

MOTION ANALYSIS

**Prepared for the Televised Course
"Physical Science: The Threshold
of Scientific Literacy"**

**Presented by
The University of Virginia**

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This manual accompanies a video titled “Motion Analysis” that demonstrates each of the experiments listed. Each experiment is shown initially in real time, then in a series of slower, single frame shots. Students are encouraged to cover the television with blank overhead transparency sheets and record positions of the various objects in motion during the single frame shots. Using this method, students can learn about different types of motion without the need for lots of expensive equipment. This manual contains sample data from each of the experiments and is designed to guide teachers through the exercises. Normally students should obtain their own data from analysis of the video. These data and analyzes here are intended for teacher use only. For a copy of the “Motion Analysis” video, please contact the demonstration lab at the University of Virginia Department of Physics at (434) 924-6800.

Determining Air Track Calibration Factor

In all of the experiments in this manual, a distance measured on the television screen will not be equivalent to the distance the object has traveled. In other words, 1 meter in the lab will not be the same as 1 meter measured on the television screen (unless you have an enormous television). In order for our measurements to make sense, we know how to convert distances measured on the screen to their actual distances in the lab. To accomplish this we need to measure something on the screen in each experiment for which we know the actual length. For some of the experiments (ie Free Fall), it is clear what the distance is (two large white marks, one meter apart). In the air track experiments, there are 10 visible posts that support the air track. Measure the distance between any two of the posts and use the chart below to make the conversion.

Post #	Position (m)	Difference (m)
0	0	0
1	0.305	0.305
2	0.660	0.305
3	0.965	0.305
4	1.270	0.305
5	1.575	0.305
6	1.880	0.305
7	2.185	0.305
8	2.490	0.305
9	2.795	0.305

The posts are numbered from the left side of the screen, with the left end post labeled here as 0. As you see, the separations are very uniform except for the one between post 0 and post 1 at the left end.

For example, if you measured the distance between posts 2 and 3 to be 6 cm on the television the calibration factor would be found as follows:

$$\frac{0.305 \text{ m (lab)}}{0.06 \text{ m (tv)}} = 5.08 \text{ m in the lab for every 1 m measured on the television}$$

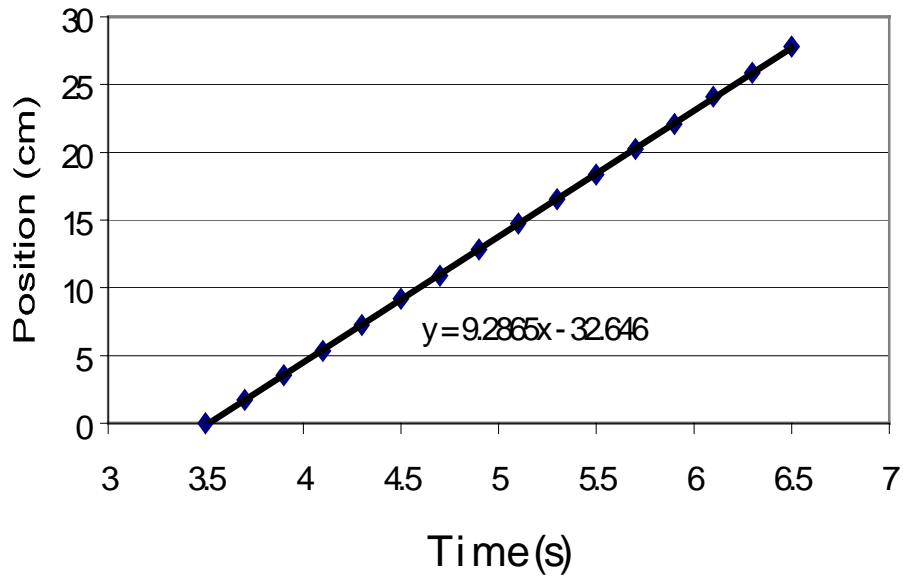
screen.

Remember that this calibration factor is not only different for every television, but may also vary vertically and horizontally on the same television.

#1. Car on Air Track: Constant Velocity

In this demonstration we look at the uniform motion of a body that has constant velocity. The plot of position versus time should be linear as the equation of motion is $x = x_0 + v_0 t$ (where we take $x_0 = 0$).

<u>Time (s)</u>	<u>Position (cm)</u>
3.5	0.00
3.7	1.73
3.9	3.55
4.1	5.35
4.3	7.25
4.5	9.20
4.7	10.90
4.9	12.85
5.1	14.75
5.3	16.55
5.5	18.35
5.7	20.25
5.9	22.10
6.1	24.10
6.3	25.85
6.5	27.80



Using the first and last points of the graph we can estimate the average velocity to be:

$$v_0 = \frac{27.8 \text{ cm} - 0 \text{ cm}}{6.5 \text{ s} - 3.52 \text{ s}} = 9.33 \text{ cm/s}$$

Using a linear best-fit trend line, the slope yields an average velocity of 9.29 cm/s.

#2. Propeller Driven Car on Air Track: Uniform Acceleration

In this demonstration we measure the motion of a body under a constant force. A propeller mounted on top drives the air car along a horizontal track. We can map out the position versus time to see the parabolic nature of the curve. This reflects the constant acceleration of the body. To determine the value of the acceleration, we plot velocity versus time by calculating the average velocity during each time interval using the relation:

$$v_{\text{average}} = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

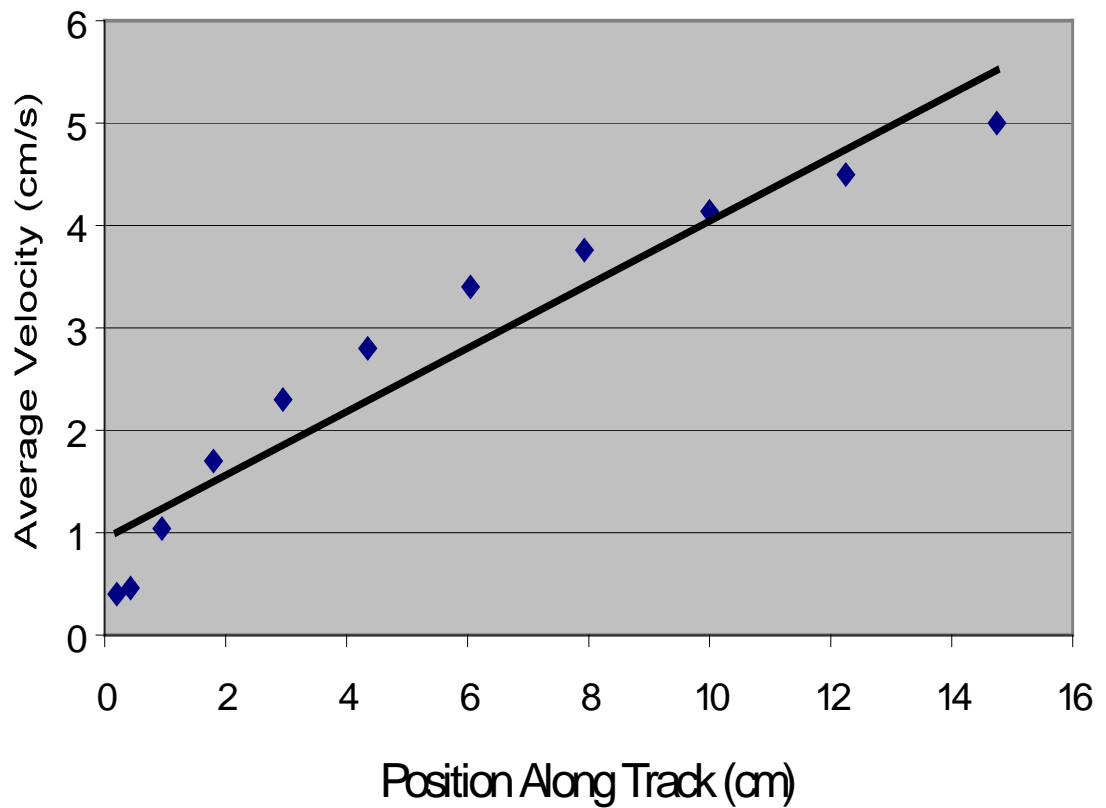
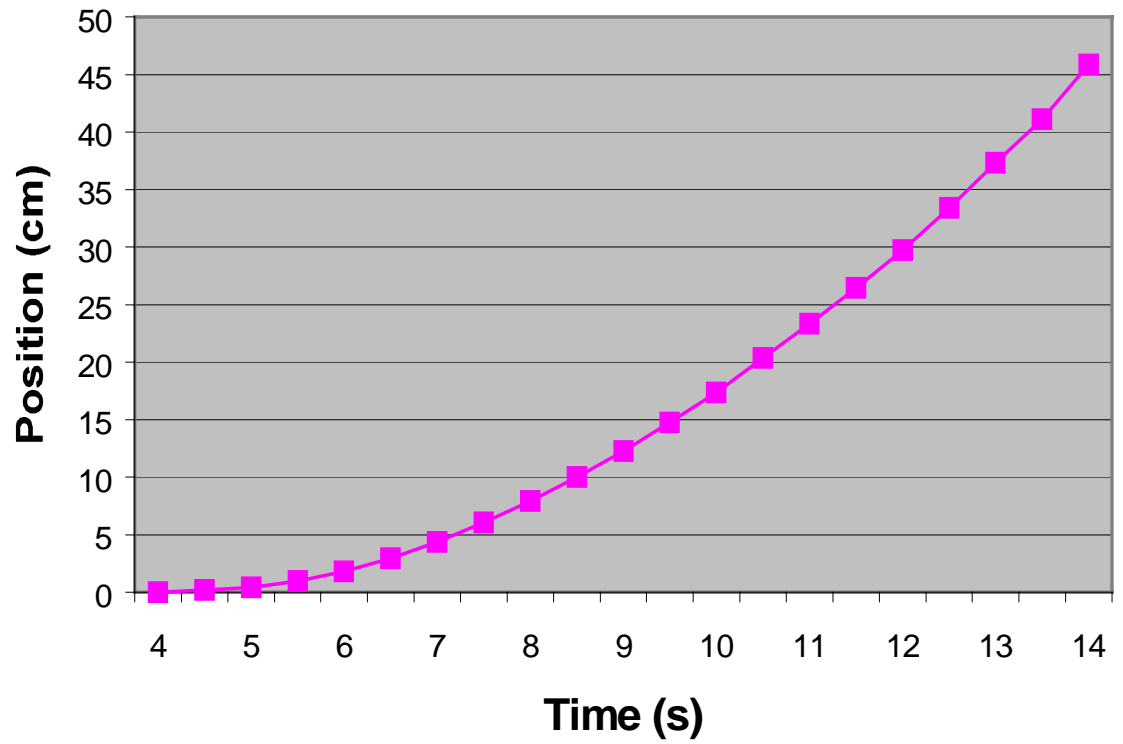
Then the acceleration is just the slope of v versus time as $v = v_0 + at$.

