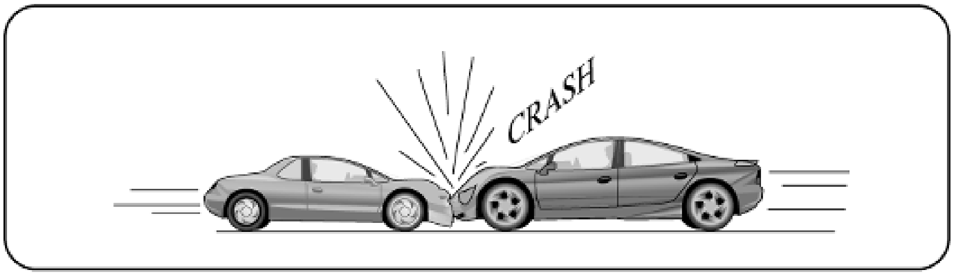
Name____________________________Date__________________Partners____________________________

COLLISIONS AND MOMENTUM - NEWTON'S THIRD LAW



In any system of bodies which act on each other, action and reaction, estimated by momentum gained and lost, balance each other according to the laws of equilibrium.

–Jean de la Rond D'Alembert

OBJECTIVES

- To understand the definition of momentum and its vector nature as it applies to one-dimensional collisions.
- To develop the concept of impulse to explain how forces act over time when an object undergoes a collision.
- To study the interaction forces between objects that undergo collisions.
- To examine the relationship between impulse and momentum experimentally in both *elastic* (bouncy) and *inelastic* (sticky) collisions.
- To examine the consequences of *Newton's third law* as applied to interaction forces between objects.
- To explore conservation of momentum in one-dimensional collisions.

OVERVIEW

In this lab we explore the forces of interaction between two objects and study the changes in motion that result from these interactions. We are especially interested in studying collisions and explosions in which interactions take place in fractions of a second. Early investigators spent a considerable amount of time trying to observe collisions and explosions, but they encountered difficulties. This is not surprising, since observation of the details of such phenomena requires the use of instruments–such as high-speed cameras–that were not yet invented.

However, the principles describing the outcomes of collisions were well understood by the late seventeenth century when several leading European scientists, including Isaac Newton, developed the concepts to describe both *elastic* collisions, in which objects bounce off each other, and *inelastic* collisions, in which objects stick together.

We will begin our study of collisions by exploring the relationship between the forces experienced by an object and its momentum change. It can be shown mathematically from Newton's laws and experimentally from our own observations that the change in momentum of an object is equal to a quantity called *impulse*. Impulse takes into account both the magnitude of the applied force at each

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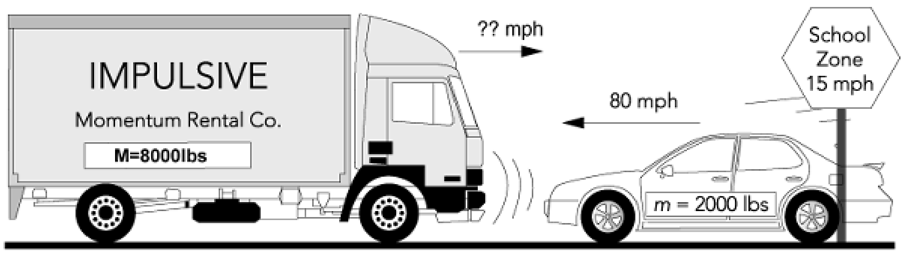
instant in time and the time interval over which this force acts. The statement of equality between impulse and momentum change is known as the *impulse—momentum theorem*.

Since interactions like collisions and explosions never involve just one object, we turn our attention to the mutual forces of interaction between two or more objects. This will lead us to a very general law known as *Newton's third law*, which relates the forces of interaction exerted by two objects on each other. This will in turn lead us to one of the most important laws of interactions between objects, the conservation of momentum law.

INVESTIGATION 1: MOMENTUM AND MOMENTUM CHANGE

In this investigation we are going to develop the concept of momentum to predict the outcome of collisions. But you don't officially know what momentum is because we haven't defined it yet. We want to define momentum to help us describe collisions in mathematical terms.



