

Name \_\_\_\_\_ Date \_\_\_\_\_ Partners \_\_\_\_\_

## Projectile Motion

### OVERVIEW

We learn in our study of kinematics that two-dimensional motion is a straightforward extension of one-dimensional motion. Projectile motion under the influence of gravity is a subject with which we are all familiar. We learn to shoot basketballs in an arc to swish through the basket or to bounce off the backboard. We learn how to lob in volleyball and tennis. These are examples of projectile motion. The force of gravity acts in the vertical direction, and air resistance acts in both the horizontal and vertical directions, but we often neglect air resistance for small objects.

In this experiment we will explore how the motion depends upon the body's initial velocity and elevation angle. Consider a body with an initial speed  $v_0$  at angle  $\alpha$  with respect to the horizontal axis. We analyze the body's motion in two independent coordinates  $x$  (horizontal) and  $y$  (vertical). [As we are free to choose this origin of our coordinate system, we choose  $x=0$  and  $y=0$  when  $t=0$  so as to simplify our calculations.] Fig. 1 then shows that the components of the velocity vector along the  $x$ - and  $y$ -axes are respectively:

$$v_{0x} = v_0 \cos \alpha \quad \text{and} \quad v_{0y} = v_0 \sin \alpha \quad (1)$$

If we neglect air resistance, the only force affecting the motion of the object is gravity, which near the Earth's surface acts purely in the vertical direction ( $\mathbf{F}_g = m\mathbf{g} = -mg\hat{y}$ ). There is no force at all in the horizontal direction.

Since there is no horizontally applied force, there is no acceleration in the horizontal direction; hence the  $x$ -direction of the velocity will remain unchanged forever. The horizontal position of the body is then described by the expression for constant velocity:

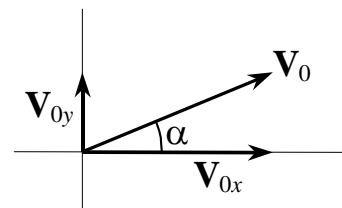


Fig. 1.

$$x(t) = v_{0x}t = (v_0 \cos \alpha)t \quad (2)$$

The force in the  $y$  (vertical) direction is gravitation ( $F_y = -mg$ ). Since  $\mathbf{F} = m\mathbf{a}$ ,  $a_y = -g$ . Integrating this with respect to time yields the vertical component of the velocity:

$$v_y(t) = v_{0y} - gt = v_0 \sin \alpha - gt. \quad (3)$$

The vertical component of the position can be obtained by integrating Eq. (3) with respect to time  $t$ , yielding the result:

$$y(t) = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin \alpha)t - \frac{1}{2}gt^2. \quad (4)$$

We can use these equations to determine the actual trajectory of the body in terms of the  $x$  and  $y$  variables, with no explicit reference to the time  $t$ . The equation has a parabolic form

$$y = C_1x - C_2x^2 \quad (5)$$

where  $C_1 = \tan(\alpha)$  and  $C_2 = g / 2v_0^2 \cos^2(\alpha)$ . We are interested in the **range**  $R$ , the horizontal distance that the projectile travels on level ground. We can get the range from Eq. (5) by noting that  $y$  starts at zero when  $x = 0$  and is once again zero when  $x = R$ . We get

$$R = \frac{v_0^2 \sin(2\alpha)}{g} \quad (6)$$

Although we won't measure them in this experiment, we can similarly show that the time of flight ( $T$ ) is given by

$$T = \frac{2v_0 \sin(\alpha)}{g} \quad (7)$$

and the maximum height (reached when  $t = T/2$ ) is given by

$$h = \frac{v_0^2 \sin^2(\alpha)}{2g} \quad (8)$$

### Before coming to class

**Read** Appendix B: Graphical Analysis; Appendix C: Timers; Appendix D: The Accuracy of Measurements, and Appendix E: Vectors.

## INVESTIGATION 1: MEASUREMENT OF RANGE

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You will need the following materials:

- projectile motion apparatus
- optical bench
- meter stick
- plastic ruler
- level
- landing pad
- digital photogate timer
- masking tape
- 3 m tape measure
- spark (pressure-sensitive) paper

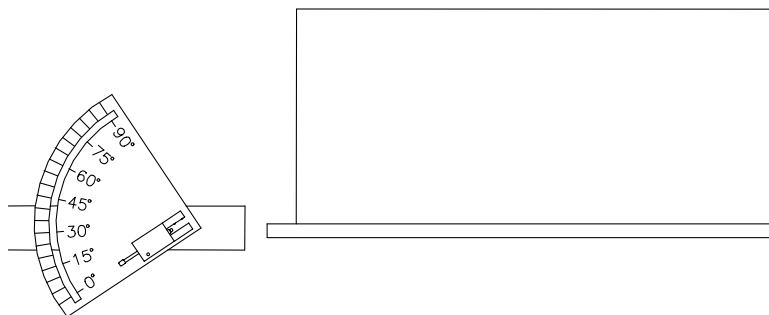


Fig. 2. Projectile motion apparatus and landing pad.