Time (s)	x (cm)	Average Time (s)	Average v (cm/s)
4.00	0.00	-	-
4.50	0.20	4.25	0.40
5.00	0.43	4.75	0.46
5.50	0.95	5.25	1.04
6.00	1.80	5.75	1.70
6.50	2.95	6.25	2.30
7.00	4.35	6.75	2.80
7.50	6.05	7.25	3.40
8.00	7.93	7.75	3.76
8.50	10.00	8.25	4.14
9.00	12.25	8.75	4.50
9.50	14.75	9.25	5.00
10.00	17.37	9.75	5.24
10.50	20.35	10.25	5.96
11.00	23.35	10.75	6.00
11.50	26.45	11.25	6.20
12.00	29.75	11.75	6.60
12.50	33.40	12.25	7.30
13.00	37.30	12.75	7.80
13.50	41.10	13.25	8.20
14.00	45.88	13.75	8.96

By drawing a best fit line through the velocity versus time points we obtained an acceleration of 0.87 cm/s^2 : a computer fit to the x versus t plot estimated an acceleration of 0.84 cm/s^2 .

The spread in the data as seen in the non-linear velocity plot may arise from the non-uniform "horizontal" air track and the jitter of the television image: this reflects the errors that occur in real experimental data.

#2. Propeller Driven Car on Air Track: Uniform Acceleration (continued)



#3. Inclined Plane: Uniform Acceleration

In this demonstration we will place a car on an air track that is inclined with respect to the horizontal. The increase in height of one support is about 10.1 cm while the distance between supports is 3.8 m: this results in an angle to the horizontal of 1.52° . By plotting the displacement versus time we can see the parabolic nature that results from $x = x_0 + v_0 t + \frac{1}{2} a t^2$. By estimating the instantaneous velocity by dividing the difference in successive positions by the time interval, we can obtain the acceleration as the slope of velocity versus time plot.

Time (s)	Position (cm)	Avg Time (s)	Avg Vel (cm/s)
2.50	0.00	-	-
2.80	0.85	2.65	2.83
3.10	2.05	2.95	4.00
3.40	3.75	3.25	5.67
3.70	5.65	3.55	6.33
4.00	8.10	3.85	8.17
4.30	11.05	4.15	9.83
4.60	14.35	4.45	11.00
4.90	17.85	4.75	11.67
5.20	21.80	5.05	13.17
5.50	26.20	5.35	14.67
5.80	30.95	5.65	15.83
6.10	35.90	5.95	16.50
6.40	41.50	6.25	18.67
6.70	47.45	6.55	19.83

From the plot of velocity vs. time we can estimate the average value of the acceleration as:

$$\frac{19.83 \text{ cm/s} - 2.83 \text{ cm/s}}{6.70 \text{ s} - 2.80 \text{ s}} = 4.36 \text{ cm/s}^2. \text{ By measuring the length of a meter stick on the screen, 16.7}$$

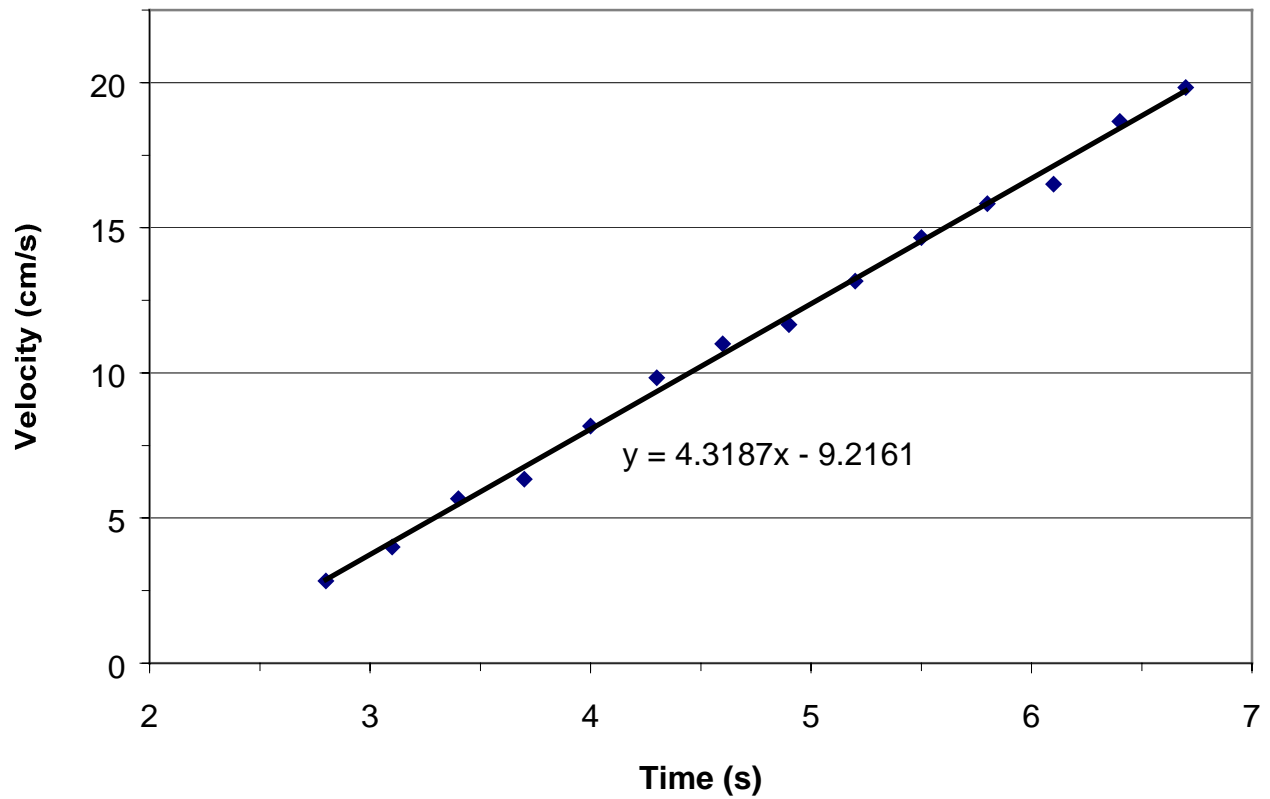
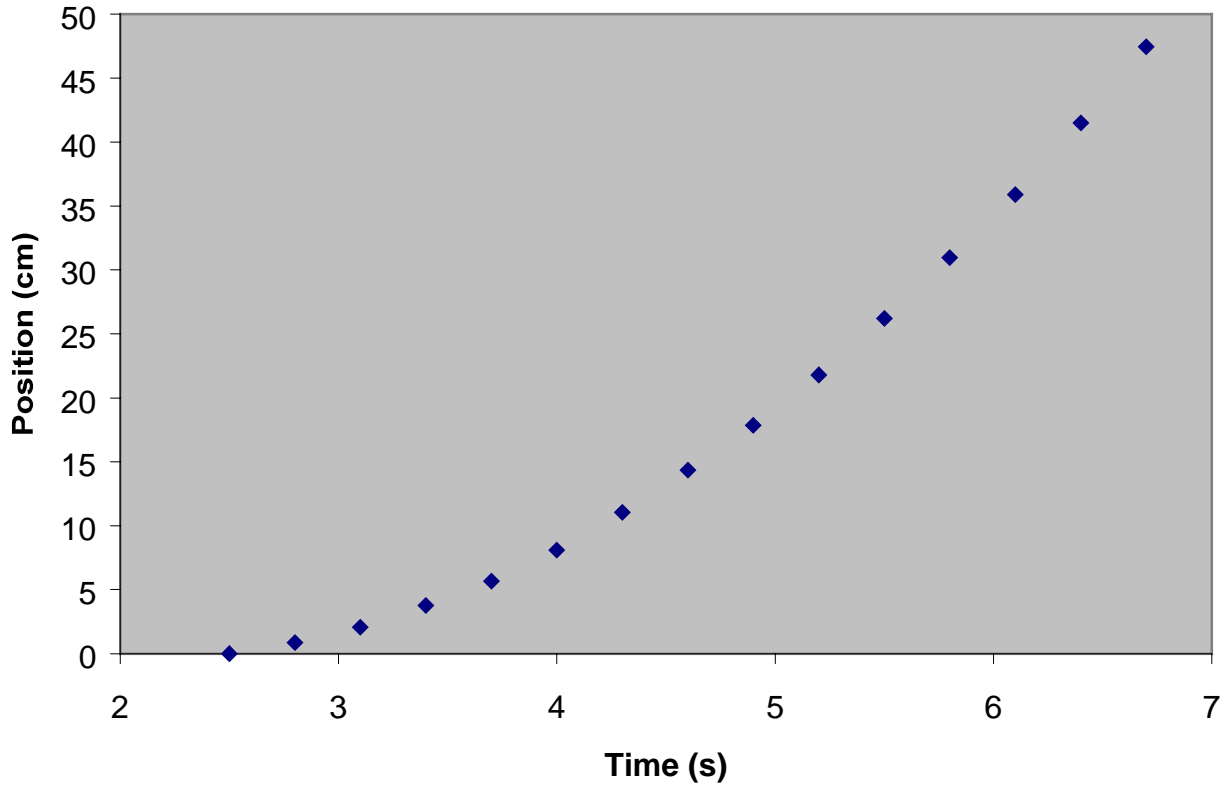
cm, we can obtain the acceleration in the lab frame, 0.261 m/s^2 . Does this give a reasonable value for g , the gravitational acceleration? Using the expression for the acceleration of an inclined plane, $a = g \sin \theta$, the value of g then should be:

$$g = a / \sin \theta = 0.261 \text{ m/s}^2 / \sin(1.52^\circ) = 9.84 \text{ m/s}^2$$

This differs from the actual value of g (9.82 m/s^2) by 0.2%.