It's early fall and you are driving along a two-lane highway in a rented moving van. It's full of all of your possessions so you and the loaded truck weigh 8000 lb. You have just slowed down to 15 mph because you're in a school zone. It's a good thing you thought to do that, because a group of first graders are just starting to cross the road. Just as you pass the children you see a 2000-lb sports car in the other lane heading straight for the children at about 80 mph.

A desperate thought crosses your mind. You just have time to swing into the other lane and speed up a bit before making a head-on collision with the sports car. *You want your truck and the sports car to crumple into a heap that sticks together and doesn't move*. Can you save the children or is this just a suicidal act?

To simulate this situation you can observe two carts of different mass set up to stick together in trial collisions. You will need

- 2 low-friction carts with Velcro on one end
- 4 – ½ kg masses to put on carts
- 2-m motion track
- level

Activity 1-1: Can You Stop the Car?

Prediction 1-1: How fast would you have to be going to completely stop the sports car? Explain the reasons for your prediction.

1. Verify that the 2-m motion track is level.

2. Try some head-on collisions with the carts of different mass to simulate the event on a small scale. Be sure that the carts stick together after the collision.

3. Observe *qualitatively* what combinations of velocities cause the two carts to be at rest after the collision.

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Question 1-1: What happens when the less massive cart is moving much faster than the more massive cart? Much slower?

Originally, Newton did not use the concept of acceleration or velocity in his laws. Instead, he used the term *motion*, which he defined as the product of mass and velocity. We now call that quantity momentum:

$$\mathbf{p} \equiv m\mathbf{v}$$

where the symbol $\equiv$ is used to designate "defined as."

NEWTON'S FIRST TWO LAWS OF MOTION

1. Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it.
2. The net force on an object can be calculated as the product of its mass and its acceleration, where the directions of the force and of the resulting acceleration are the same.

Now let's test your intuition about momentum and forces. You are sleeping in your sister's room while she is away at college. Your house is on fire and smoke is pouring into the partially open bedroom door. To keep the smoke from coming in, you must close the door. The room is so messy that you cannot get to the door. The only way to close the door is to throw either a blob of clay or a superball at the door–there isn't time to throw both.

Prediction 1-2: Assuming that the clay blob and the superball have the same mass, and that you throw them with the same velocity, which would you throw to close the door–the clay blob, which will stick to the door, or the superball, which will bounce back at almost the same speed as it had before it collided with the door? Give reasons for your choice using any notions you already have or any new concepts developed in physics, such as force, energy, momentum, or Newton's laws. If you think that there is no difference, justify your answer. Remember, your life depends on it!

It would be nice to be able to use Newton's formulation of the *second law* of motion to find collision forces, but it is difficult to measure the rate of change of momentum during a rapid collision without special instruments. However, measuring the momenta of objects just before and just after a collision is usually not too difficult. This led scientists in the seventeenth and eighteenth centuries to concentrate on the overall changes in momentum that resulted from collisions. They then tried to relate changes in momentum to the forces experienced by an object during a collision.

In the next activity you are going to explore the mathematics of calculating momentum changes for the two types of collisions–the *elastic* collision, where the ball bounces off the door, and the *inelastic*

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collision, where the ball sticks to the door.

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Activity 1-2: Momentum Changes

Prediction 1-3: Which object undergoes the greater *momentum change* during the collision with a door–the clay blob or the superball? Explain your reasoning carefully.

Comment: Recall that momentum is defined as *a vector* quantity; that is, it has both *magnitude* and *direction*. Mathematically, momentum change is given by the equation

$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

where $\mathbf{p}_i$ is the initial momentum of the object just before a collision and $\mathbf{p}_f$ is its final momentum just after. Remember, in one dimension, the *direction* of a vector is indicated by its *sign*.

Just looking at the *maximum* force exerted on the ball does not tell the whole story. You can see this from a simple experiment tossing raw eggs. We will do it as a thought experiment to avoid the mess! Suppose somebody tosses you a raw egg and you catch it. (In physics jargon, one would say that the egg and the hand have undergone an *inelastic collision.*)

Question 1-2: If you catch an egg of mass *m* that is heading toward your hand at speed *v*, what magnitude momentum *change* does it undergo? (**Hint**: The egg is at rest after you catch it.)

