Lecture 9 Magnetic Fields due to Currents Ch. 30

- Cartoon - Shows magnetic field around a long current carrying wire and a loop of wire
- Opening Demo - Iron filings showing B fields around wires with currents
- Warm-up problem
- Topics
  - Magnetic field produced by a moving charge
  - Magnetic fields produced by currents. Big Bite as an example.
  - Using Biot-Savart Law to calculate magnetic fields produced by currents.
  - Examples: Field at center of loop of wire, at center of circular arc of wire, at center of segment of wire.
  - Amperes' Law : Analogous to Gauss' L:aw in electrostatics, Useful in symmetric cases.
  - Infinitely long straight wire of radius a. Find B outside and inside wire.
  - Solenoid and Toroid Find B field.
  - Forces between current carrying wires or parallel moving charges
- Demos
  - Torque on a current loop(galvanometer)
  - Iron filings showing B fields around wires with currents.
  - Compass needle near current carrying wire
  - Big Bite as an example of using a magnet as a research tool.
  - Force between parallel wires carrying identical currents.

Reminders

Ungraded problem solutions are on website?

UVa email activated

Quiz tomorrow

Class in afternoon 1:00 PM

Grades are on WebAssign

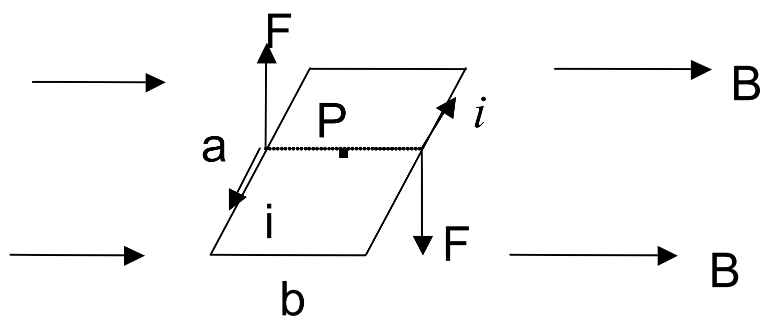
Magnetic Fields due to Currents

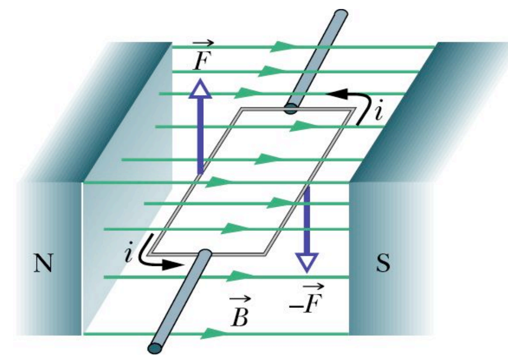
Torque on a coil in a magnetic field demo– left over from last time

- So far we have used permanent magnets as our source of magnetic field. Historically this is how it started.
- In early decades of the last century, it was learned that moving charges and electric currents produced magnetic fields.
- How do you find the Magnetic field due to a moving point charge?
- How do you find the Magnetic field due to a current?
  - Biot-Savart Law – direct integration
  - Ampere's Law – uses symmetry
- Examples and Demos

Torques on current loops

Electric motors operate by connecting a coil in a magnetic field to a current supply, which produces a torque on the coil causing it to rotate.





Above is a rectangular loop of wire of sides *a* and *b* carrying current i.

B is in the plane of the loop and ^ to *a*.

Equal and opposite forces $F = iaB$ are exerted on the sides *a*

No forces exerted on *b* since $i \| B$

Since net force is zero, we can evaluate τ (torque) about any point. Evaluate it about P (cm).

