Newton's Law of Cooling

A container of hot water at temperature, T, placed in a room of lower temperature T_{room}, will result in an exchange of heat from the hot water to the room. The water will eventually cool to the same temperature as the room. You observe this cooling process every time you wait for a hot drink to cool. In this experiment you will examine the cooling of hot water, with the goal of creating a model that describes the process. You can also predict the time it takes for the hot water to cool to room temperature.

Isaac Newton modeled the cooling process by assuming that the rate at which thermal energy moved from one body to another is proportional (by a constant k) to the difference in temperature between the two bodies, T_{diff}. In the case of a sample of water cooling in room temperature air

$$\text{cooling rate} = -kT_{diff}$$

From this simple assumption he showed that the temperature change is exponential in time and can be predicted by

$$T_{diff} = T_0 e^{-kt}$$

where T_0 is the initial temperature difference. Exponential changes are common in science. Systems in which a rate of change is proportional to the changing quantity show exponential behavior.

To complete this experiment in a short time, you will use a small quantity of hot water, at a temperature about 30°C above room temperature. A temperature probe connected to a CBL and calculator will record the water's temperature as it cools.

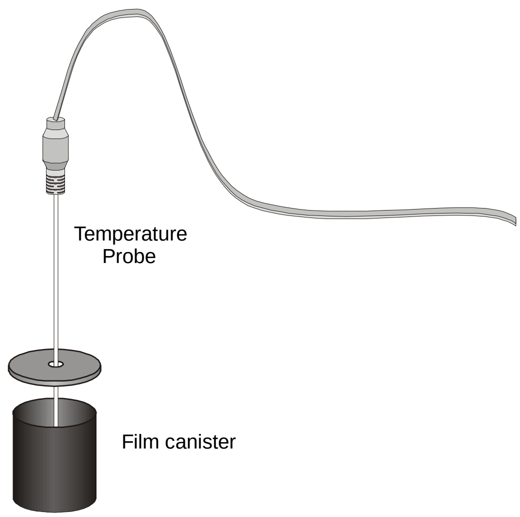

Figure 1

Objectives

- Use a Temperature Probe to record the cooling process of hot water.
- Test Newton's law of cooling using your collected water temperature data.
- Use Newton's law of cooling to predict the temperature of cooling water at any time.

Materials

TI-82, 83, 86, 89, 92, or 92 Plus
CBL System
PHYSICS program loaded in calculator
Vernier or TI Temperature Probe
35-mm film canister with top
hot water

Procedure

1. Connect a Temperature Probe to the CH1 input on the CBL. Use the black link cable to connect the CBL unit to the calculator. Firmly press in the cable ends.

2. Turn on the CBL unit and the calculator. Start the PHYSICS program and proceed to the MAIN MENU.

3. Set up the calculator and CBL for the Temperature Probe.
   - Select SET UP PROBES from the MAIN MENU.
   - Select ONE as the number of probes.
   - Select TEMPERATURE (TI Temperature Probe or Vernier Direct-Connect Temperature Probe), VERN STD TEMP (Vernier Standard Temperature Probe) or VERN QIK TEMP (Vernier Quick-Response Temperature Probe) from the SELECT PROBE menu.
   - Confirm that the probe is attached to CH1, and press ENTER to continue.
   - (Skip if using TI probe.) Select USE STORED from the CALIBRATION menu.

4. Determine room temperature. To do this, hold the probe in the air with nothing touching the probe tip.
   - Select COLLECT DATA from the MAIN MENU.
   - Select MONITOR INPUT from the DATA COLLECTION menu.
   - Observe the temperature reading on the calculator. When it is stable, record the value in your Data Table as the room temperature.
   - Press + to leave the monitor mode.

5. Push the Temperature Probe through the hole in the cap so that the end of the probe will be submerged in the water when the cap is on the canister. Do not let the end of the probe rest against the bottom of the canister.

6. Obtain some water at about 55°C. You should be able to get water this hot from a hot water faucet. If necessary, heat water to this temperature.