Question 1-3: Does the total momentum change differ if you catch the egg more slowly or is it the same?

In bringing an egg to rest, the change in momentum is the same whether you use a large force during a short time interval or a small force during a long time interval. *Of course, which one you choose makes a lot of difference in whether or not the egg breaks!*

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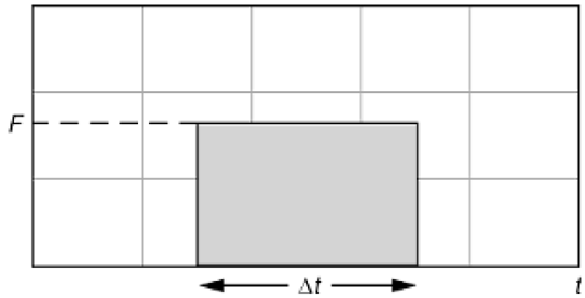
A quantity called *impulse* has been defined for you in lecture or in your textbook. It combines the applied force and the time interval over which it acts. In one dimension, for a *constant* force F acting over a time interval Δt , as shown in the graph below, the impulse J is

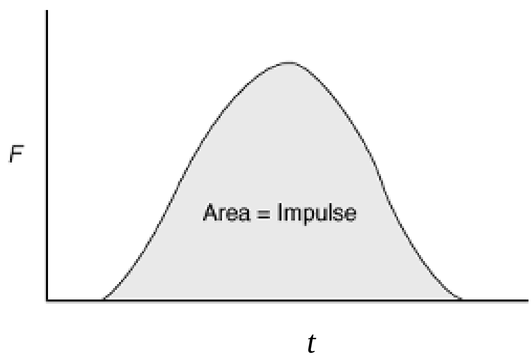
$$J = F\Delta t$$



As you can see, a large force acting over a short time and a small force acting over a long time can have the same impulse.

Note that $F\Delta t$ is the area of the rectangle, that is, the area under the force vs. time curve. If the applied force is not constant, then the impulse can still be calculated as the *area under the force vs. time graph.* In other words,

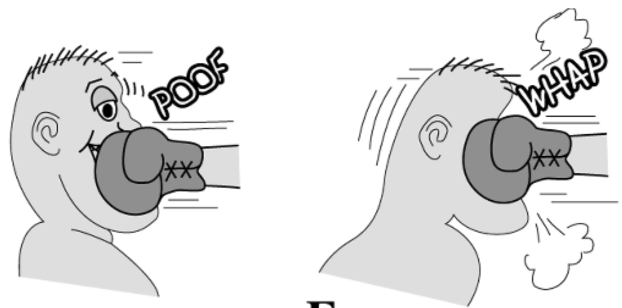
$$\mathbf{J} = \int \mathbf{F} dt$$



It is the impulse that equals the change in momentum.

$$\mathbf{J} = \Delta \mathbf{P}$$

Question 1-4: Suppose the time you take to bring the egg to a stop is Δt . Would you rather catch the egg in such a way that Δt is small or large? Why?



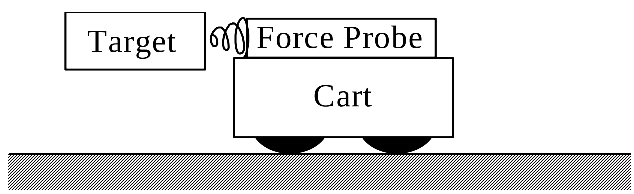
$F\Delta t$ = change in momentum $F\Delta t$ = change in momentum

On the left the boxer moves backwards during the blow, which occurs during a relatively long time Δt, while on the right the boxer moves forward just as the blow hits him. Although the Δt is shorter, the force is greater on the right. The greater forces on the right side cause more damage to be done.