$$\tau = Fb\sin\theta$$

$$F = iaB$$

$$\tau = iabB\sin\theta$$

For N loops we mult by N

$$\tau = NiabB\sin\theta = NiAB\sin\theta$$

A=ab=area of loop

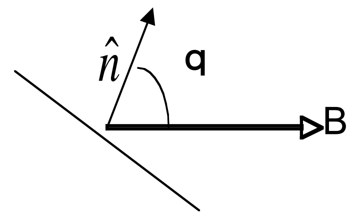
Torque on a current loop

$$\tau = NiAB\sin\theta$$

$$\mu = NiA = \text{magnetic dipole moment}$$

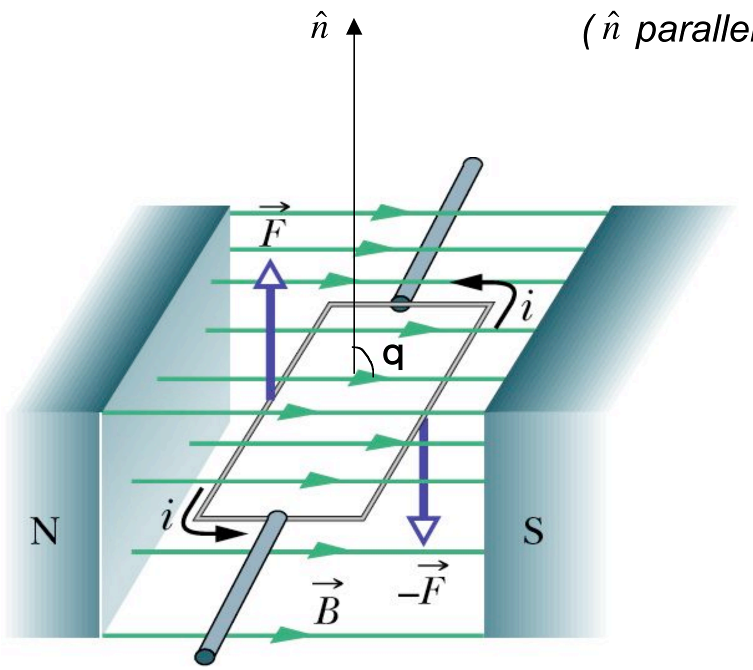
$$\tau = \mu B\sin\theta$$

$$\tau = \mu \times B$$



Torque tends to rotate loop until plane is ˆ to B

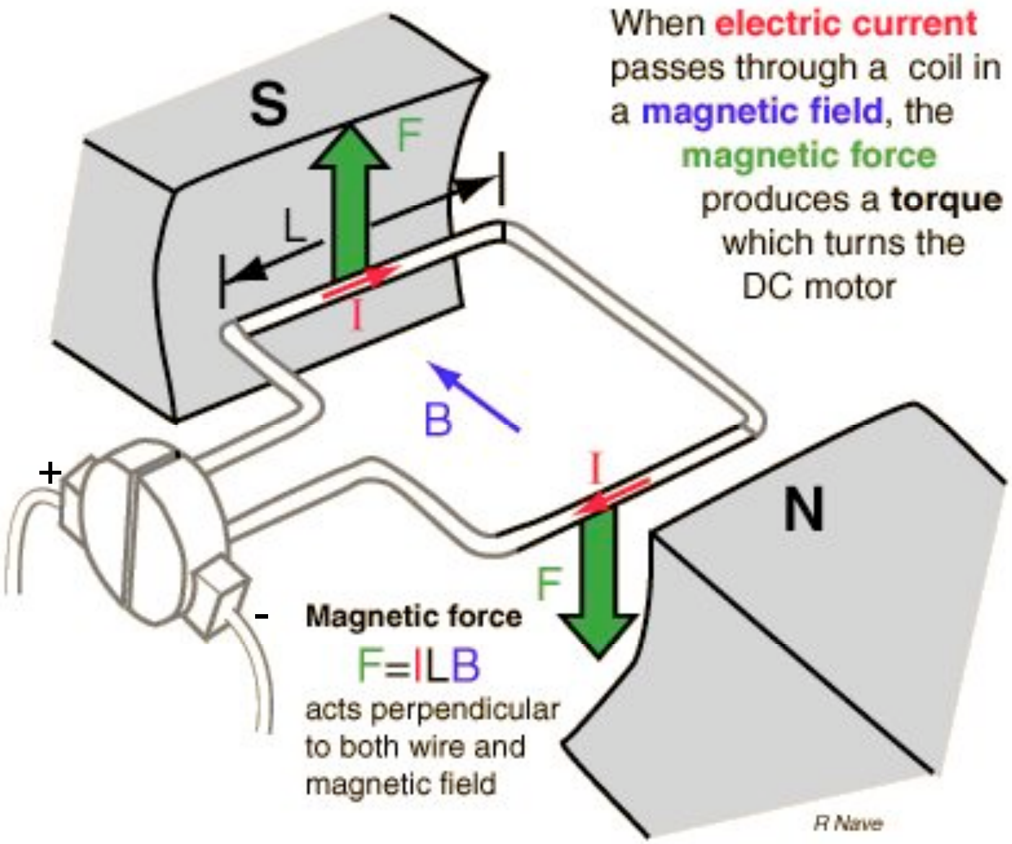
($\hat{n}$ parallel to B).



Must reverse current using commutator to keep loop turning.

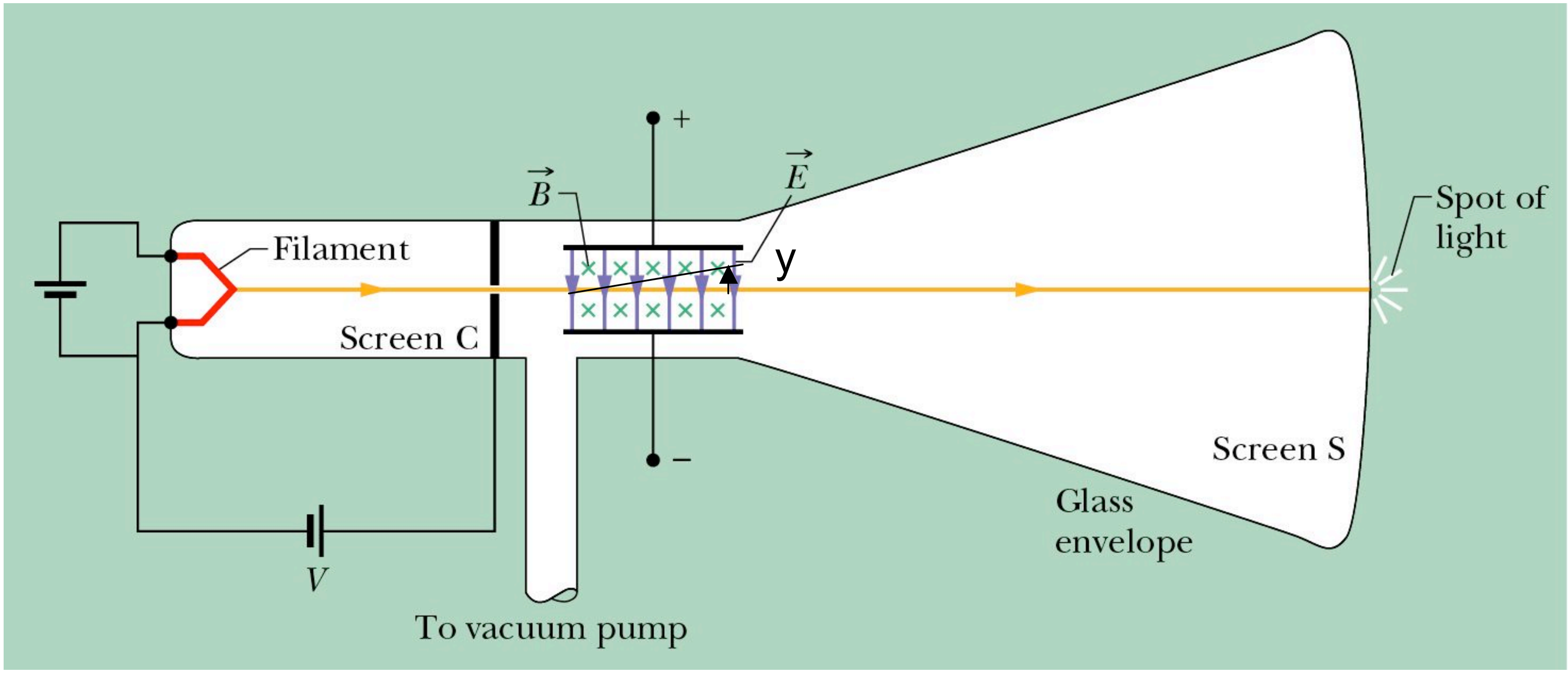
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motdc.html#c4