7. Carefully fill the canister about three-fourths full with the hot water. Place the cap containing the probe onto the canister and press until it is sealed with a click.

8. Set up the calculator and CBL for data collection.
   - Select TIME GRAPH from the DATA COLLECTION menu.
   - Enter "15" as the time between samples in seconds.
   - Enter "80" as the number of samples.
   - Press ENTER, and select USE TIME SETUP to continue. If you want to change the sample time or sample number, select MODIFY SETUP instead.
   - Select NON-LIVE DISPL from the TIME GRAPH menu.

9. Wait about 10 seconds for the temperature probe to reach the temperature of the water. Collect your cooling data:
   - Press ENTER to begin data collection. Data will be collected for 20 minutes.
   - After the CBL shows DONE on its screen, the calculator will have turned itself off. Turn it back on and press ENTER.
   - Select RETRIEVE DATA from the MAIN MENU, and press ENTER.
   - Press ENTER to see your graph. Sketch or print your graph.
   - Press ENTER and select NO to return to the MAIN MENU.

Data Table

Room Temperature (°C)	

A	
B	
k	

Analysis

1. Since the model for Newton's law of cooling use the difference between the sample temperature and room temperature, you must subtract the room temperature from the measured temperature before comparing data to the model. To do this:
   - Select QUIT from the MAIN MENU.
   - TI-89/92/92 Plus only: Press F5 to return to the home screen.
   - Press "L2 − (room temperature) STO▶ L2 " where (room temperature) is the numerical value you determined in Step 4 of the Procedure. This step replaces the measured water temperatures with the temperature above room temperature.
   - Restart the PHYSICS program and proceed to the MAIN MENU.

2. Fit the exponential function $y = A\ e^{-B*x}$ to your temperature difference *vs.* time data.
   - Select ANALYZE from the MAIN MENU.
   - Select CURVE FIT from the ANALYZE MENU.
   - Select EXPONENT L1, L2 from the CURVE FIT menu.
   - Record the fit parameters A and B in your Data Table.
   - Press ENTER to see a graph of your data with the fitted function.

3. Newton's law of cooling was given above as

$$T_{diff} = T_0\, e^{-kt}$$

Since you subtracted room temperature from the measured water temperatures, your graph shows the difference T_{diff} directly. The calculator fits the function $y = A\ e^{-B*x}$ to your data. The parameter k then corresponds to the value of B. Enter your value for k in the Data Table.

4. When $t = 0$, what is the value of e^{-kt}?

5. When t is very large, what is the value of temperature difference? What is the temperature of the water at this time?

6. What could you do to your experimental apparatus to decrease the value of k in another run? What quantity does k measure?

7. Use your equation to calculate the temperature after 800 seconds. Compare your calculated value with the actual data value.

8. Use your equation to predict the time it takes the water to reach a temperature 1°C above room temperature.

9. If the starting temperature difference is cut in half, does it take half as long to get to 1°C above room temperature?

Extensions

1. Take data for a longer period of time so that the water cools to nearly room temperature. This may take more than 30 minutes. Does the exponential model still fit the data?

2. A coffee drinker is faced with the following dilemma. She is not going to drink her hot coffee with cream for ten minutes, but wants it to still be as hot as possible. Is it better to immediately add the room-temperature cream, stir the coffee, and let it sit for ten minutes, or is it better to let the coffee sit for ten minutes and *then* add and stir in the cream? Which results in a higher temperature after ten minutes? Use your Temperature Probe to examine this dilemma. Explain your results in terms of the assumptions Newton made about cooling.

3. Use the Temperature Probe to experiment with coffee cups made of different material. Does a drink cool faster in a ceramic cup than in a Styrofoam cup? What variables must you hold constant in order to guarantee that the difference in the data is due to the cup? What part of the exponential equation is related to the cup?

4. The mathematical model for the cooling of a liquid can also be used to explain other phenomena in nature. For example, radioactivity and RC circuits behave in a similar fashion. Find other phenomena that are modeled by exponential functions. If possible make a measurement of the phenomenon in your physics lab.