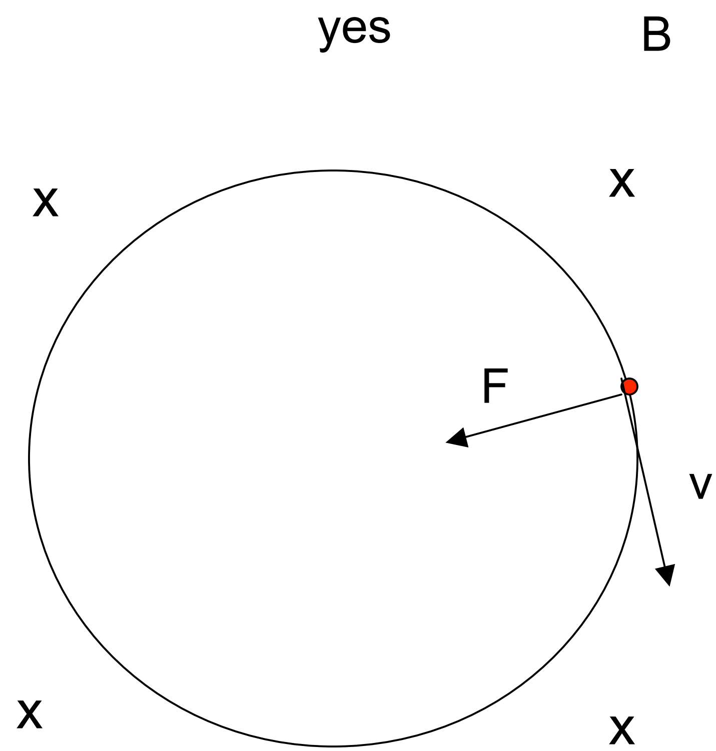
Negative Charge



Rotate **v** into **B** through the smaller angle ϕ and the force **F** will be in the opposite Direction a right handed screw will move.

Suppose we have an electron . Which picture is correct?





Example of the force on a fast moving proton due to the earth's magnetic field. (Already we know we can neglect gravity, but can we neglect magnetism?) Magnetic field of earth is about 0.5 gauss. Convert to Tesla. 1 gauss=10^{-4} Tesla

Let v = 10^7 m/s moving North.

What is the direction and magnitude of F?

Take B = $0.5\text{x}10^{-4}$ T and v$\perp$ B to get maximum effect.

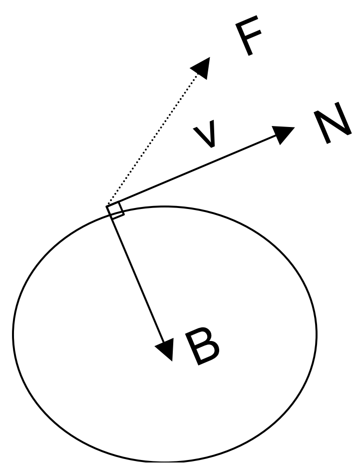
$$\left|\vec{F}_m\right| = qvB = 1.6\times10^{-19}\,C\cdot10^7\,\tfrac{m}{s}\cdot0.5\times10^{-4}\,T$$

$$\boxed{\left|\vec{F}_m\right| = 8\times10^{-17}\,N}$$

(a very fast-moving proton)

$$\left|\vec{F}_e\right| = qE = 1.6\times10^{-19}\,C\cdot100\,\tfrac{\text{volts}}{\text{meter}}$$