Using a linear best-fit line, the slope of the line yields an average acceleration of 4.32 m/s^2 . The calculated value for g from this number is 9.75 m/s^2 , an error of 0.7%.

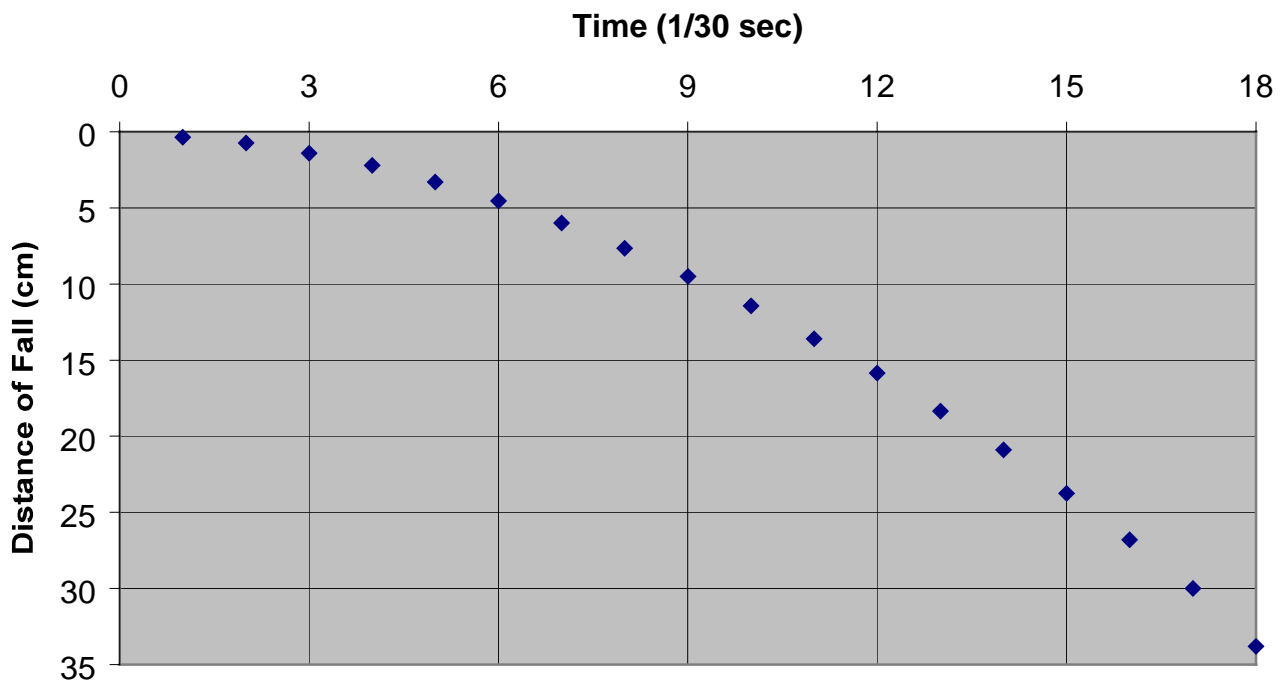
#3. Inclined Plane: Uniform Acceleration (continued)



#4. Free Fall: Uniform Gravitational Acceleration

In this demonstration we record the path of a ball dropped from rest: under the attraction of gravity $x = \frac{1}{2} g t^2$, where $g = 9.82 \text{ m/s}^2$. By plotting x versus t we can see the parabolic nature of the path. By calculating the average velocity as in earlier demos we can measure the value of g from the slope of v versus time.

Time (1/30 s)	Position (cm)	Average Time (1/30 s)	Average Velocity (cm/s)
1	0.35	0.5	10.5
2	0.73	1.5	11.4
3	1.40	2.5	20.1
4	2.20	3.5	24.0
5	3.30	4.5	33.0
6	4.55	5.5	37.5
7	6.00	6.5	43.5
8	7.65	7.5	49.5
9	9.50	8.5	55.5
10	11.43	9.5	57.5
11	13.60	10.5	65.1
12	15.85	11.5	67.5
13	18.35	12.5	75.0
14	20.90	13.5	76.5
15	23.75	14.5	85.5
16	26.80	15.5	91.5
17	30.00	16.5	96.0
18	33.80	17.5	114.0

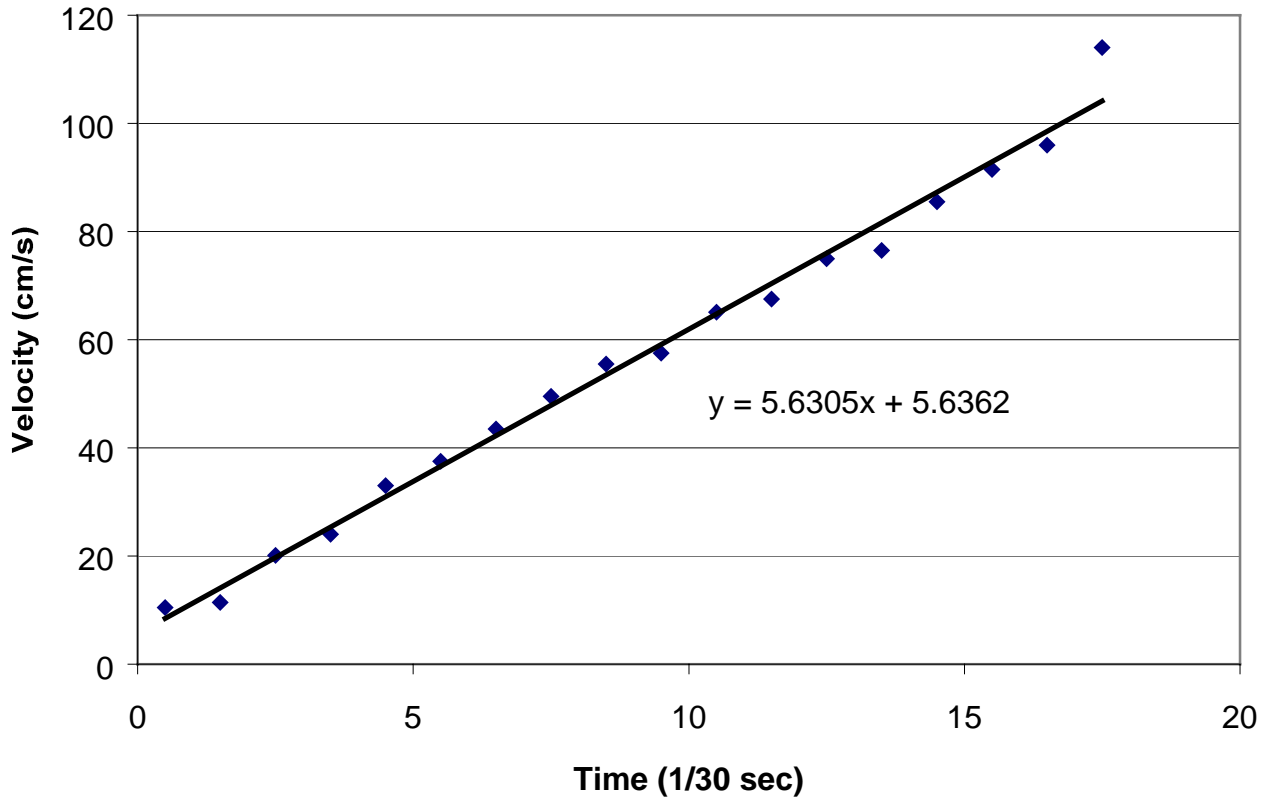


#4. Free Fall: Uniform Gravitational Acceleration (continued)

We can find acceleration by using a linear best-fit line:

$$a = \text{slope} * (30 \text{ frames per second}) = 5.631 \text{ cm/s}^2 * 30 = 168.92 \text{ cm/s}^2$$

By measuring the meter stick on the television monitor to be 16.9 cm, we can convert the above acceleration in the monitor frame to the lab frame by dividing by 16.9. We obtain a value of 9.99 m/s², a 1.7% difference from the true value of 9.8 m/s².



Note: we can also use these data to check conservation of energy. The change in kinetic energy is equal to the change in potential energy: $\frac{1}{2} mv^2 = mgh$. Thus $v^2 = 2gh$. For the last "good point" on the v versus t graph, we have $v^2 = 32.3 \text{ m}^2/\text{s}^2$ and $2gh = 34.8 \text{ m}^2/\text{s}^2$ (after converting to the lab frame), a difference of 7%.

Note: Data are difficult to take from later frames due to the blurring of the image during the exposure of the frame – the image shape goes from a circle to a hot dog oval, we used the center of the image for the location of the ball.

