

15. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40 \text{ rev/s})(2\pi \text{ rad/rev})}{2\pi} = 40 \text{ Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-44, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the “circuit” will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ , is

$$A = A_0 + \frac{\pi a^2}{2} \cos \theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi f t$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since \vec{B} is uniform (in space) and constant (in time), Faraday’s law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B \frac{dA}{dt} = -B \frac{d\left(A_0 + \frac{\pi a^2}{2} \cos \theta\right)}{dt} = -B \frac{\pi a^2}{2} \frac{d \cos(2\pi f t)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f \sin(2\pi f t)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020 \text{ T})\pi^2 (0.020 \text{ m})^2 (40/\text{s}) = 3.2 \times 10^{-3} \text{ V}.$$