

61. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\max} = i_{d\max} = 7.60 \mu\text{A}$.

(b) Since $i_d = \epsilon_0 (d\Phi_E/dt)$,

$$\left(\frac{d\Phi_E}{dt} \right)_{\max} = \frac{i_{d\max}}{\epsilon_0} = \frac{7.60 \times 10^{-6} \text{ A}}{8.85 \times 10^{-12} \text{ F/m}} = 8.59 \times 10^5 \text{ V} \cdot \text{m/s}.$$

(c) According to problem 13,

$$i_d = C \frac{dV}{dt} = \frac{\epsilon_0 A}{d} \frac{dV}{dt}.$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \epsilon_m \sin \omega t$ and $dV/dt = \omega \epsilon_m \cos \omega t$. Thus, $i_d = (\epsilon_0 A \omega \epsilon_m / d) \cos \omega t$ and $i_{d\max} = \epsilon_0 A \omega \epsilon_m / d$. This means

$$d = \frac{\epsilon_0 A \omega \epsilon_m}{i_{d\max}} = \frac{(8.85 \times 10^{-12} \text{ F/m}) \pi (0.180 \text{ m})^2 (130 \text{ rad/s}) (220 \text{ V})}{7.60 \times 10^{-6} \text{ A}} = 3.39 \times 10^{-3} \text{ m},$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates, $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so \vec{B} is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. Thus,

$$2\pi r B = \mu_0 \left(\frac{r^2}{R^2} \right) i_d \Rightarrow B = \frac{\mu_0 i_d r}{2\pi R^2}.$$

The maximum magnetic field is given by

$$B_{\max} = \frac{\mu_0 i_{d\max} r}{2\pi R^2} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(7.6 \times 10^{-6} \text{ A})(0.110 \text{ m})}{2\pi (0.180 \text{ m})^2} = 5.16 \times 10^{-12} \text{ T}.$$