INVESTIGATION 2: IMPULSE, MOMENTUM, AND COLLISIONS

Let's first see qualitatively what an impulse curve might look like in a real collision in which the forces change over time during the collision. To explore this idea you will need

- motion cart
- 2-m motion track
- "dartboard" target
- rods, clamps, etc.
- force probe
- motion detector
- level



Activity 2-1: Observing Collision Forces That Change with Time

1. Mount the force probe on the cart and screw in the spring on the end of the force probe.

2. Use the table clamps and other accessories to rigidly mount the target to the table so that the spring on the force probe will collide directly with the target when the cart is rolling down the motion track. Ask your instructor for help if this is not clear.

3. Start pushing the cart about 0.75 meters away from the target, let the cart coast and collide with the target several times and observe what happens to the spring.

Question 2-1: If friction is negligible, what is the net force exerted on the cart ***just before*** it starts to collide?

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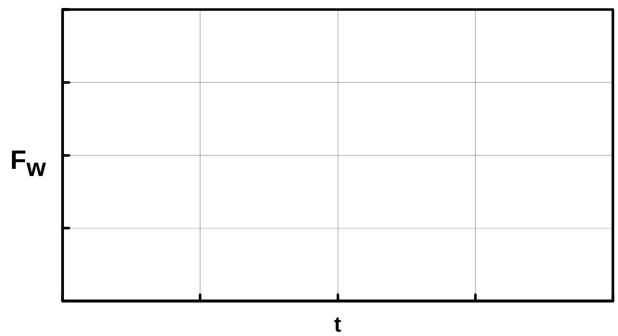
Question 2-2: When is the magnitude of the net force on the cart maximum during the collision?

Question 2-3: Estimate roughly how long the collision process takes? Half a second? Less time? Several seconds?

Prediction 2-1: Draw a rough sketch below of the shape of the force F_w the spring exerts on the cart as a function of time during the collision.



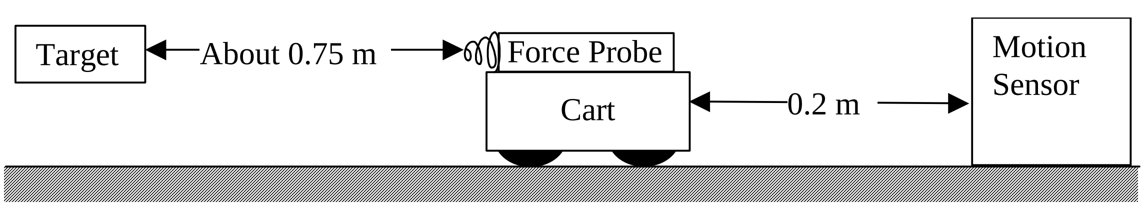
During the collision the force is not constant. To measure the impulse and compare it to the change in momentum of the cart, you must (1) plot a force—time graph and find the area under it, and (2) measure the velocity of the cart before and after the collision with the wall. Fortunately, the force probe, motion detector, and motion software will allow you to do this.

The force probe is mounted on the cart to measure the *force applied to* the cart. You will collide the cart into the target mounted near the track.

Activity 2-2: Examining the Impulse—Momentum Theorem in an *Elastic* Collision

In an elastic collision between a cart and fixed target, the cart would recoil with the same magnitude of momentum that it had before the collision.

1. Set up the motion detector as shown. *Be sure that the ramp is level*.



2. Measure the mass of the cart and force probe combination. You may need to mass them separately and add the masses.

Mass of cart plus force probe:____________kg

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3. Open the experiment file called **Impulse and Momentum L7.2-2**. This experiment has been set up to record force and motion data at 50 data points per second. Because the positive direction is toward the right in the diagram above, the software has been set up to record a *push* on the force probe as a *positive* force, and velocity *toward the motion detector as positive*.

4. Be sure that the wire from the force probe is taped out of the way, so the motion detector won't see it.

5. Practice pushing the cart toward the target and watching it bounce off. Find a way to push without putting your hand between the motion detector and the cart.

6. When you are ready, **zero** the force probe and then **start the computer**. As soon as you hear the clicking of the motion detector, give the cart a push toward the target, release it, and let it collide.