Split ring commutator



Electron moving with speed v in a crossed electric and magnetic field in a cathode ray tube.

$$\vec{F} = qE + q\vec{v}x\vec{B}$$



Discovery of the electron by J.J. Thompson in 1897

1. E=0, B=0 Observe spot on screen

2. Set E to some value and measure y the deflection

$$y = \frac{qEL^2}{2mv^2}$$

3. Now turn on B until spot returns to the original position

$$qE = qvB$$

$$v = \frac{E}{B}$$

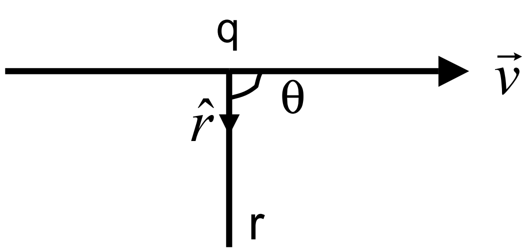
4 Solve for

$$\frac{m}{q} = \frac{B^2L^2}{2yE}$$

This ratio was first measured by Thompson to be lighter than hydrogen by 1000

Show demo of CRT

Topic: Moving charges produce magnetic fields



$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}}$$

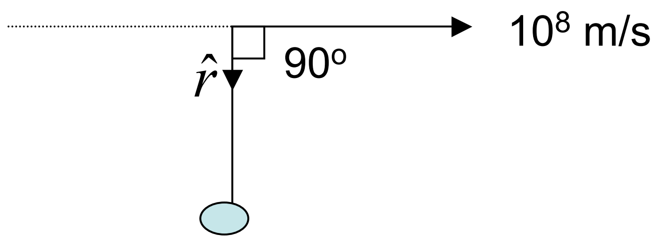
Determined fom
Experiment

1. Magnitude of B is proportional to q, $\vec{v}$, and $1/r^{2}$.
2. B is zero along the line of motion and proportional to sin at other points.
3. The direction is given by the RHR rotating $\vec{v}$ into $\hat{r}$
4. Magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$$

Example:

A point charge q = 1 mC (1×10^{-3}C) moves in the x direction with v = 10^8 m/s. It misses a mosquito by 1 mm. What is the B field experienced by the mosquito?



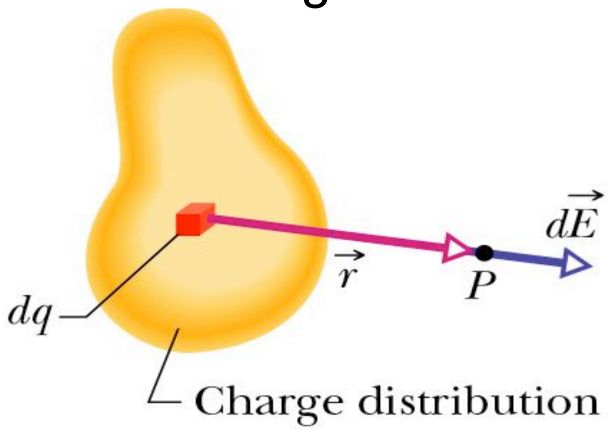
$$B = \frac{\mu_0}{4\pi} q \frac{v}{r^2}$$

$$B = 10^{-7} \tfrac{N}{A^2} \times 10^{-3} C \times 10^8 \tfrac{m}{s} \times \frac{1}{10^{-6} m^2}$$

$$\boxed{B = 10^4 T}$$

Topic: A current produces a magnetic field

Recall the **E** field of a charge distribution

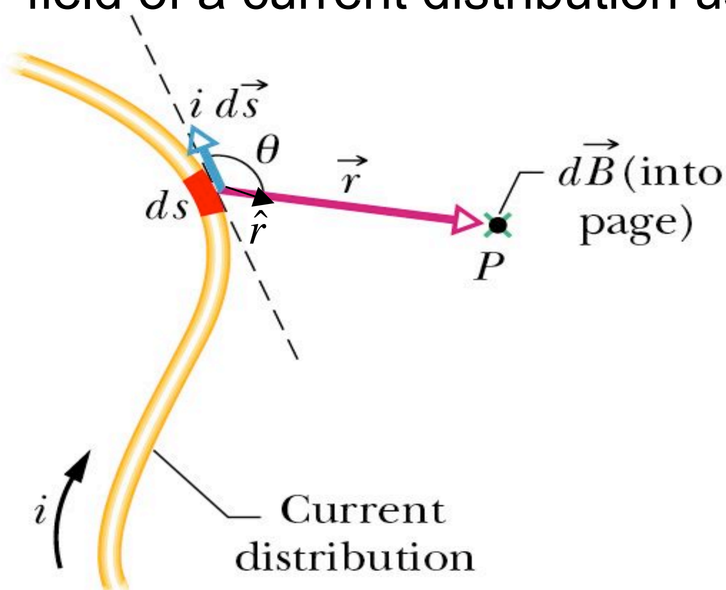


(a)

"Coulombs Law"

$$d\vec{E} = \frac{kdq}{r^2}\hat{r}$$

To find the $\vec{B}$ field of a current distribution use:



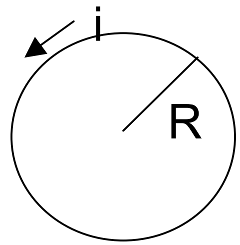
(b)

Biot-Savart Law

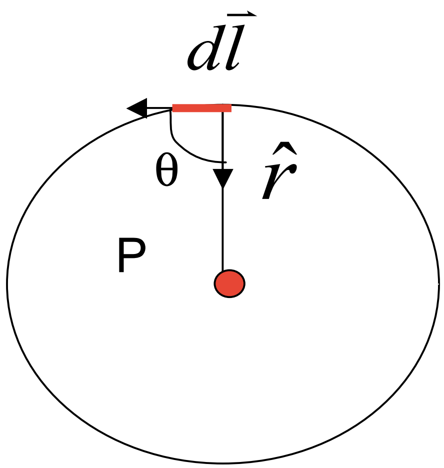
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2}$$

This Law is found from experiment

Find B field at center of loop of wire lying in a plane with radius R and total current i flowing in it.



$$B = \int \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \hat{r}}{r^2}$$



$d\vec{l} \times \hat{r}$ is a vector coming out of the paper The angle between dl and r is constant and equal to 90 degrees.