$$\boxed{\left|\vec{F}_e\right| = 1.6\times10^{-17}\,N}$$

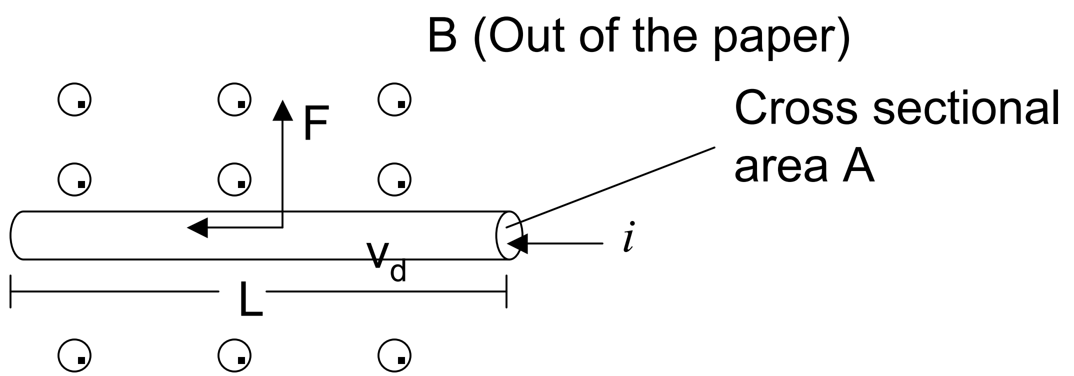


V x **B** is into the paper (west). Check with globe

Earth

Force on a current-carrying wire

v_d is the drift velocity of the electrons.



When a wire carries current in a magnetic field, there is a force on the wire that is the sum of the forces moving charges that carry the current.

n = density of mobile charges

Number of charges = nAL

$$\vec{F} = (q\vec{v} \times \vec{B})(nAL) \qquad \mathbf{v} \perp \mathbf{B}$$

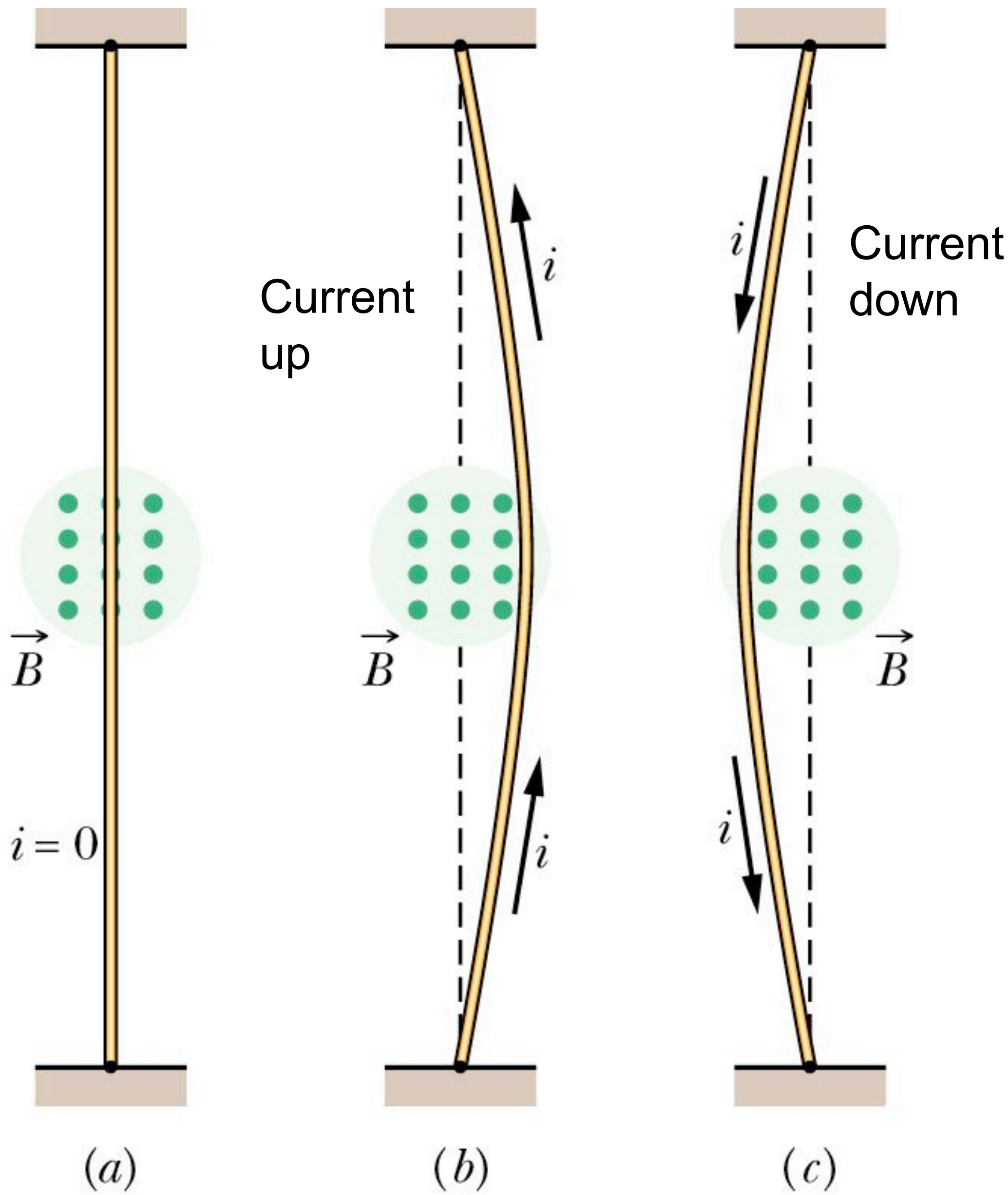
$$|F| = nqvALB \qquad Current, i = nqvA$$