Repeat until you get a good set of graphs, that is, a set in which the motion detector saw the relatively constant velocities of the cart as it moved toward the spring target and as it moved away, and also the maximum force is no more than 50 N. We have found that a force of 10N is adequate. (With too large a force, the spring will bottom out.)

Question 2-4: Does the shape of the force—time graph agree with your Prediction 2-1? In what way is it similar? In what way does it differ?

7. Use the **statistics features** in the software to measure the average velocity of the cart as it approached the target, and the average velocity as it moved away from the target. Don't forget to include a sign.

Average velocity just before the collision:___________m/s

Average velocity just after the collision:___________m/s

8. Calculate the change in momentum of the cart. Show your calculations.

$\Delta p =$ ___________kg m/s

9. Use the **area routine (statistics)** in the software to find the area under the force—time graph–the impulse. (The area under a curve is the same as the *integral* of force vs. time.)

$J =$ ____________N • s

10. **Print** out one set of graphs for your group report.

Question 2-5: Did the calculated change in momentum of the cart equal the measured impulse applied to it by the spring during the nearly elastic collision? If they differ by more than 15%, explain what may have happened.

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Activity 2-3: A Larger Momentum Change

Suppose a more massive cart collided elastically with the target. What would the impulse be? You can add mass to the cart and find out.

Prediction 2-2: If the cart had twice the mass and collided with the target elastically moving at the same velocity as in Activity 2-2, how large do you think the impulse would be? The same as before? Larger? Smaller? Why?

1. Test your prediction. Add ½-kg masses to your cart to make the total mass approximately twice as large. It does not need to be exactly twice.

New mass of cart:___________kg

2. You can use the same experiment file **Impulse and Momentum L7.2-2**, as in Activity 2-2. **Zero** the force probe, and then collide the cart with the target again. Try several times until you get the initial velocity about the same as in Activity 2-2. Find the average velocities as in Activity 2-2 and calculate the change in momentum.

Average velocity toward the target: __________m/s

Average velocity away from the target: __________m/s

Δp = ____________kg • m/s

3. Find the impulse as in **Activity 2-2. Print** out one set of graphs for your group report.

J =____________N • s

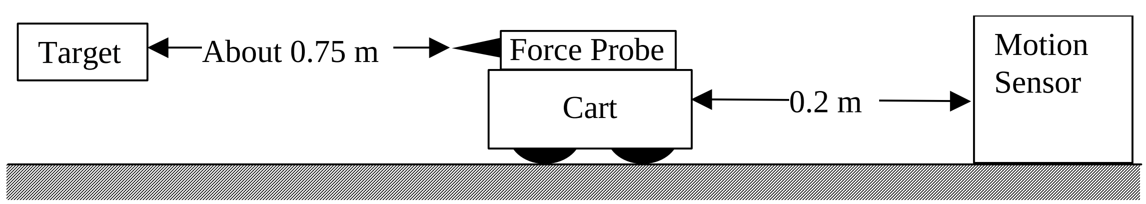
Question 2-6: Discuss how well this agrees with your prediction.

Question 2-7: Were the impulse and change in momentum equal to each other? If different by more than 15%, explain why you think this happened.

Activity 2-4: Impulse—Momentum Theorem in an Inelastic Collision

It is also possible to examine the impulse—momentum theorem in a collision where the cart sticks to the target and comes to rest after the collision. This can be done by replacing the spring with a dart tip at the end of the force probe.

1. Leave the extra mass on your cart so that its mass is the same as in Activity 2-3. Attach the dart tip to the force probe. The rest of the setup is as in Activity 2-2.



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Prediction 2-3: Now when the cart hits the target, it will come to rest. What do you predict about the impulse? Will it be the same, larger, or smaller than in the nearly elastic collision? What do you predict now about the impulse and momentum? Will they equal each other, or will one be larger than the other?

2. You can use the same experiment file, **Impulse and Momentum L7.2-2,** as in Activity 2-2.

3. **Zero** the force probe, and then collide the cart with the target. Try several times until you get the initial velocity about the same as in Activity 2-2.