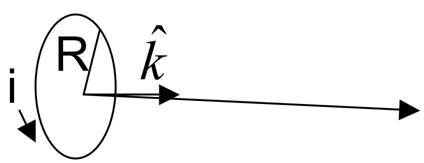
$$d\vec{l} \times \hat{r} = dl \sin\theta\ \hat{k}$$

$$\sin\theta = \sin 90 = 1$$

$$d\vec{l} \times \hat{r} = dl\ \hat{k}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\hat{k}\int \frac{i dl}{R^2} = \frac{\mu_0}{4\pi}\hat{k}\frac{i}{R^2}\int dl = \frac{\mu_0}{4\pi}\hat{k}\frac{i}{R^2} 2\pi R$$

Magnitude of B field at center of loop. Direction is out of paper.

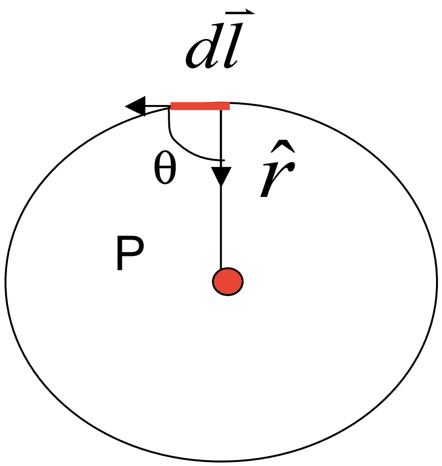


$$\boxed{\vec{B} = \frac{\mu_0 i}{2R}\hat{k}}$$

Magnitude is $\dfrac{\mu_0 i}{2R}$

Direction is $\hat{k}$

Integration Details



$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\hat{k}\int \frac{idl}{R^2}$$

$$= \frac{\mu_0}{4\pi}\frac{i}{R^2}\hat{k}\int_{l_i}^{l_f} dl = \frac{\mu_0}{4\pi}\frac{i}{R^2}\hat{k}\, l\Big|_{l_i}^{l_f}$$

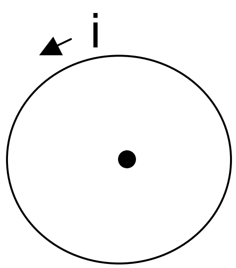
$$= \frac{\mu_0}{4\pi}\frac{i}{R^2}\hat{k}(l_f - l_i)$$

$$= \frac{\mu_0}{4\pi}\frac{i}{R^2}\hat{k}(2\pi R - 0)$$

$$= \frac{\mu_0 i}{2R}\hat{k}$$

Example

Loop of wire of radius R = 5 cm and current i = 10 A. What is B at the center? Magnitude and direction



$$B = \frac{\mu_0 i}{2R}$$

$$B = 4\pi \times 10^{-7} \frac{N}{A^2} \frac{10A}{2(.05m)}$$

$$B = 1.2 \times 10^{-6} \cdot 10^2 T$$

$$B = 1.2 \times 10^{-4} T = 1.2 \quad Gauss$$

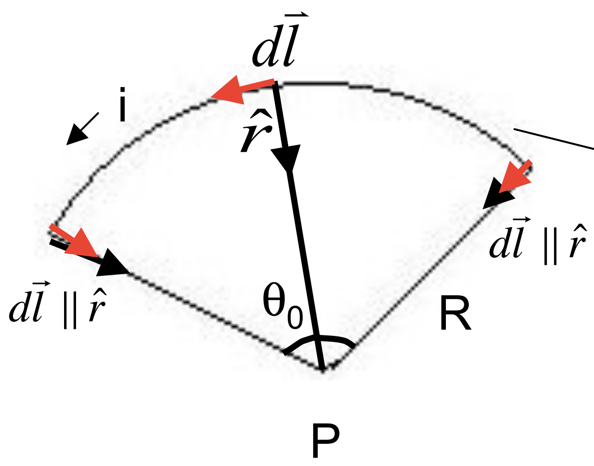
Direction is out of the page.

What is the B field at the center of a segment or circular arc of wire?

$$B = \int \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \frac{i}{R^2} \hat{k} \int dl$$

$$= \frac{\mu_0}{4\pi} \frac{i}{R^2}\ \hat{k}\ S$$



Total length of arc is S.

where S is the arc length $S = R\theta_0$

θ_0 is in radians (not degrees)

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} \frac{i}{R}\ \theta_0 \hat{k}}$$

$$d\vec{l} \times \vec{r} = r\,dl \sin\theta\ \hat{k}$$

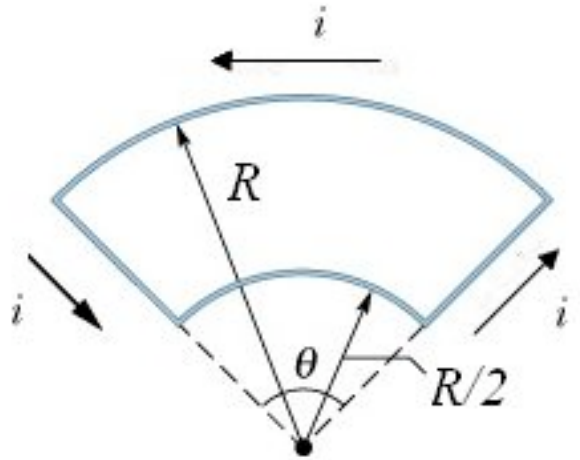
$$\sin\theta = \sin 0 = 0$$

$$\theta_0 = 100 \text{ deg} = 100 \text{ deg} \times \frac{6.28 \text{ rad}}{360 \text{ deg}} = 100 \times 0.0174 = 1.74 \text{ rad}$$

Why is the contribution to the B field at P equal to zero from

the straight section of wire?