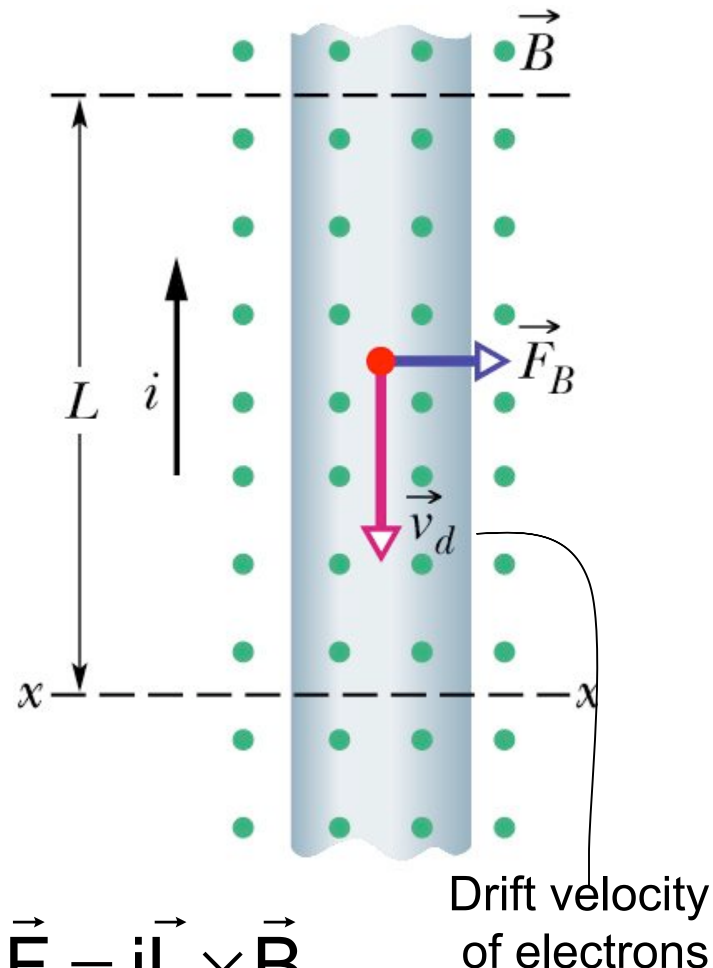
$$\mathrm{F} = \mathrm{iLB} \quad \text{or} \quad \vec{\mathrm{F}} = \mathrm{i}\vec{\mathrm{L}} \times \vec{\mathrm{B}}$$

Also $d\vec{F} = i\,d\vec{L} \times \vec{B}$

L is a vector in the direction of the **current i** with magnitude equal to the length of the wire.