#5. Relativity Car

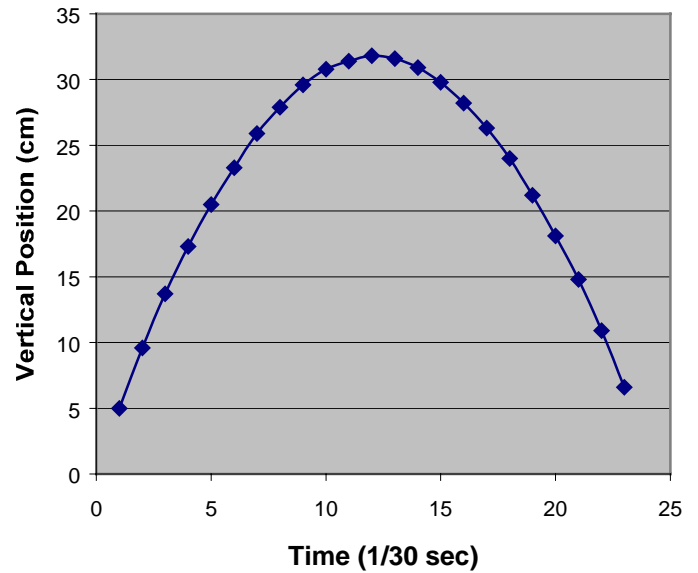
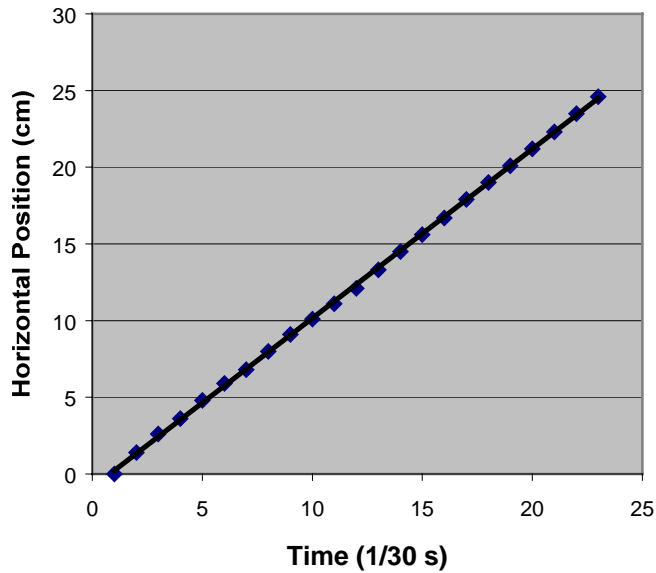
The relativity car is equipped with a spring-loaded gun. The car rides along a track until it hits a trigger, which causes a ball bearing to be launched vertically from the gun barrel. The gun gives the ball bearing a vertical velocity relative to the car, and the ball bearing also has a horizontal velocity component equal to that of the cart: the velocity of the ball bearing in the lab frame is the sum of the two components.

Since the ball bearing will be moving in two dimensions (vertically and horizontally) we will need to record its position in both dimensions. Be careful, use enough plastic sheets to cover the entire range of motion in both directions on the TV screen. We used the top of the funnel as a convenient "zero line". Mark it at the left of the screen and again at the right to define a horizontal baseline.

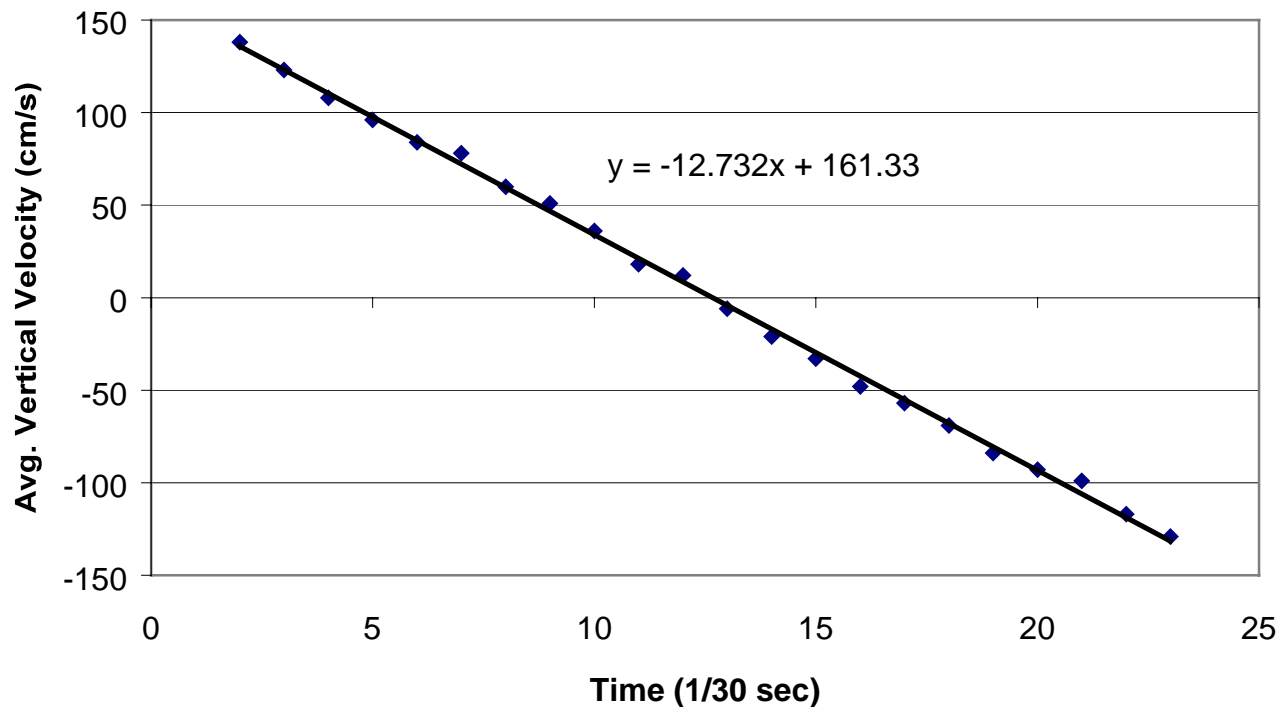
Time (1/30 s)	x Position (cm)	Y Position (cm)	$\Delta y/\Delta t$ (cm/s)
1	0.0	5.0	-
2	1.4	9.6	138
3	5.6	13.7	123
4	3.6	17.3	108
5	4.8	20.5	96
6	5.9	23.3	84
7	6.8	25.9	78
8	8.0	27.9	60
9	9.1	29.6	51
10	10.1	30.8	36
11	11.1	31.4	18
12	12.1	31.8	12
13	13.3	31.6	-6
14	14.5	30.9	-21
15	15.6	29.8	-33
16	16.7	28.2	-48
17	17.9	26.3	-57
18	19.0	24.0	-69
19	20.1	21.2	-84
20	21.2	18.1	-93
21	22.3	14.8	-99
22	23.5	10.9	-117
23	24.6	6.6	-129

It's interesting to plot y versus t and x versus t on two separate graphs. We find that the x plot is linear with a slope of about 33 cm/s (screen velocity). The y plot looks parabolic, indicating that it will be interesting to plot velocity in the y dimension $\left(\frac{\Delta y}{\Delta t} \right)$ versus t .

#5. Relativity Car (continued)



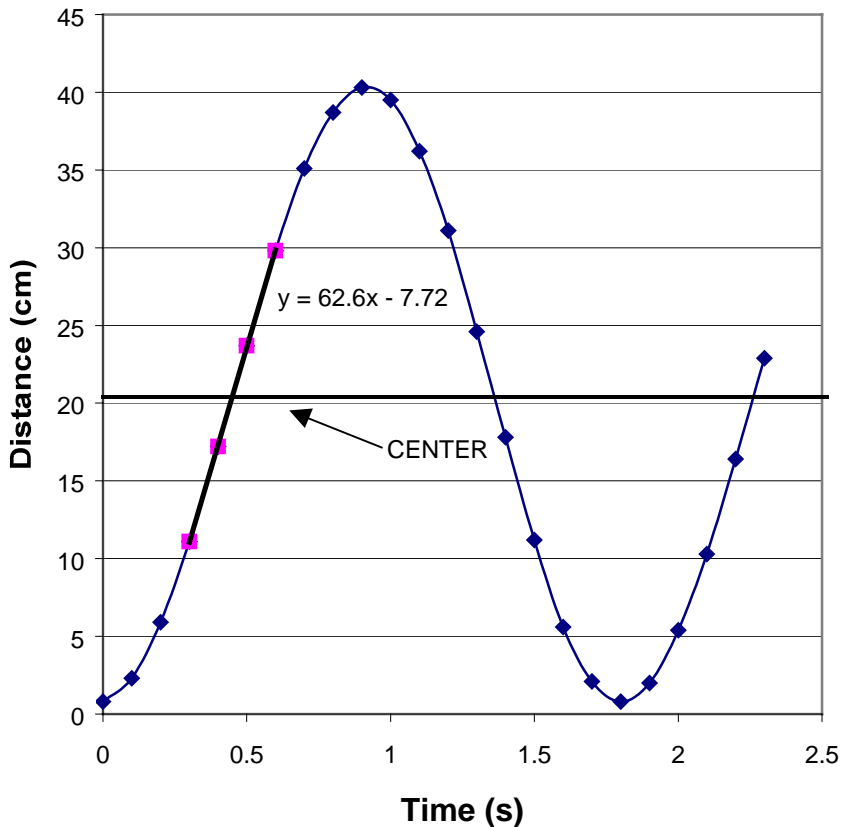
This gives a more or less straight line, thereby indicating that the acceleration in the y direction is constant. The slope of this line is $-12.732 \text{ cm/s}^2 * (30 \text{ frames per second}) = -381.96 \text{ cm/s}^2$. I measured the actual maximum height of the trajectory in the room to be about 90 cm, while on the TV (see data for maximum value) the height was 31.8 cm. Using this conversion, the acceleration becomes $-1081 \text{ cm/s}^2 = -10.81 \text{ m/s}^2$, a difference of 10.1% from the accepted value of -9.8 m/s^2 – not bad for such a crude measurement.



#6. Projection of Circular Motion

This is the end-on projection of a ball attached to a phonograph turntable moving in a circle. What we see is the ball moving back and forth across the screen horizontally as the turntable rotates around. We arbitrarily located a zero mark on the left side of the screen, marked the ball's location through a little more than one cycle, and got the following data:

Time (sec)	Location (cm)
0.0	0.8
0.1	2.3
0.2	5.9
0.3	11.1
0.4	17.2
0.5	23.7
0.6	29.8
0.7	35.1
0.8	38.7
0.9	40.3
1.0	39.5
1.1	36.2
1.2	31.1
1.3	24.6
1.4	17.8
1.5	11.2
1.6	5.6
1.7	2.1
1.8	0.8
1.9	2.0
2.0	5.4
2.1	10.3
2.2	16.4
2.3	22.9



Graphing these data gave us a very nice sinusoidal shape oscillating around the turntable's spindle location (which on our screen was at 19.7 cm). The period of oscillation was measured to be $T = 1.81$ s which agrees nicely with the speed of the turntable, 33 1/3 rpm.

$$T = (60 \text{ s/min}) / (33.333 \text{ rpm}) = 1.80 \text{ s}$$

It's also interesting to find the slope of the graph (which is the velocity) at the center and compare this to the predicted $\left(v = \frac{2\pi r}{T} \right)$ velocity. We get a slope of 62.6 cm/s and a radius of

19.75 cm, so $\frac{2\pi r}{T} = 68.6$ cm/s (8.7% difference).