Suppose you had the following loop.
Find magnetic field at center of arc length



What is the magnitude and direction of B at the origin?

$$B = \frac{\mu_0}{4\pi}\frac{i}{R^2}S$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{R}\theta_0$$

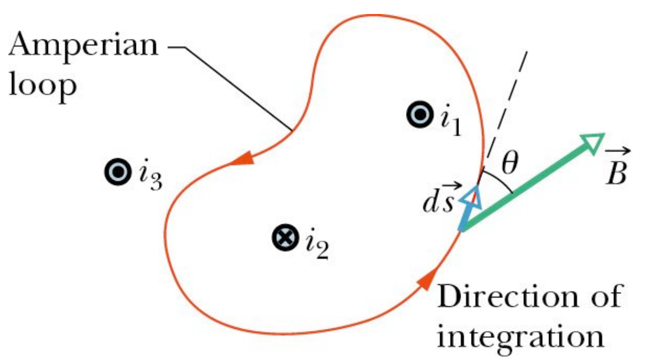
$$B = \frac{\mu_0}{4\pi}\left(\frac{i}{R}\theta_0 - \frac{i}{R/2}\theta_0\right) = -\frac{\mu_0}{2\pi}\frac{i}{R}\theta_0$$

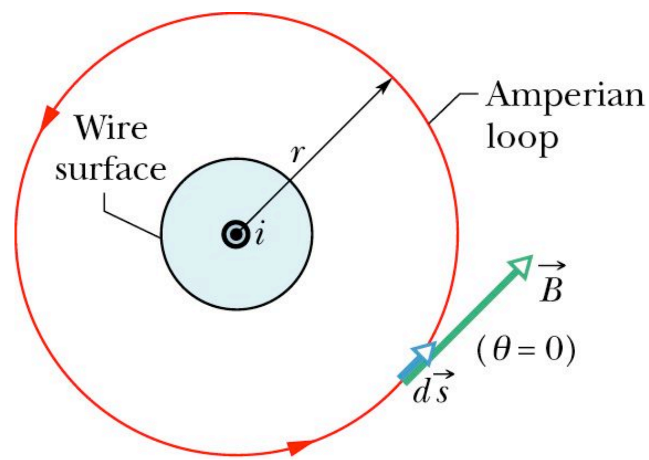
Next topic: **Ampere's Law**

Allows us to solve certain highly symmetric current problems for the magnetic field as Gauss' Law did in electrostatics.

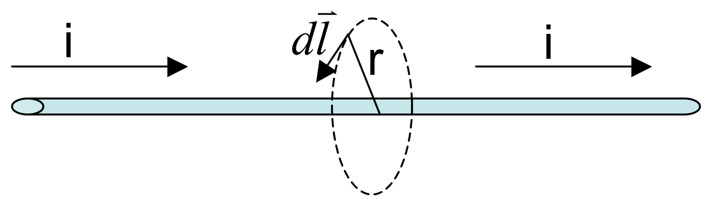
Gauss's Law $$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}}$$ Charge enclosed by surface

Ampere's Law is $$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_c}$$ Current enclosed by the path





Example: Use Ampere's Law to find B near a very long, straight wire. B is independent of position along the wire and only depends on the distance from the wire (symmetry).



By symmetry $\vec{B} \parallel d\vec{l}$

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \int dl = B 2\pi r = \mu_0 i$$

$$\boxed{B = \frac{\mu_0}{2\pi} \frac{i}{r}}$$

Suppose i = 10 A

R = 10 cm

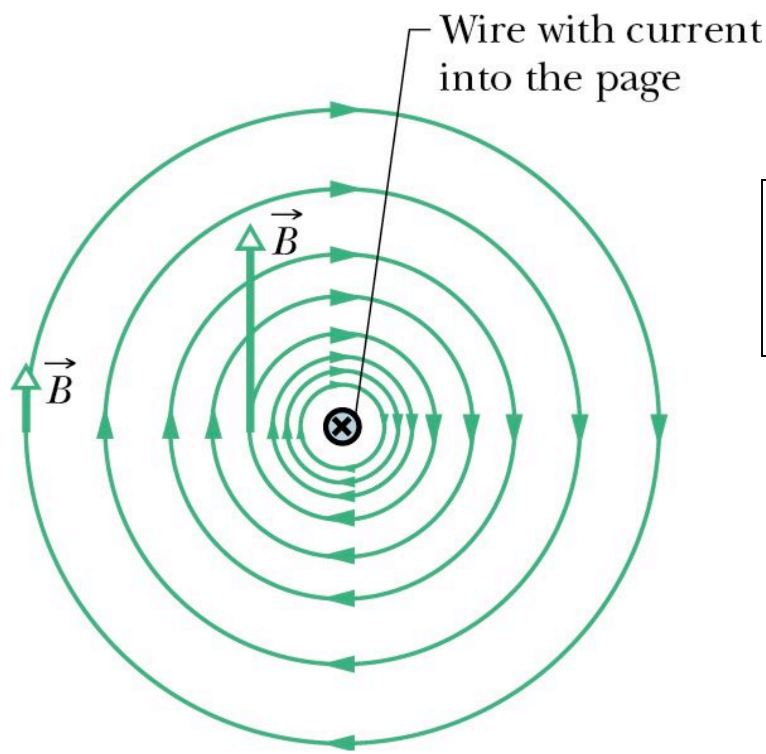
$$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$$