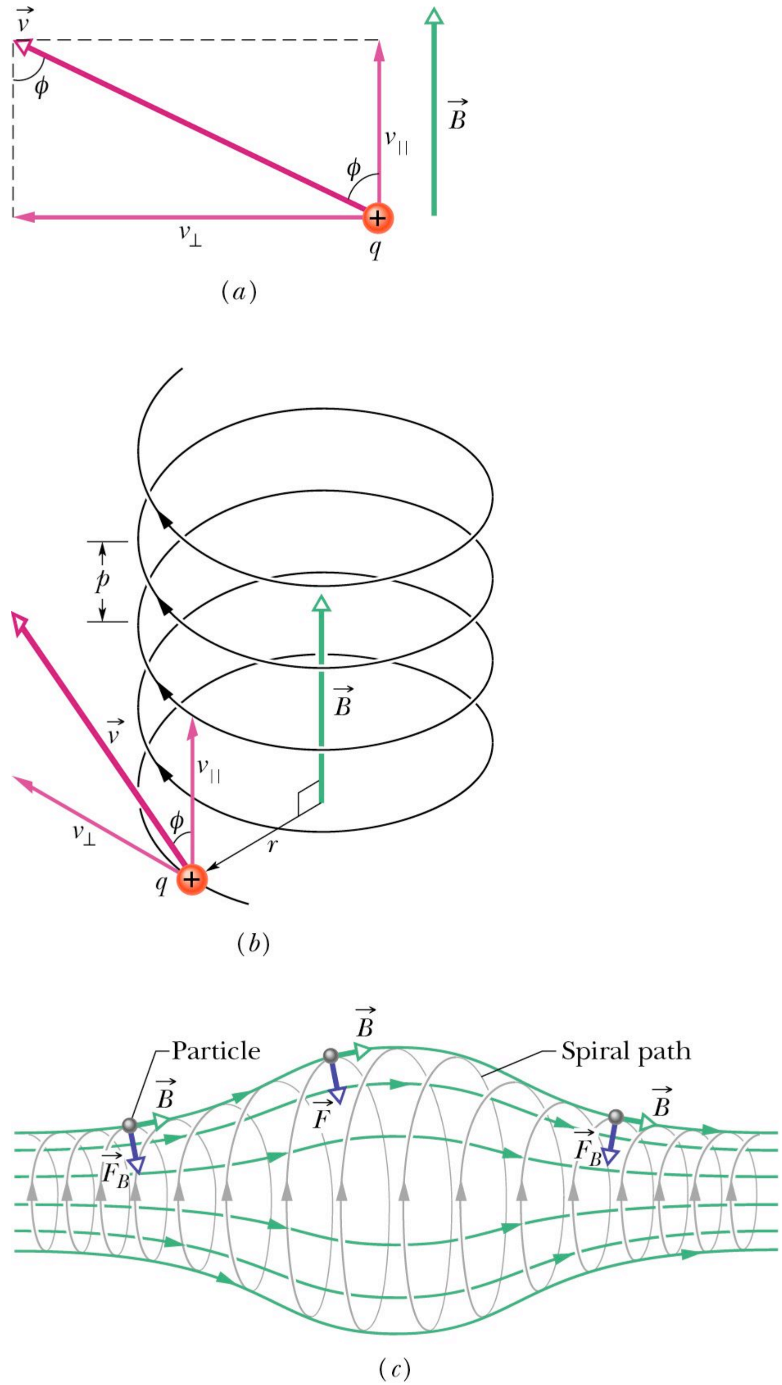
Show force on a wire in a magnetic field