1. Before starting the measurements, familiarize yourself with the apparatus. The central feature of the setup is a “spring-loaded gun” mounted on a plate marked in degrees, as shown in Fig. 2. The plate can be rotated around a horizontal axis to set the initial direction of motion: a desired angle can be set by loosening the knurled screw attached to the plate, aligning the required angle mark with the horizontal mark on the stand and then tightening the screw. The initial speed of the projectile (a steel ball) can be set to three different values by drawing back the spring-loaded firing pin until it is locked in one of the three preset positions. Notice the cable-release trigger and make sure that its plunger is fully withdrawn (this is achieved by pressing on the small ring near the end of the trigger cable), then try drawing back the firing pin and listen to the three distinct “clicks” corresponding to the three positions. Unfortunately, sometimes the cable-release becomes disconnected, something you want to avoid, because it is difficult to reconnect it! Press the trigger’s plunger and the pin will “fire”. Now you can practice launching the projectile: try different angles and different velocities to get a feeling of which

angular range will land the projectile onto the table. When doing your measurements, record the landing spot of the ball on the table by means of a pressure sensitive paper taped to the board. **Do not tape the paper in position until you are ready to start your measurements, because it is expensive, and we can't afford to waste any.**

- Now you will want to verify that the apparatus is level and that the landing table is at the same height as the pivot point (which is where we believe the ball also leaves the spring). First, place the level on the back of the gun apparatus and, if needed, use the two screws holding up the apparatus to level the gun apparatus. Use a meter stick and the level to make sure that the table top surface is at the same height as the pivot point of the angle mark (the timing apparatus can easily be pulled out to see where the pivot point actually is) **and** that the table is level. Ask your instructor for assistance if this procedure is not clear.

### Activity 1-1: Determining the Initial Speed

- As the final preparation, we want to measure the initial speed of the projectile. As it leaves the gun, the projectile crosses the light beam of two “light bulbs” (actually light emitting diodes or LED’s), shining onto two photocells, which are connected to a timer unit. When in *pulse* mode, the timer will start counting when the first light beam is interrupted, and it will keep counting until the second beam is obscured. Make sure the timer is set to 0.1 ms resolution and set the angle to 0°. Withdraw the firing pin to one of the three positions and place the ball into the gun. If the timer is running, press the RESET button. Launch the ball and use a Styrofoam cup to catch the ball. The timer will record the time it took the projectile to go from the first to the second light. Knowing the distance  $d$  between the two (it is marked on the apparatus), you can determine the average speed of the ball as it travels between the lights. Take one half of the least significant digit in  $d$  as your estimate of your uncertainty in  $d$ ,  $\sigma_d$ .

$d$  \_\_\_\_\_  $\sigma_d$  \_\_\_\_\_

**Question 1-1:** Can you think of a better way to measure the initial speed? Discuss among your group why we use this procedure (0° angle) to determine the initial speed. Write your conclusions here.

Measure the time ( $t$ ) for at least four trials for each “click” setting . Enter your data into Table 1-1. Use these data and the position between the photocells to determine the speed.

For each setting calculate the average time, ( $\bar{t}$ ) and the standard deviation,  $\sigma_t$ . Let your uncertainty in  $\bar{t}$  ( $\sigma_{\bar{t}}$ ), be the **larger** of the “standard error in the mean” or one-half of the least significant digit (0.05 ms) calculate the initial speed,  $v_0 = d/\bar{t}$ , and its uncertainty,  $\sigma_{v_0}$ , based on your estimates of  $\sigma_d$  and  $\sigma_{\bar{t}}$ .