$$B = \frac{2 \times 10^{-7} \times 10}{10^{-1}}$$

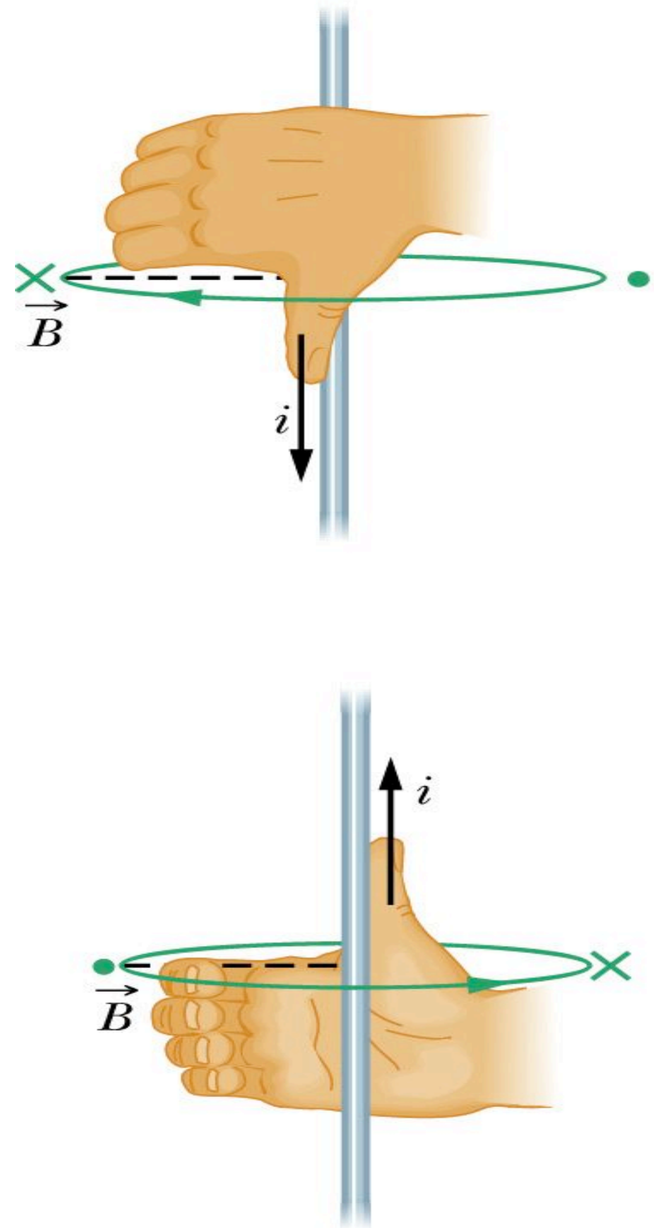
$$B = 2 \times 10^{-5} \text{ T}$$

Show Fe fillings around a straight wire with current, current loop, and solenoid.

Rules for finding direction of B field from a current flowing in a wire

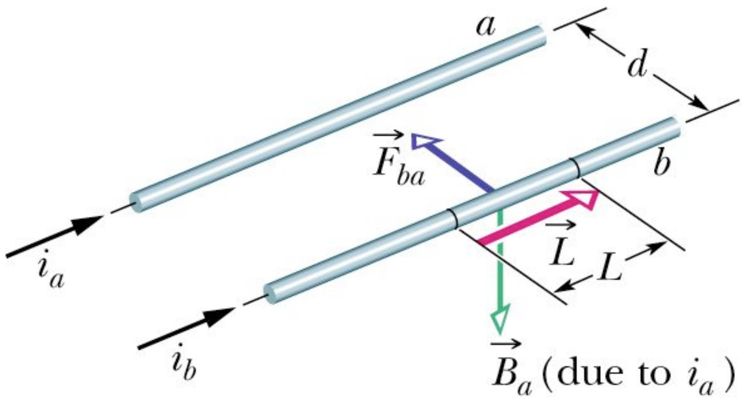


$$B = \frac{\mu_0 i}{2\pi r}$$



Force between two current carrying wires

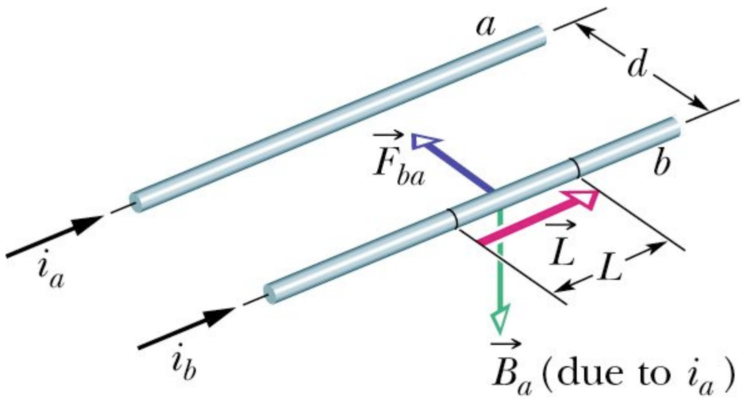
Find the force due to the current element of the first wire and the magnetic field of the second wire. Integrate over the length of both wires. This will give the force between the two wires.



$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

Force between two current carrying wires

Find the force due to the current element of the first wire and the magnetic field of the second wire. Integrate over the length of both wires. This will give the force between the two wires.



From experiment we find

$$F_{ba} = L\frac{i_a i_b \mu_0}{2\pi d}$$

$$\frac{F_{ba}}{L} = \frac{i_a i_b \mu_0}{2\pi d}$$

$$B_a = \frac{\mu_0 i_a}{2\pi d}$$ (Given by Ampere's Law)

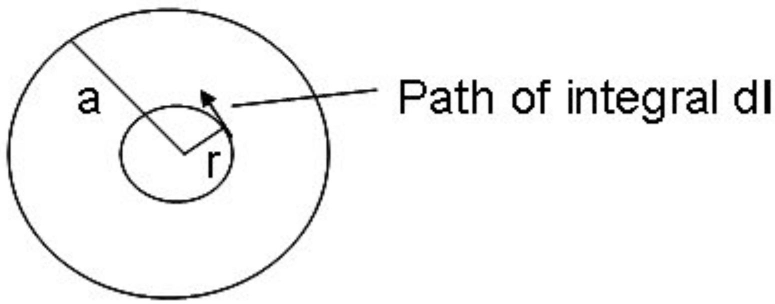
$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a = i_b L \frac{\mu_0 i_a}{2\pi d}$$

$\vec{L} \perp \vec{B}$ and directed towards wire a (wires are attracted).

Suppose one of the currents is in the opposite direction? What direction is $\vec{F}_{ba}$?