#7. Bowling Ball Pendulum: Periodic Motion

In this demonstration we look at the periodic motion of a pendulum formed by a bowling ball suspended from a wire. The length of the wire is 5.43 m, so we can compare the expected period of motion:

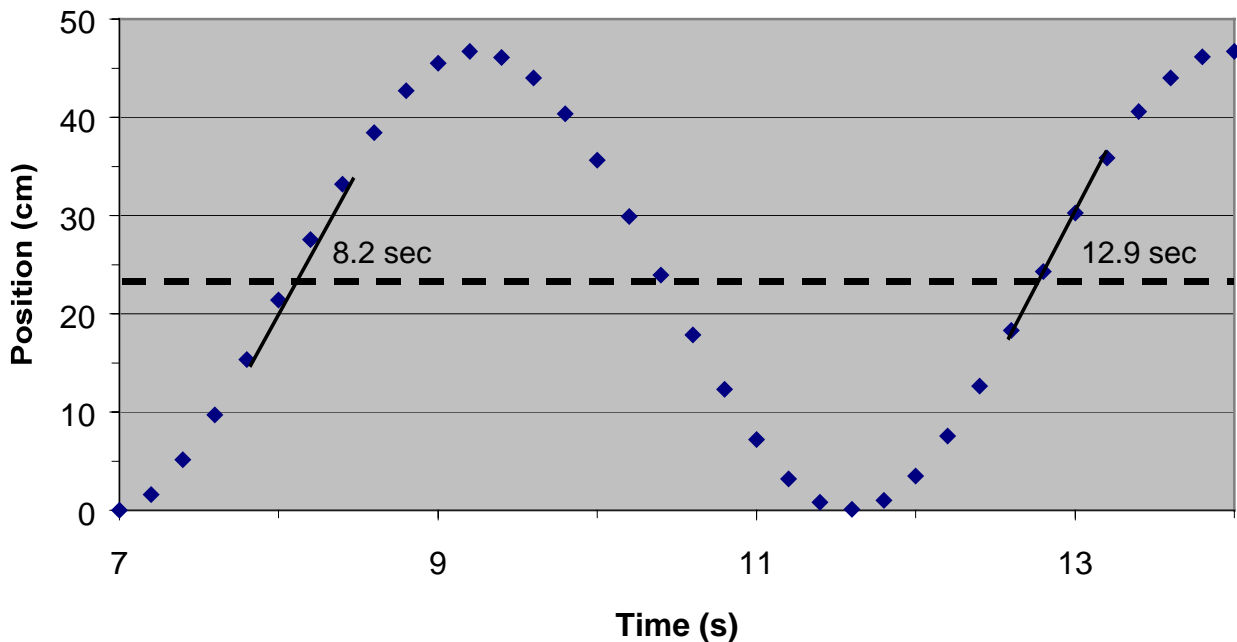
$$T = 2\pi\sqrt{l/g} = 4.67 \text{ s}$$

with what we can measure from the plot of position versus time.

From the data tape we can measure the position as:

Time (s)	Position (cm)	Time (s)	Position (cm)	Time (s)	Position (cm)
7.0	0.00	9.4	46.10	11.8	1.00
7.2	1.60	9.6	44.00	12.0	3.50
7.4	5.15	9.8	40.35	12.2	7.55
7.6	9.70	10.0	35.65	12.4	12.65
7.8	15.35	10.2	29.90	12.6	18.30
8.0	21.40	10.4	23.95	12.8	24.30
8.2	27.55	10.6	17.85	13.0	30.25
8.4	33.20	10.8	12.30	13.2	35.85
8.6	38.45	11.0	7.20	13.4	40.60
8.8	42.70	11.2	3.20	13.6	44.00
9.0	45.50	11.4	0.80	13.8	46.15
9.2	46.70	11.6	0.10	14.0	46.70

By comparing two areas of similar slope that are spaced one period apart, and by drawing a horizontal line between them to select the same point, one period apart, we find the period of our plot to be $12.9 \text{ s} - 8.2 \text{ s} = 4.7 \text{ s}$, a 0.6% error when compared with the calculated value.



#8. Bowling Ball Pendulum: Conservation of Energy

In this demonstration we use the bowling ball pendulum to examine the relation between potential and kinetic energies. In a conserved system the total energy is constant, thus any change in potential energy, mgh , is balanced by a change in the kinetic energy, $\frac{1}{2}mv^2$. Equating these two yields $v^2 = 2gh$. We can measure the average maximum height of the bowling ball and measure the displacement versus time during its maximum rate of motion at the bottom of its swing to prove the above equation.

From the data tape we observe:

The height of the ball above the table is 6.65 cm at maximum and 3.15 cm at minimum on the television monitor: a 1 meter stick measures 11.8 cm. Thus the change in height is $(6.65 - 3.15) / 11.8 = 29.7$ cm in the lab frame.

We measure the distance versus time for a few oscillations to obtain an average value of maximum speed:

Time interval (s)	Δx (cm)
3.3 - 3.6	10.03
5.7 - 6.0	9.83
8.1 - 8.4	9.63
10.4 - 10.7	9.47
12.8 - 13.1	9.40
15.2 - 15.5	9.45
17.6 - 17.9	9.25

The average spacing is 9.58 cm. Note however that the horizontal calibration of the meter stick is 13.05 cm, not the 11.8 cm of the vertical direction: televisions usually have slightly different horizontal and vertical calibrations. Using the horizontal calibration we can obtain we can

obtain an average velocity = $\frac{9.58 \text{ cm}}{13.05 \text{ cm/m} * 0.3 \text{ s}} = 2.45 \text{ m/s}$.

Using the relation $v^2 = 2gh$, the value of 2.45 m/s would predict a maximum height of 30.5 cm, a 2.7% difference with the measured value of 29.7 cm determined above.

#9. Cart on Air Track with Springs: Harmonic Motion

In this demo we look at the fairly frictionless motion of a cart on an air track as it oscillates under the force of a pair of springs. We want to show the period of the motion is independent of the amplitude of the path and to show that it is a sinusoidal function.

The data points are:

Time (s)	x (cm)	Time (s)	x (cm)	Time (s)	x (cm)
4.8	-8.50	-	-	127.5	-5.55
5.0	0.05	48	-5.05	127.7	-0.85
5.2	8.65	48.2	1.05	127.9	4.00
5.4	16.25	48.4	7.20	128.1	8.50
5.6	21.95	48.6	12.40	128.3	11.85
5.8	25.10	48.8	16.25	128.5	14.15
6.0	25.40	49	18.20	128.7	14.55
6.2	22.25	49.2	17.75	128.9	13.15
6.4	16.85	49.4	15.15	129.1	10.30
6.6	9.35	49.6	10.75	129.3	6.20
6.8	1.10	49.8	5.20	129.5	1.55
7.0	-7.30	50	-0.85	129.7	-3.10
7.2	-15.10	50.2	-6.95	129.9	-7.70
7.4	-21.25	50.4	-12.30	130.1	-11.20
7.6	-24.65	50.6	-16.45	130.3	-13.45
7.8	-24.65	50.8	-18.20	130.5	-13.75
8.0	-21.55	51	-17.80	130.7	-12.40
8.2	-15.65	51.2	-15.25	130.9	-9.45
8.4	-8.00	51.4	-10.60	131.1	-5.30
8.6	0.03	51.6	-4.75	131.3	-0.70
8.8	8.45	51.8	1.25	131.5	4.00

By measuring the time differences for the three oscillations we can obtain the period:

$$1^{\text{st}} \quad T = 8.6 - 5.0 = 3.6 \text{ s}$$

$$2^{\text{nd}} \quad T = 51.8 - 48.2 = 3.6 \text{ s}$$

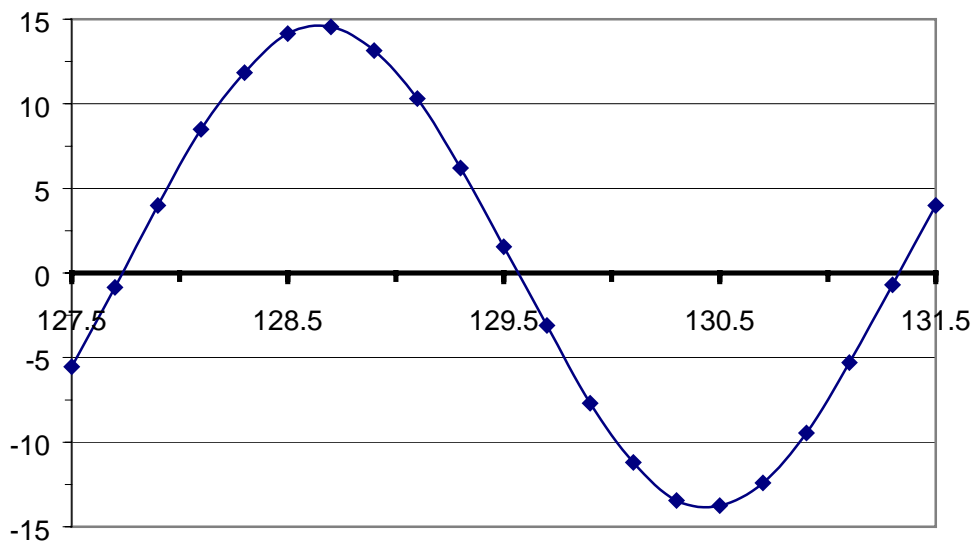
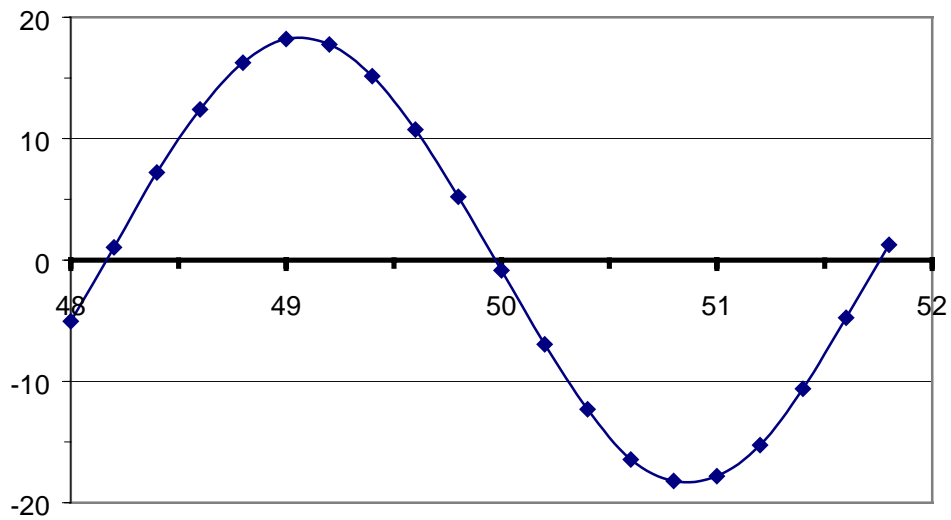
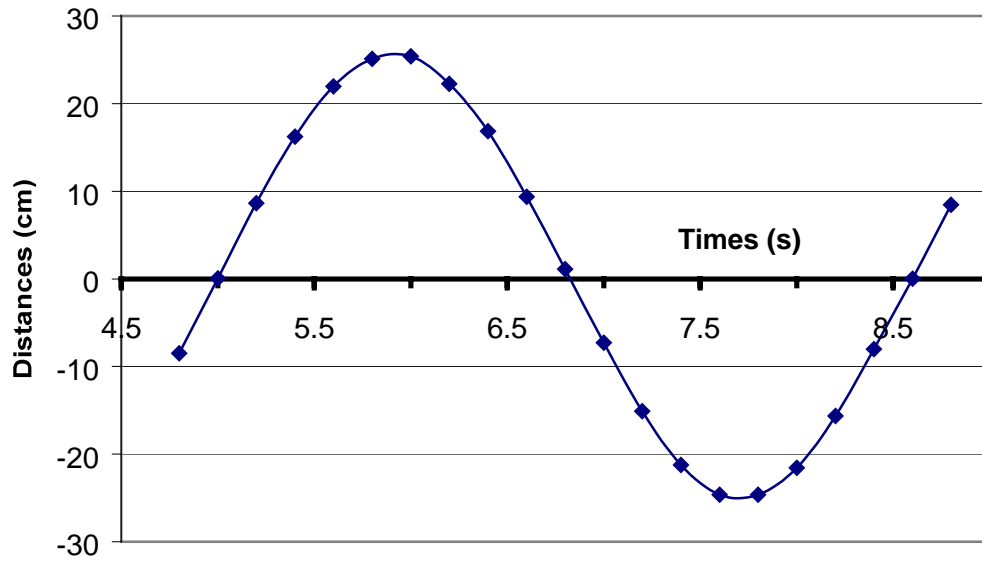
$$3^{\text{rd}} \quad T = 131.3 - 127.7 = 3.6 \text{ s}$$

Thus the average value of the period is 3.6 seconds. Does the curve of position versus time behave as a sine wave? If we approximate the zero-crossing point to be at 5 seconds, and approximate the average amplitude for the first oscillation to be 25.25 cm, we can roughly fit the data to:

$$y = 25.25 \sin\left(\frac{360^\circ}{36 \text{ s}}(t - 5.0 \text{ s})\right) \text{ cm}$$

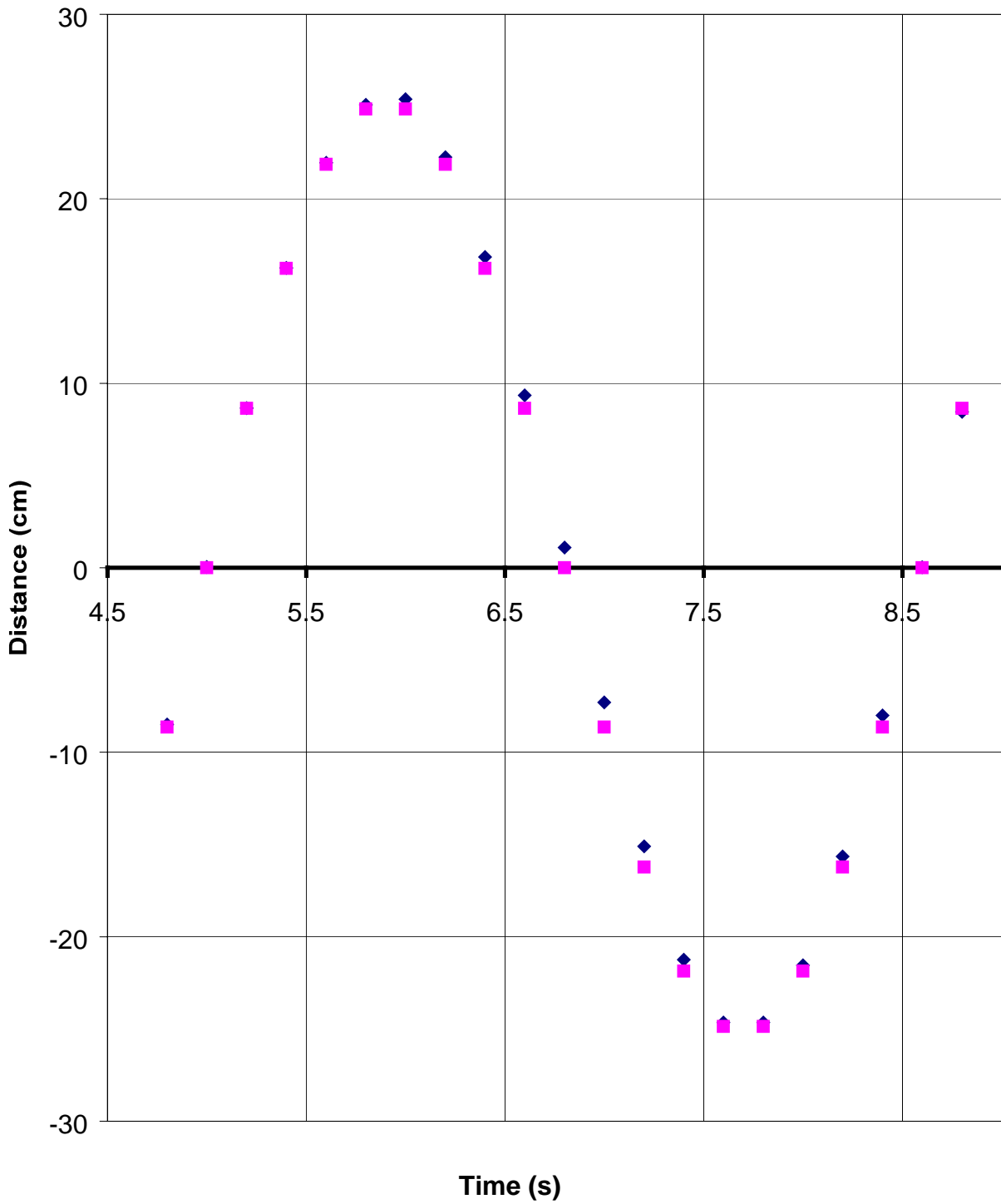
Examination of the plot of this fit and the data from the 1st run shows a fairly consistent agreement: note that the fit depends on the amplitude, the period, and the phase of the sine wave.

#9. Cart on Air Track with Springs: Harmonic Motion (continued)



#9 Cart on Air Track: Harmonic Motion

◆ Plot of 1st Data ■ $25.25 \sin(100 * (t - 5))$



#10. Mass on a Vertical Spring: Harmonic Motion

This segment starts with a spring hanging from a rod. A 1 kg mass is attached to the spring and allowed to stretch the spring to the equilibrium position where the force of gravity on the mass is countered by the force of the stretched spring. Then the mass is displaced downward and released and the resulting periodic motion is shown.

The first thing we need to do is to measure the spring constant, k , of the equation of the force of an expanded (or contracted) ideal spring: $\vec{F} = -k \Delta\vec{x}$. Using the meter stick in the picture, measure the location of the end of the spring as it hangs freely; then measure again after the mass has been attached and is hanging at rest. At this point you should also note the equilibrium position of the yellow mark on the mass, about 35.5 cm on the meter stick. This will be useful later.