4. Find the average velocity, as in Activity 2-2, and calculate the change in momentum.

Average velocity toward the target: ___________m/s

Δp = __________kg • m/s

5. Find the impulse as in Activity 2-2. **Print** out one set of graphs for your group report.

J = ___________N • s

Question 2-8: Were the impulse and change in momentum equal to each other for the inelastic collision? Explain why you think the results came out the way they did.

INVESTIGATION 3: FORCES BETWEEN INTERACTING OBJECTS

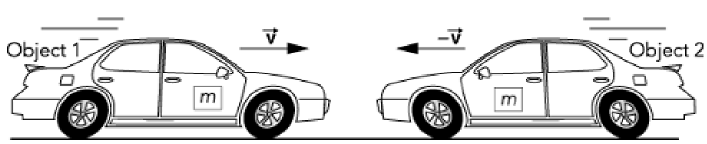
There are many situations where objects interact with each other, for example, during collisions. In this investigation we want to compare the forces exerted by the objects on each other. In a collision, both objects might have the same mass and be moving at the same speed, or one object might be much more massive, or they might be moving at very different speeds. What factors might determine the forces the objects exert on each other? Is there some general law that relates these forces?

ACTIVITY 3-1: COLLISION INTERACTION FORCES

What can we say about the forces two objects exert on each other during a collision?

Prediction 3-1: Suppose two objects have the same mass and are moving toward each other at the same speed so that $m_1 = m_2$ and $\mathbf{v_1} = -\mathbf{v_2}$ (same speed, opposite direction).

$$m_1 = m_2 \quad \text{and} \quad \mathbf{v_1} = -\mathbf{v_2}$$



Predict the relative magnitudes of the forces between object 1 and object 2 during the collision. Place a check next to your prediction.

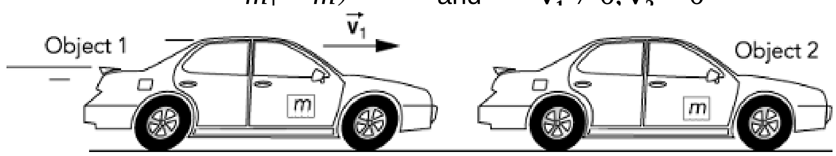
______Object 1 exerts a larger force on object 2.

______The objects exert the same size force on each other.

______Object 2 exerts a larger force on object 1.

Prediction 3-2: Suppose the masses of two objects are the same and that object 1 is moving toward object 2, but object 2 is at rest.

$m_1 = m_2$ and $\mathbf{v}_1 \neq 0, \mathbf{v}_2 = 0$



Predict the relative magnitudes of the forces between object 1 and object 2 during the collision.

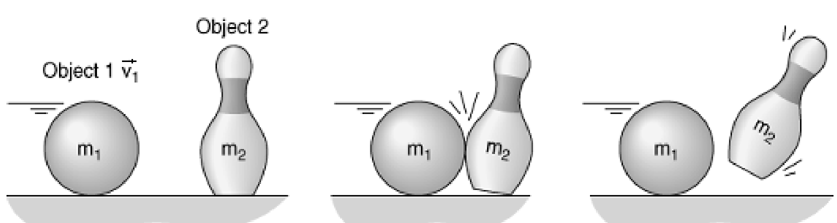
______Object 1 exerts a larger force on object 2.

______The objects exert the same size force on each other.

______Object 2 exerts a larger force on object 1.

Prediction 3-3: Suppose the mass of object 1 is greater than that of object 2 and that it is moving toward object 2, which is at rest.

$m_1 > m_2$ and $\mathbf{v}_1 \neq 0, \mathbf{v}_2 = 0$



Predict the relative magnitudes of the forces between object 1 and object 2 during the collision.