**Table 1-1 Initial Speed**

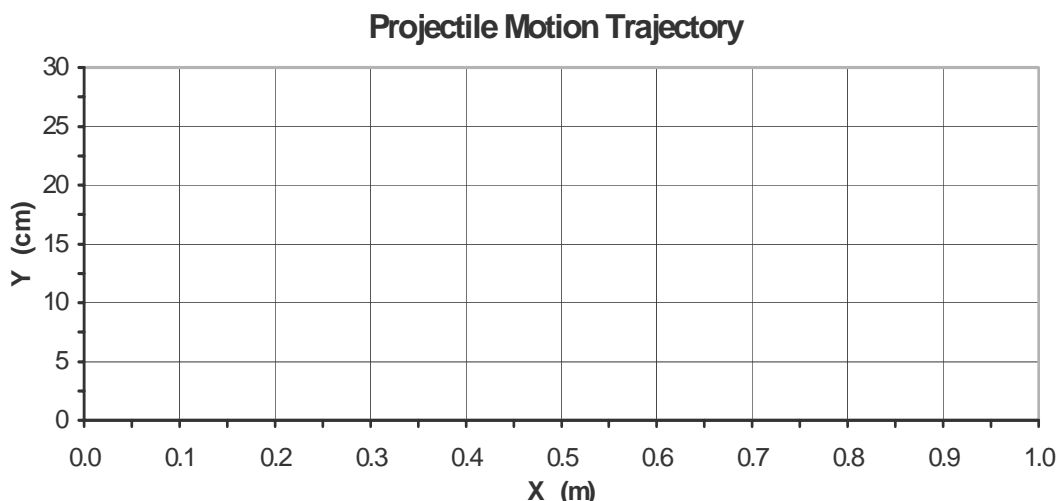
Position Click	$t$ (ms)				$\bar{t}$ (ms)	$\sigma_t$ (ms)	$\sigma_{\bar{t}}$ (ms)	$v_0$ (m/s)	$\sigma_{v_0}$ (m/s)
	Trial 1	Trial 2	Trial 3	Trial 4					
1									
2									
3									

### Activity 1-2: Range as a Function of Initial Angle

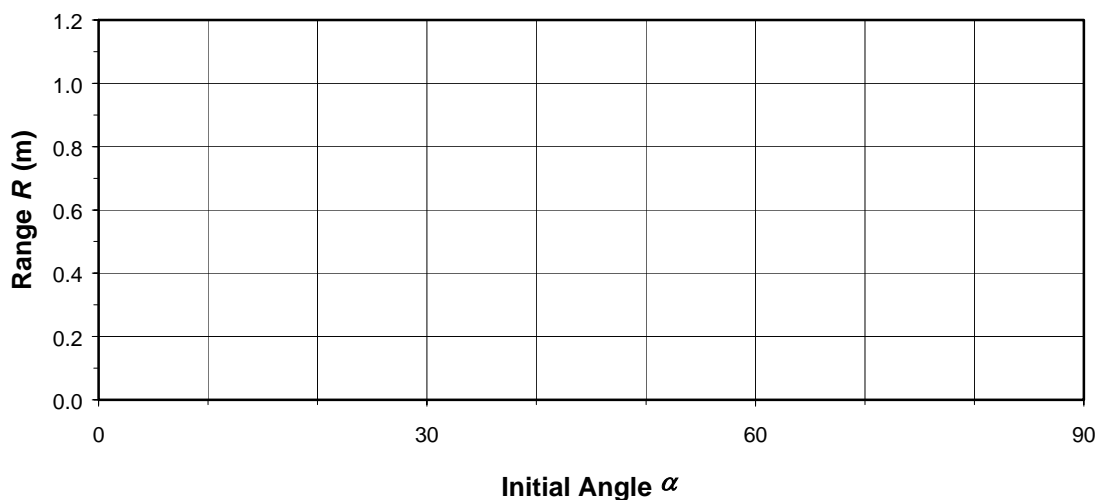
For this activity you will choose **one** of the three values of the launch speed and execute a set of launches for at least five different values of the launch angle, say  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $75^\circ$ . We suggest that you take data with highest initial speed.

**Question 1-2:** Write down two reasons why it might be best to use the highest speed to take these data.

**Question 1-3:** Fix your coordinate system such that the  $x$  and  $y$  coordinates are both zero when the ball passes through the pivot and adjust the top of the landing board to that level ( $y = 0$ ). The ball then starts at  $y = 0$  and reaches  $y = 0$  again when landing on the board. The range will simply be the  $x$  coordinate where it hits. Draw your predictions for the trajectory on the following graph for each of your chosen angles.



**Question 1-4:** Draw your theoretical predictions for the range  $R$  as a function of the initial angle  $\alpha$  on the following graph.