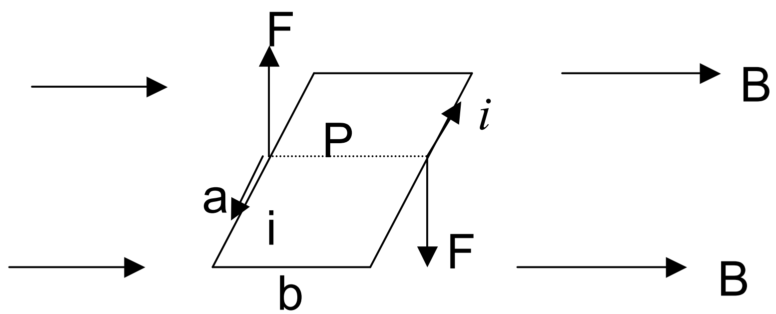
$$\vec{F} = i\vec{L} \times \vec{B}$$

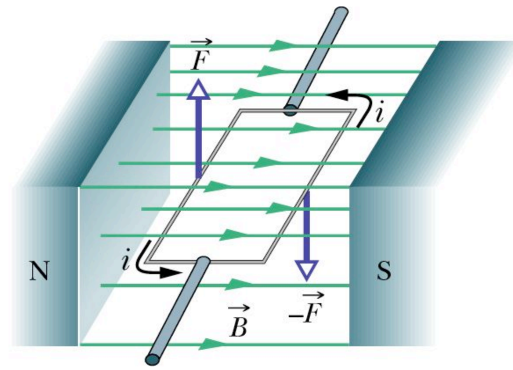
Magnetic bottle. The charge is trapped inside and spirals back and forth



Torques on current loops

Electric motors operate by connecting a coil in a magnetic field to a current supply, which produces a torque on the coil causing it to rotate.





Above is a rectangular loop of wire of sides *a* and *b* carrying current i.

B is in the plane of the loop and ⊥ to *a*.

Equal and opposite forces $F = iaB$ are exerted on the sides *a*.

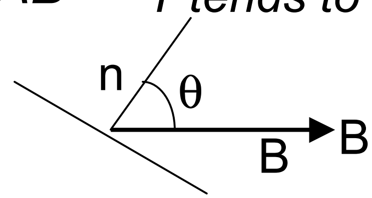
No forces exerted on *b* since $i \| B$

Since net force is zero, we can evaluate T (torque) at any point. Evaluate it at P.

$$T = Fb = iaBb = iAB$$

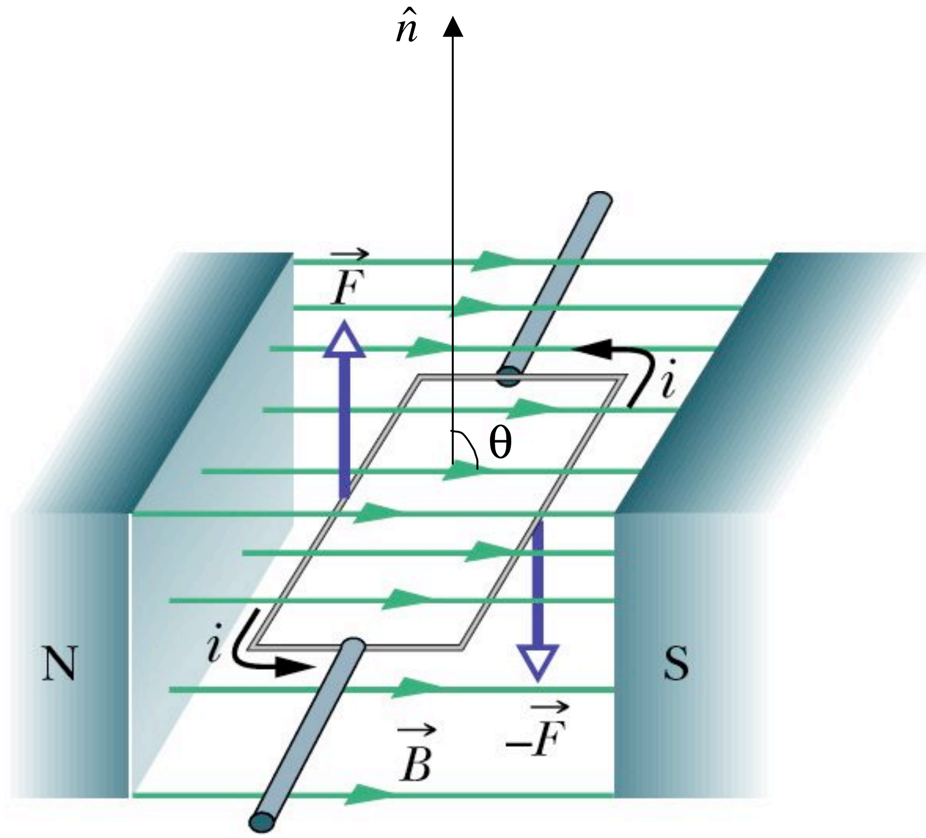
T tends to rotate loop until plane is ⊥ to B.