______Object 1 exerts a larger force on object 2.

______The objects exert the same size force on each other.

______Object 2 exerts a larger force on object 1.

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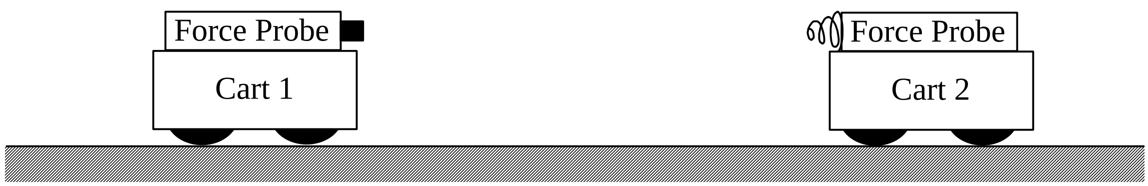
To test the predictions you made you can study *gentle* collisions between two force probes attached to carts. You can add masses to one of the carts so it has significantly more mass than the other. To make these observations of interactions you will need the following equipment:

- two force probes; one with a rubber stopper and one with a spring
- two PASCO motion carts
- masses to place on one of the carts to double and triple its mass
- 2-m motion track
- level

1. Set up the apparatus as shown in the following diagram. The force probes should be securely fastened to the carts. The rubber stopper and spring should be *carefully aligned* so that they will collide head-on with each other.



2. Open the experiment file called **Collisions L7.3-1**. The software will be set up to measure the forces applied to each probe with a very fast data collection rate of 2000 points per second. (This allows you to see all of the details of the collision which takes place in a very short time interval.) The software will also be set up to be **triggered**, so that data collection will not start until the carts actually collide. Make sure you keep the force well below the 50 N maximum.

3. The sign of the Force in probe 1 should be reversed, because a push on it is negative (toward the left). [This should already be done for you in the experiment file.]

4. Use the two carts to explore various situations that correspond to the predictions you made about interaction forces. Your goal is to find out under what circumstances one cart exerts more force on the other.

Try collisions (a)—(c) listed below.

Be sure to **zero** the force probes before each collision. Also be sure that the forces during the collisions do not exceed 50 N.

5. **Print** out one set of graphs for your group report. Be sure to label your graphs.

6. For each collision use the **area routine** to find the values of the impulses exerted by each cart on the other (i.e., the areas under the force-time graphs). Record these values in the spaces below and carefully describe what you did and what you observed.

(a) Two carts of the same mass moving toward each other at about the same speed.

(b) Two carts of the same mass, one at rest and the other moving toward it.

(c) One cart twice or three times as massive as the other, moving toward the other cart, which is at rest.

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Question 3-1: Did your observations agree with your predictions? What can you conclude about forces of interaction during collisions?

Question 3-2: How does the impulse due to cart 1 acting on cart 2 compare to the impulse of cart 2 acting on cart 1 in each collision? Are they the same in magnitude or different? Do they have the same sign or different signs?

INVESTIGATION 4: NEWTON'S LAWS AND MOMENTUM CONSERVATION

Your previous work should have shown that interaction forces between two objects are equal in magnitude and opposite in sign (direction) on a moment by moment basis for all the interactions you might have studied. This is a testimonial to the seemingly universal applicability of *Newton's third law* to interactions between objects.

As a consequence of the forces being equal and opposite at each moment, you should have seen that the impulses of the two forces were always equal in magnitude and opposite in direction. This observation, along with the impulse—momentum theorem, is the basis for the derivation of the *conservation of momentum law*. (The impulse—momentum theorem is really equivalent to *Newton's second law* since it can be derived mathematically from the *second law*.) The argument is that the impulse $\mathbf{J}_1$ acting on cart 1 during the collision equals the change in momentum of cart 1, and the impulse $\mathbf{J}_2$ acting on cart 2 during the collision equals the change in momentum of cart 2:

$$\mathbf{J}_1 = \Delta\mathbf{p}_1 \qquad \mathbf{J}_2 = \Delta\mathbf{p}_2$$

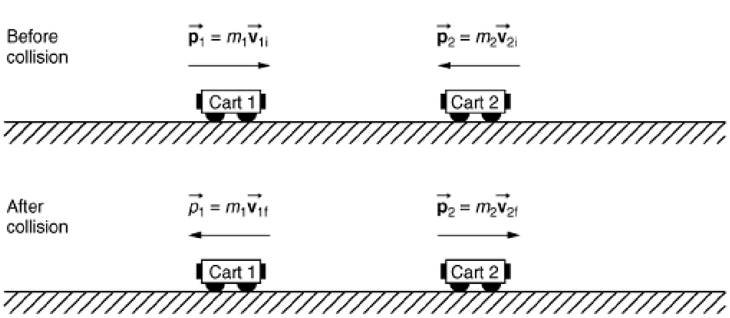
But as you have seen, if the only forces acting on the carts are the interaction forces between them, then $\mathbf{J}_1 = -\mathbf{J}_2$. Thus, by simple algebra