**Question 1-5:** If you wanted to plot the range  $R$  versus a quantity that depends on angle (that is, you are going to vary the angle) and **that would result in a straight line** for all five initial angles, what would you choose for the independent function variable (function variable plotted along the horizontal axis)? Explain.

- For each launch, the range of the projectile will be recorded by a dot on the pressure-sensitive paper. You will notice that even if you keep the initial speed and inclination fixed, the landing positions will be spread over several millimeters. The actual range you will use for plotting your data will be the average of the measured points. To avoid confusions due to spurious marks on the paper caused by bounces of the projectile, it is advisable, first, to label each mark as the ball makes it and, second, to perform the measurements with the longest ranges first. You might, for example, circle each mark to distinguish it from the newer ones. **Do not take data until instructed to do so!**

**Question 1-6:** Can you determine from the formula you derived earlier which angle corresponds to the longest range? Explain. That should be your first angle.

- Record your initial speed:

$v_0$  \_\_\_\_\_

Now take your data for this same  $v_0$  and insert your values into the following table:

**Table 1-2**  
**Range (in cm) for each projectile launch**

Trial	Angle				
	15°	30°	45°	60°	75°
1					
2					
3					
4					
5					
$\bar{R}$					
$\sigma_R$					
$\sigma_{\bar{R}}$					

- For each angle, calculate the average range,  $\bar{R}$ , and the standard deviation,  $\sigma_R$ . Let your uncertainty in  $\bar{R}$ ,  $\sigma_{\bar{R}}$ , be the larger of the “standard error in the mean” or one half of the least significant digit.

**Question 1-7:** For which angle is the range the largest? Does this agree with your prediction? Explain why or why not.

**Question 1-8:** Does it appear from your data that the ranges for any of the angles are the same (within the probable errors)? If so, for which sets of data are this true? Explain your results.

**Question 1-9:** From the spread of your experimental data for each angle above, does it appear that you have a statistical error larger than 5%? Is this reasonable? Explain.

5. Now that you have your data, compare them with the theoretical predictions. You want to plot your data using the function you decided in Question 1-5 that produces a **straight line**. For each angular setting, use the expression you derived earlier to compute the expected projectile range as some function of angle. Produce the line for the theoretical predictions (see  $R = [v_0^2 \sin(2\alpha)] / g$ ) and use your measured values of  $v_0$  and the known value of  $g$ . Calculate  $R$  for each angle and put the result into Excel. Then you can connect the data points with a line, but choose in Excel to not display the calculated data points. Show only the theoretical line and the experimental data versus a suitable parameter that depends on the initial angle  $\alpha$  (see Question 1-5) using Excel. **The result should be a linear plot. Print** out and include the graph with your report.
6. By hand, add “error bars” to the measured  $\bar{R}$  data on your graph. These should be short lines of length  $\sigma_{\bar{R}}$  that extend above and below the data point. Often these lines have “T” like ends.

**Question 1-10:** How well do your measurements agree with the theory? Are the deviations consistent to within the accuracy you expect of your measurements? Explain why or why not. Discuss possible sources of systematic errors that may be present.

### Activity 1-3: Range as a Function of Initial Speed

1. Now you want to choose one suitable angle and measure the range  $R$  for each of the three initial speeds that you determined earlier. Write down your chosen angle:

Launch angle \_\_\_\_\_

**Question 1-11:** What criteria did you use for choosing your launch angle? Would it better to use a shallow or high angle? Does it matter for any reason?

2. Take at least four readings for each of the three initial speeds. Insert your results into Table 1-3.

**Table 1-3**  
**Range (in cm) as a Function of Initial Speed**

Trial	Initial Velocity		
	1:	2:	3:
1			
2			
3			
4			
5			
$\bar{R}$			
$\sigma_R$			
$\sigma_{\bar{R}}$			

**Question 1-12:** It would be instructive to plot your results again on a graph that produces a linear line. Look at the equation for range  $R$  and determine what you should plot the range versus. State your choice and give an explanation:

3. Plot your experimental values for the range on a graph versus the parameter you chose in the previous question and compare them as before with a line drawn by the theoretical values predicted by the expression you derived. Use Excel or MathCad. **Print** out your graph, add error bars and attach the graph to your report.

**Question 1-13:** Do your data agree reasonably well with your theoretical predictions? How many of your data points have error bars that overlap your theoretical calculation? Is this what you expect? Do you believe your errors are statistical or systematic? Explain.