$$T = NiAB\sin\theta$$

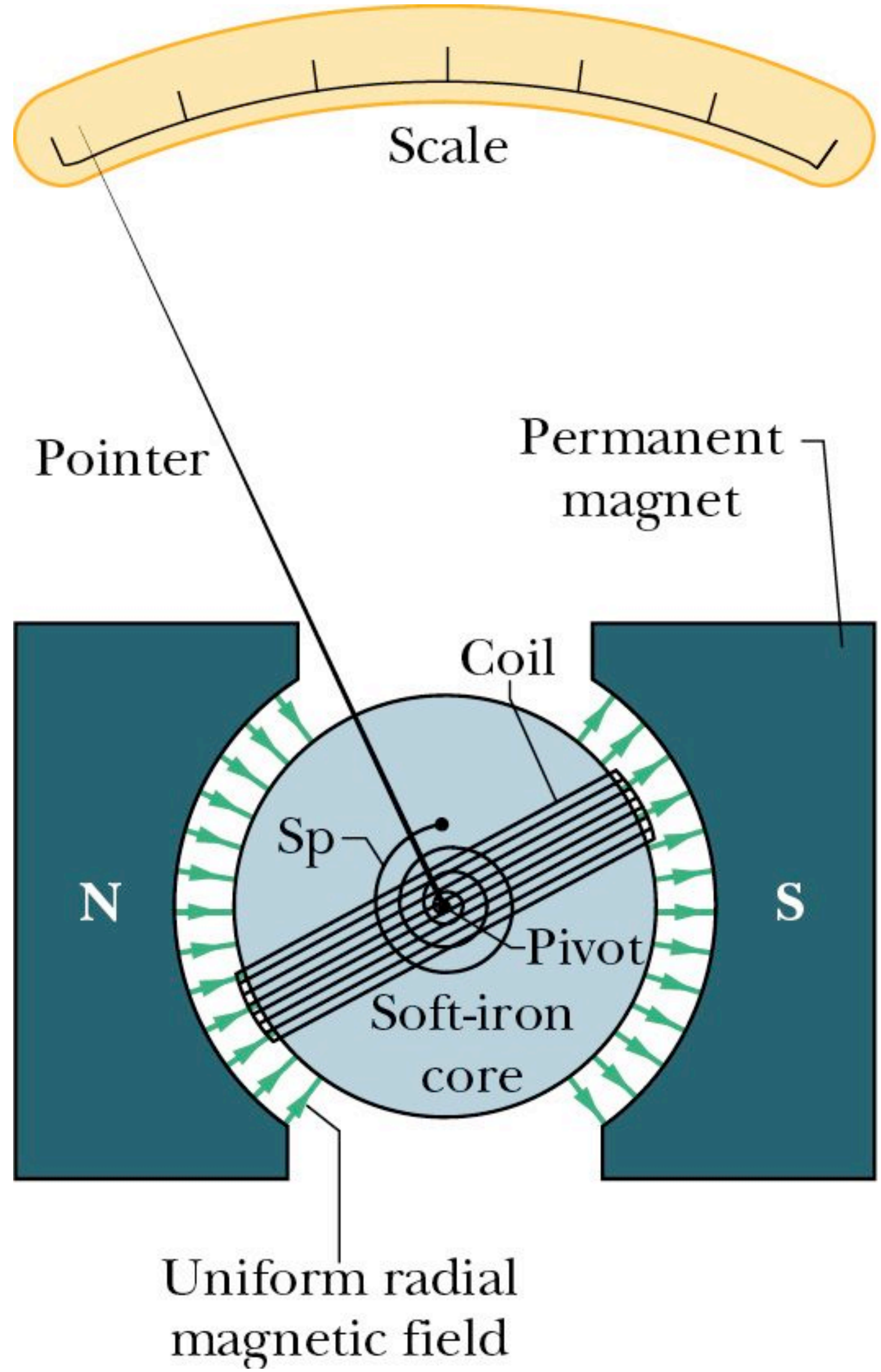


Torque on a current loop

$$\tau = NiAB\sin\theta$$
$$\mu = NiA$$
$$\tau = \mu B\sin\theta$$
$$\tau = \mu \times B$$



Galvanometer

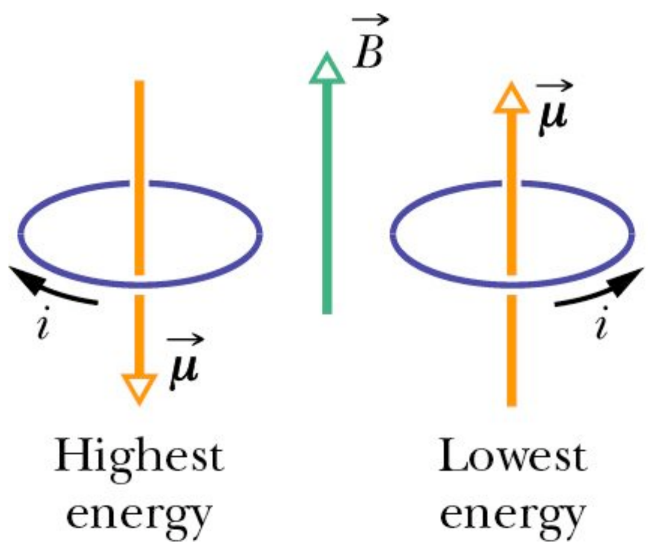


Magnetic dipole moment μ

$$\tau = NiAB\sin\theta$$
$$\mu = NiA$$
$$\tau = \mu B\sin\theta$$
$$\tau = \mu \times B$$
$$U = -\mu \cdot B$$

Recall that for Electric dipole moment p

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$U = -\vec{p} \cdot \vec{E}$$



<u>Demo</u>: show torque on current loop (galvanometer)

Can you predict direction of rotation?

<u>Example</u>

A square loop has N = 100 turns. The area of the loop is 4 cm^2 and it carries a current I = 10 A. It makes an angle of 30^o with a B field equal to 0.8 T. Find he magnetic moment of the loop and the torque.

$$\mu = NiA = 100 \times 10A \times 4 \times 10^{-4} m^2 = 0.4 A.m^2$$
$$T = \mu B \sin 30^\circ = 0.4 A.m^2 \times 0.8T \times 0.5 = 0.16 N.m$$

<u>Demo</u>: Show world's simplest electric motor

(scratch off all insulation on one end)

Scratch off half on the other end

Momentum will carry it _ turn

(no opportunity for current to reverse coil direction)

Cathode Ray Tube