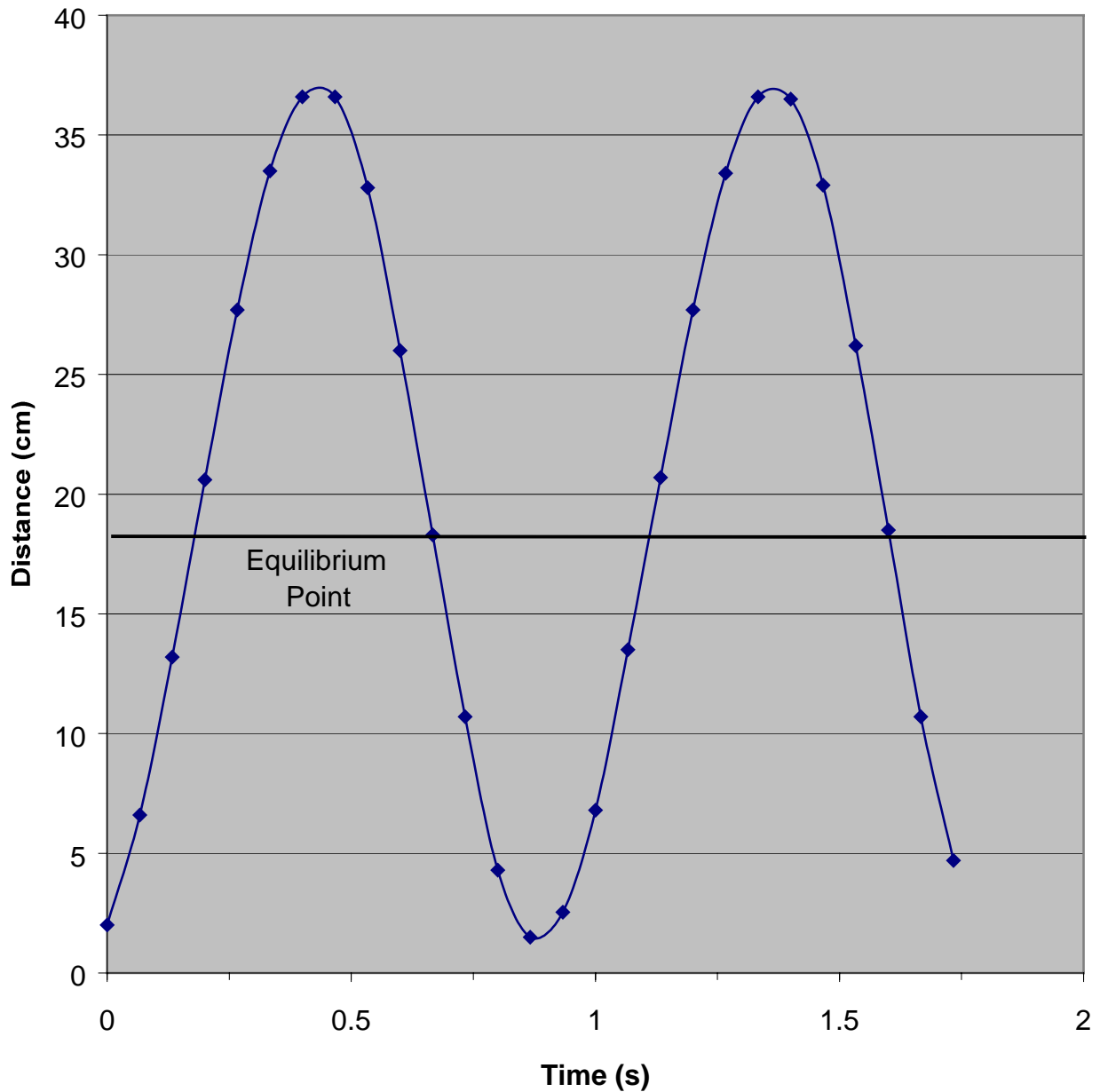
Now take the data on position versus time while the mass is in motion. When you begin you should mark your equilibrium point (35.5 cm) for later reference. Note that the stop-frame segment shows every other frame, so that the time between each point shown is $2/30$ of a second. Here are the data we obtained:

Frame	Time (1/30 s)	Location (cm)
1	0	2.00
3	2	6.60
5	4	13.20
7	6	20.60
9	8	27.70
11	10	33.50
13	12	36.60
15	14	36.60
17	16	32.80
19	18	26.00
21	20	18.30
23	22	10.70
25	24	4.30
27	26	1.50
29	28	2.54
31	30	6.80
33	32	13.50
35	34	20.70
37	36	27.70
39	38	33.40
41	40	36.60
43	42	36.50
45	44	32.90
47	46	26.20
49	48	18.50
51	50	10.70
53	52	4.70

#10. Mass on a Vertical Spring: Harmonic Motion (continued)

Plotting these data gives a nice sinusoidal curve. Locate the midpoint of the range of motion: it should be the same as the equilibrium position you measured with the mass at rest. Note as well that the motion is uniform about the midpoint: an equal amount of time is spent above and below the equilibrium point. How about the period? Does it agree with the period expected from

the equation of motion, $T = 2\pi\sqrt{\frac{m}{k}}$?



#11. Elastic Collision: $m_1 = m_2, v_{2i} = 0$

In this collision demonstration, the two cars on the air track have equal masses: $m_1 = m_2 = 0.183$ kg. M_1 has a nonzero initial velocity as well, which we can obtain from the plot of positions versus time.

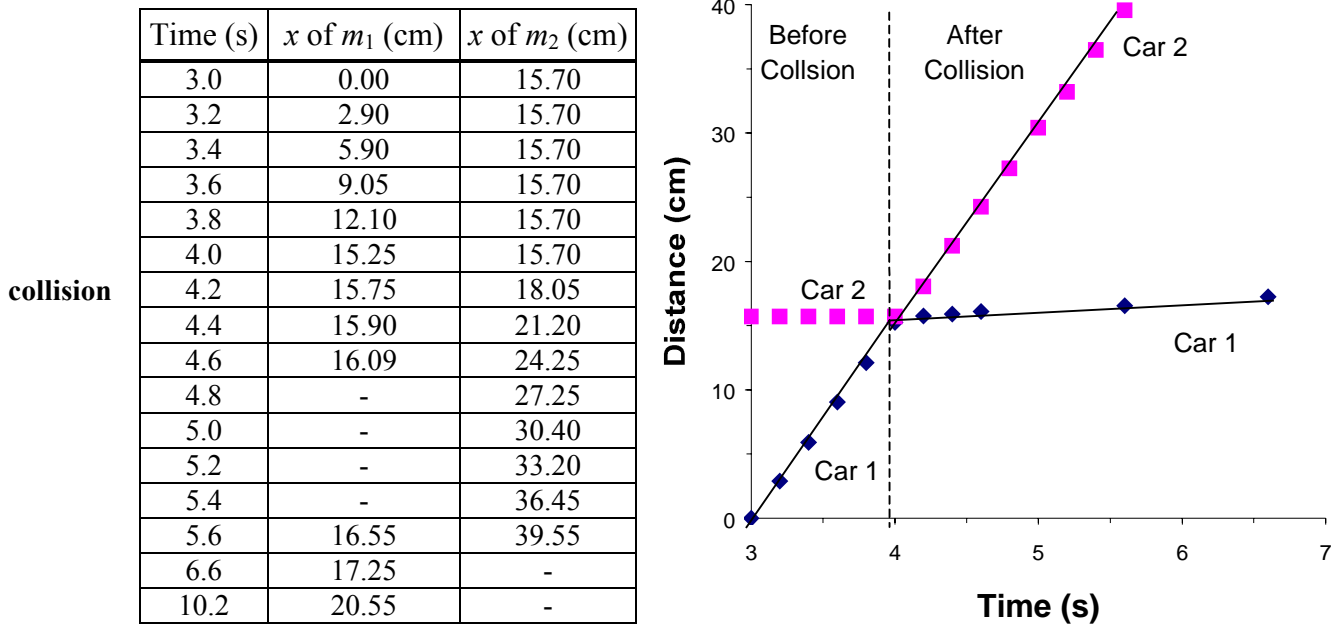
To solve for the final velocities, we use both conservation of momentum (a vector) and conservation of kinetic energy as follows:

$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$(2) \quad \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find the positions versus time for the two masses:



From the accompanying graph we can estimate the values of the velocities from the slopes of the lines as:

$$v_{1i} = \frac{15.25 \text{ cm} - 0 \text{ cm}}{4.0 \text{ s} - 3.0 \text{ s}} = 15.25 \text{ cm/s} \text{ and } v_{2i} = 0 \text{ cm/s}$$

These values, when used with the known values of the masses and the equations of momentum and energy conservation, yield $v_{1f} = 0$ cm/s and $v_{2f} = 15.25$ cm/s: measured values from the slopes of the graph are $v_{1f} = 0.8$ cm/s and $v_{2f} = 15.35$ cm/s, the latter a difference of 0.6%.

#12. Elastic Collision: $m_1 \neq m_2, v_{2i} = 0$

In this collision demonstration, the two cars on the air track have unequal masses: $m_1 = 0.988 \text{ kg}$ and $m_2 = 0.183 \text{ kg}$. M_1 has a nonzero initial velocity as well, which we can obtain from the plot of position versus time.

To solve for the final velocities, we use both conservation of momentum (a vector) and conservation of kinetic energy as follows:

$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$(2) \quad \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find the positions versus time for the two masses:

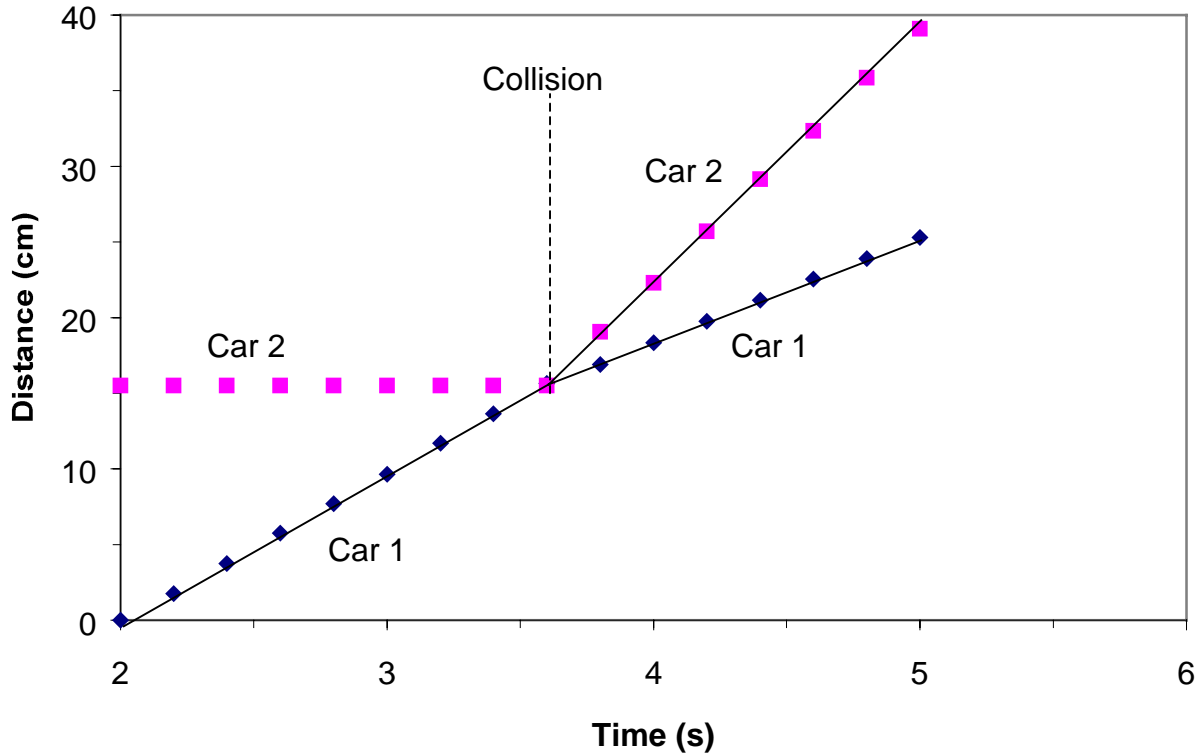
	Time (s)	x of m_1 (cm)	x of m_2 (cm)
	2.0	0.00	15.50
	2.2	1.75	15.50
	2.4	3.75	15.50
	2.6	5.75	15.50
	2.8	7.70	15.50
	3.0	9.65	15.50
	3.2	11.70	15.50
	3.4	13.65	15.50
	3.6	15.65	15.50
collision	3.8	16.90	19.05
	4.0	18.35	22.30
	4.2	19.75	25.70
	4.4	21.15	29.15
	4.6	22.55	32.35
	4.8	23.90	35.85
	5.0	25.30	39.10