Example: Find magnetic field inside a long, thick wire of radius a

Cross-sectional view



$$\oint \vec{B}\cdot d\vec{l} = \oint B dl = B\int_0^{2\pi r} dl = B2\pi r$$

$$B2\pi r = u_0 I_c$$

I_C = current enclosed by the circle whose radius is r

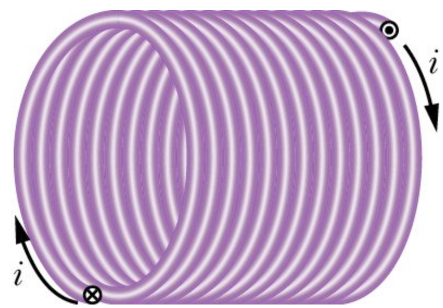
$$I_C = \frac{\pi r^2}{\pi a^2} i = \frac{r^2}{a^2} i$$

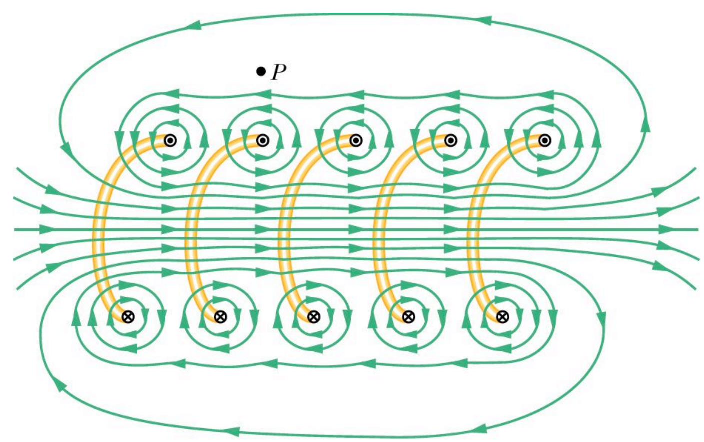
$$B = \mu_0 \frac{r^2}{a^2} i \frac{1}{2\pi r}$$

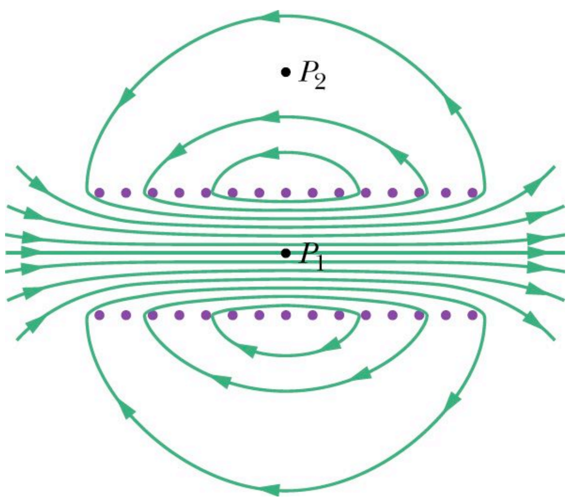
$$\boxed{B = \frac{\mu_0 i r}{2\pi a^2}}$$

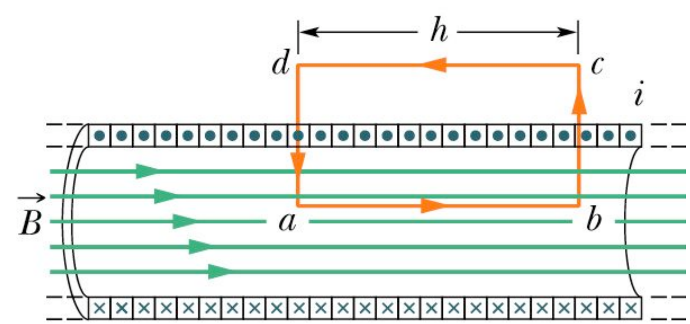
Example: Find field inside a solenoid. See next slide.

Solenoid



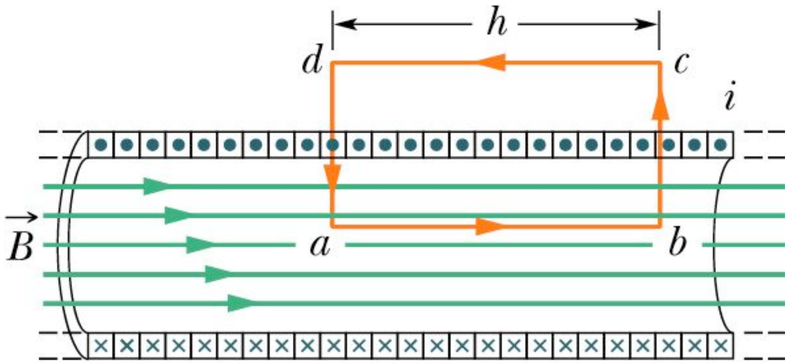


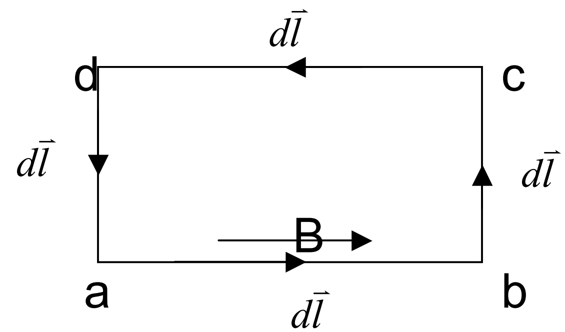




$$B = \mu_0 n i$$

n is the number of turns per meter





First evaluate the right side, it's easy

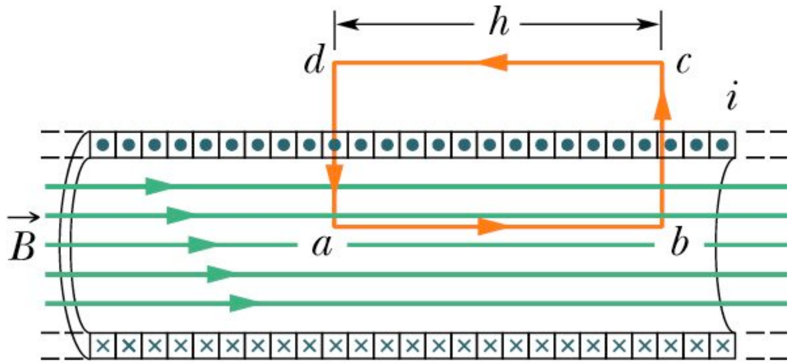
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

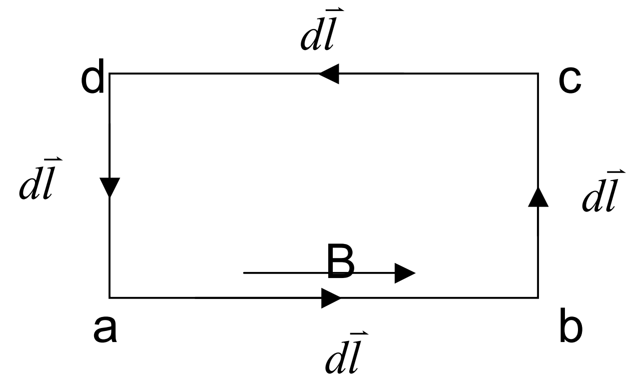
I_C is the total current enclosed by the path

$$I_c = \underbrace{n}h i$$

The number of loops of current; h is the length of one side.