$$\Delta\mathbf{p}_1 = -\Delta\mathbf{p}_2 \quad \text{or} \quad \Delta\mathbf{p}_2 + \Delta\mathbf{p}_1 = 0$$

that is, there is no change in the total momentum $\mathbf{p}_1 + \mathbf{p}_2$ of the system (the two carts).

If the momenta of the two carts before (initial–subscript i) and after (final–subscript f) the collision are represented in the diagrams below, then:

$$\mathbf{p}_f = \mathbf{p}_i$$



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where

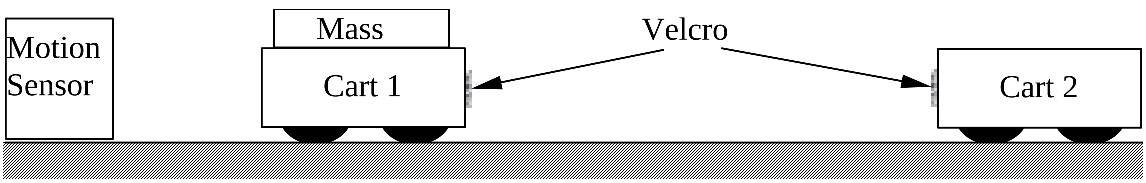
$$\mathbf{p_i} = m_1\mathbf{v_{1i}} + m_2\mathbf{v_{2i}} \qquad \mathbf{p_f} = m_1\mathbf{v_{1f}} + m_2\mathbf{v_{2f}}$$

In the next activity you will examine whether momentum is conserved in a simple *inelastic* collision between two carts of unequal mass. You will need the following:

- motion detector
- two PASCO motion carts (with Velcro ends)
- masses to place on one of the carts to double its mass
- 2-m motion track
- level

Activity 4-1: Conservation of Momentum in an Inelastic Collision

1. Set up the carts, ramp, and motion detector as shown below. If not already removed, **remove the force probes**. Make sure that the ends of the carts will stick together with the Velcro after the collision. Add masses to cart 1 so that it is about twice as massive as cart 2.



2. Measure the masses of the two carts.

m_1 = ____________kg m_2 = ____________kg

Prediction 4-1: You are going to give the more massive cart 1 a push and collide it with cart 2, which is initially at rest. The carts will stick together after the collision. Suppose that you measure the *total* momentum of cart 1 and cart 2 before and after the collision. How do you think that the total momentum after the collision will compare to the total momentum before the collision? Explain the basis for your prediction.

3. Test your prediction. Open the experiment file called **Inelastic Collision L7.4-1.** Begin with cart 1 at least 0.20 m from the motion detector. **Begin graphing**, and when you hear the clicks of the motion detector, give cart 1 a brisk push toward cart 2 and release it. *Be sure that the motion detector does not see your hand*.

4. Repeat until you get a good run when the carts stick and move together after the collision. **Print** one copy of the graph.

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5. Use the **statistics features** of the software to measure the velocity of cart 1 just before the collision and the velocity of the two carts together just after the collision. (You will want to find the average velocities over short time intervals just before and just after–but not during–the collision.)

v_{1i} = ___________m/s $v_{1f} = v_{2f}$ = ___________m/s

6. Calculate the total momentum of carts 1 and 2 before the collision and after the collision. Show your calculations below.

p_i = ______kg·m/s p_f = ______kg·m/s

Question 4-1: Was momentum conserved during the collision? Did your results agree with your prediction? Explain.

Question 4-2: Now look back at Activity 2-4 (where the cart collided in inelasticity with a fixed target). The cart clearly had momentum before the collision, but was at rest afterwards. Explain how this can be in light of Newton's third law.

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