Right side = $\mu_0 nhi$





Evaluate left side:

$$\oint \vec{B} \cdot d\vec{l} = \int_a^b B \cdot dl + \int_b^c B \cdot dl + \int_c^d B \cdot dl + \int_d^a B \cdot dl$$

$$= Bh + 0 + 0 + 0$$

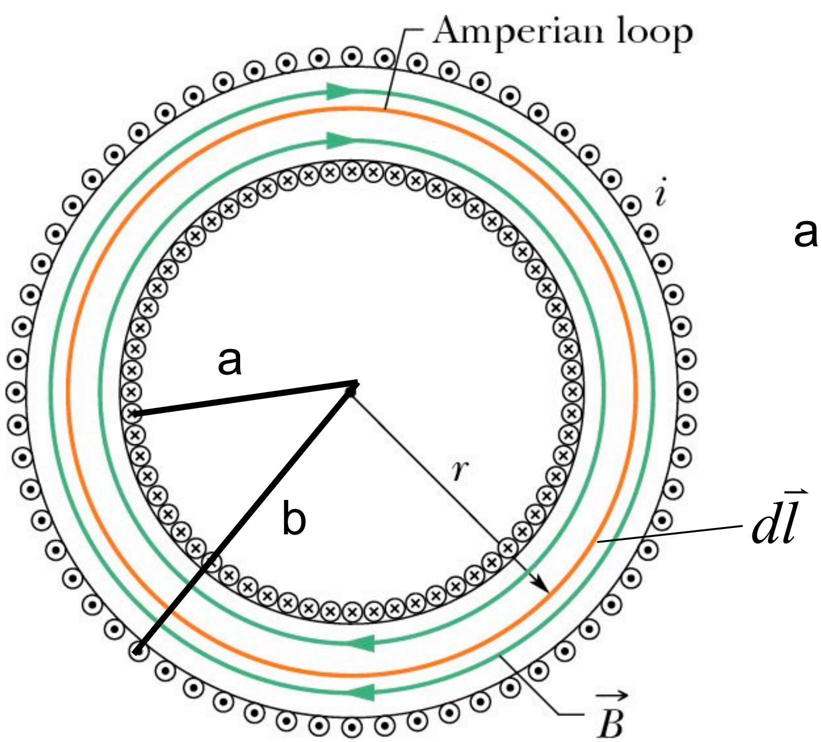
$$Bh = \mu_0 nhi$$

$$B = \mu_0 ni$$

n = the number of loops or turns per meter

Toroid

(a)



(b)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

$$\oint B dl = \mu_0 N i$$

$$B 2\pi r = \mu_0 N i$$

$$\boxed{B = \frac{\mu_0 N i}{2\pi r}}$$

N is the total number of turns

a < r < b

$$\vec{B} \cdot d\vec{l} = B dl$$

$$\cos 0^o = 1$$

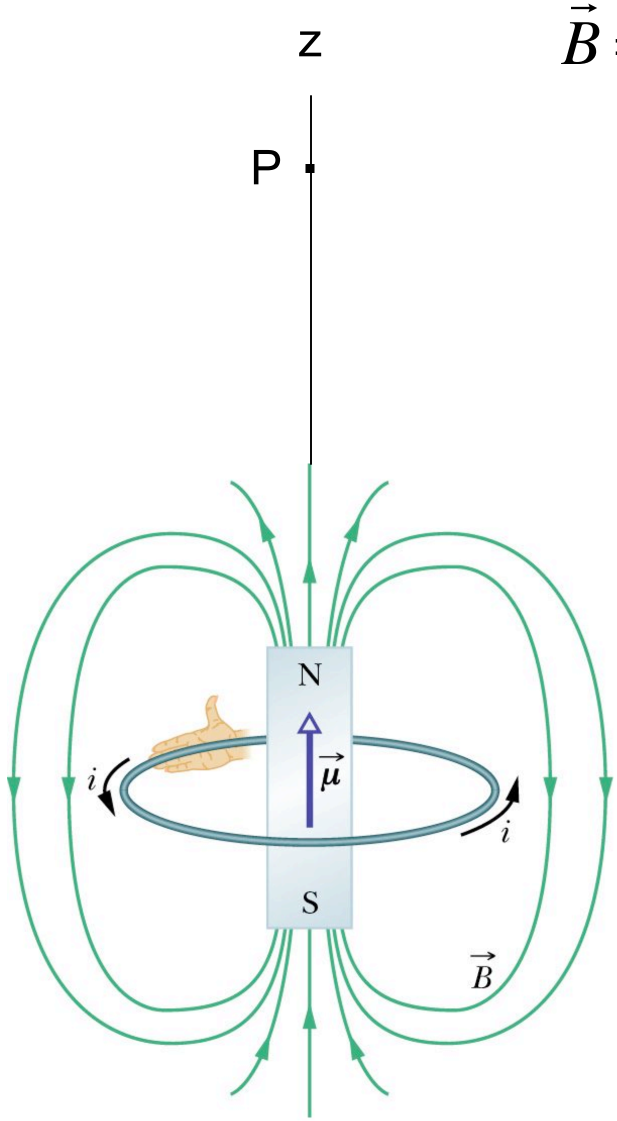
Tokamak Toroid at Princeton

I = 73,000 Amps for 3 secs

B = 5.2 T

Magnetic dipole inverse cube law

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$



Magnetic field lines for a bar magnet and a current loop are similar