From the accompanying graph we can estimate the values of the velocities from the slopes of the lines as:

$$v_{1i} = \frac{15.45 \text{ cm} - 3.75 \text{ cm}}{3.6 \text{ s} - 2.4 \text{ s}} = 9.75 \text{ cm/s} \text{ and } v_{2i} = 0 \text{ cm/s}$$

Similarly the other velocities measured from the graph are $v_{1f} = 7.0 \text{ cm/s}$ and $v_{2f} = 16.75 \text{ cm/s}$.

#12. Elastic Collision: $m_1 \neq m_2, v_{2i} = 0$ (continued)



From Newton's laws we can expect the conservation of momentum and kinetic energy as expressed in the above equations (1-2). Let's verify the "conservation laws" by comparing the left and right hand side of the two equations:

$$(1) \quad 0.988 \text{ kg} * 9.75 \text{ cm/s} + 0.183 \text{ kg} * 0 = 0.988 \text{ kg} * 7.0 \text{ cm/s} + 0.183 \text{ kg} * 16.75 \text{ cm/s}$$

$$9.63 \text{ (kg cm/s)} \quad 9.98 \text{ (kg cm/s)}$$

and

$$(2) \quad \frac{1}{2} 0.988 \text{ kg} * (9.75 \text{ cm/s})^2 + \frac{1}{2} 0.183 \text{ kg} * (0 \text{ cm/s})^2 =$$

$$\frac{1}{2} 0.988 \text{ kg} * (7 \text{ cm/s})^2 + \frac{1}{2} 0.183 \text{ kg} * (16.75 \text{ cm/s})^2$$

$$46.96 \text{ (kg cm}^2\text{/s}^2) \quad 49.88 \text{ (kg cm}^2\text{/s}^2)$$

We can see by the roughly equal values of both sides that "conservation" means the constant value of the expressions: thus before and after values are equal.

When we use the measured initial values of the masses and velocities in the equations of momentum and energy conservation, we obtain values for the final velocities of $v_{1f} = 6.71 \text{ cm/s}$ and $v_{2f} = 16.42 \text{ cm/s}$, differences of 4.3% and 2% respectively from the measured values.

#13. Elastic Collision: $m_1 \neq m_2, v_1 \neq v_2$

In this collision demonstration, the two cars on the air track have unequal masses: $m_1 = 0.183 \text{ kg}$ and $m_2 = 0.384 \text{ kg}$. They have nonzero initial velocities as well, which we can determine from the plot of positions versus time.

To solve for the final velocities, we use both conservation of momentum (a vector) and conservation of kinetic energy as follows:

$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

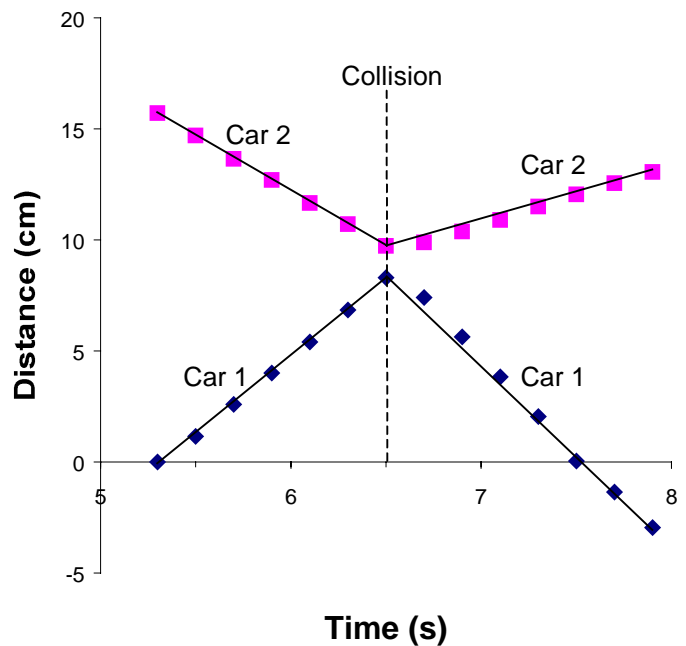
$$(2) \quad \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find the positions versus time for the two masses:

Time (s)	x of m_1 (cm)	x of m_2 (cm)
5.3	0.00	15.70
5.5	1.15	14.70
5.7	2.60	13.65
5.9	4.00	12.70
6.1	5.40	11.65
6.3	6.85	10.70
6.5	8.30	9.73
6.7	7.40	9.88
6.9	5.63	10.38
7.1	3.83	10.90
7.3	2.05	11.50
7.5	0.04	12.05
7.7	-1.35	12.55
7.9	-2.95	13.05

collision



From the accompanying graph we can estimate the values of the velocities from the slopes of the lines as:

$$v_{1i} = \frac{8.30 \text{ cm} - 0 \text{ cm}}{6.5 \text{ s} - 5.33 \text{ s}} = 7.09 \text{ cm/s} \quad \text{and} \quad v_{2i} = \frac{9.73 \text{ cm} - 15.7 \text{ cm}}{6.5 \text{ s} - 5.30 \text{ s}} = -4.98 \text{ cm/s}$$

These values, when used with the known values of the masses and the equations of momentum and energy conservation, yield $v_{1f} = -9.26 \text{ cm/s}$ and $v_{2f} = 2.81 \text{ cm/s}$: measured values from the graph are $v_{1f} = -8.52 \text{ cm/s}$ and $v_{2f} = 2.64 \text{ cm/s}$, differences of 8.7 and 6.4% respectively.

#14. Inelastic Collision: $m_1 = m_2, v_{2i} = 0$

In this collision demonstration, the two cars on the air track have equal masses and the second one has zero initial velocity, which we can obtain from the plot of position versus time.

To solve for the final velocities, we can only use conservation of momentum (a vector) as follows:

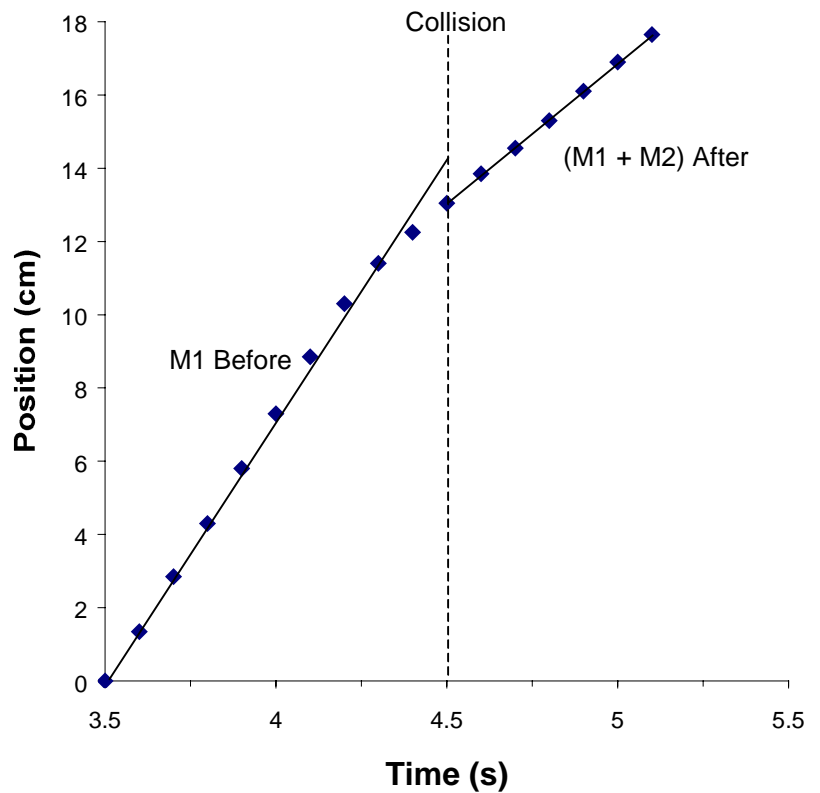
$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find the position versus time for the two masses:

Time (s)	x of m_1 (cm)
3.5	0
3.6	1.35
3.7	2.85
3.8	4.3
3.9	5.8
4.0	7.3
4.1	8.85
4.2	10.3
4.3	11.4
4.4	12.25
4.5	13.05
4.6	13.85
4.7	14.55
4.8	15.3
4.9	16.1
5.0	16.9
5.1	17.65

collision



From the accompanying graph we can estimate the values of the velocities from the slopes of the lines as:

$$v_{1i} = \frac{10.30 \text{ cm} - 0 \text{ cm}}{4.2 \text{ s} - 3.51 \text{ s}} = 14.93 \text{ cm/s} \quad \text{and} \quad v_{1f} = \frac{17.7 \text{ cm} - 11.4 \text{ cm}}{5.1 \text{ s} - 4.3 \text{ s}} = 7.88 \text{ cm/s}$$

The first value, when used with the equation of momentum conservation, yields:

$$v_{1f} = v_{1i} / 2 = 7.47 \text{ cm/s},$$

an error of 5.5%.

#15. Inelastic Collision: $m_1 \neq m_2, v_{2i} = 0$

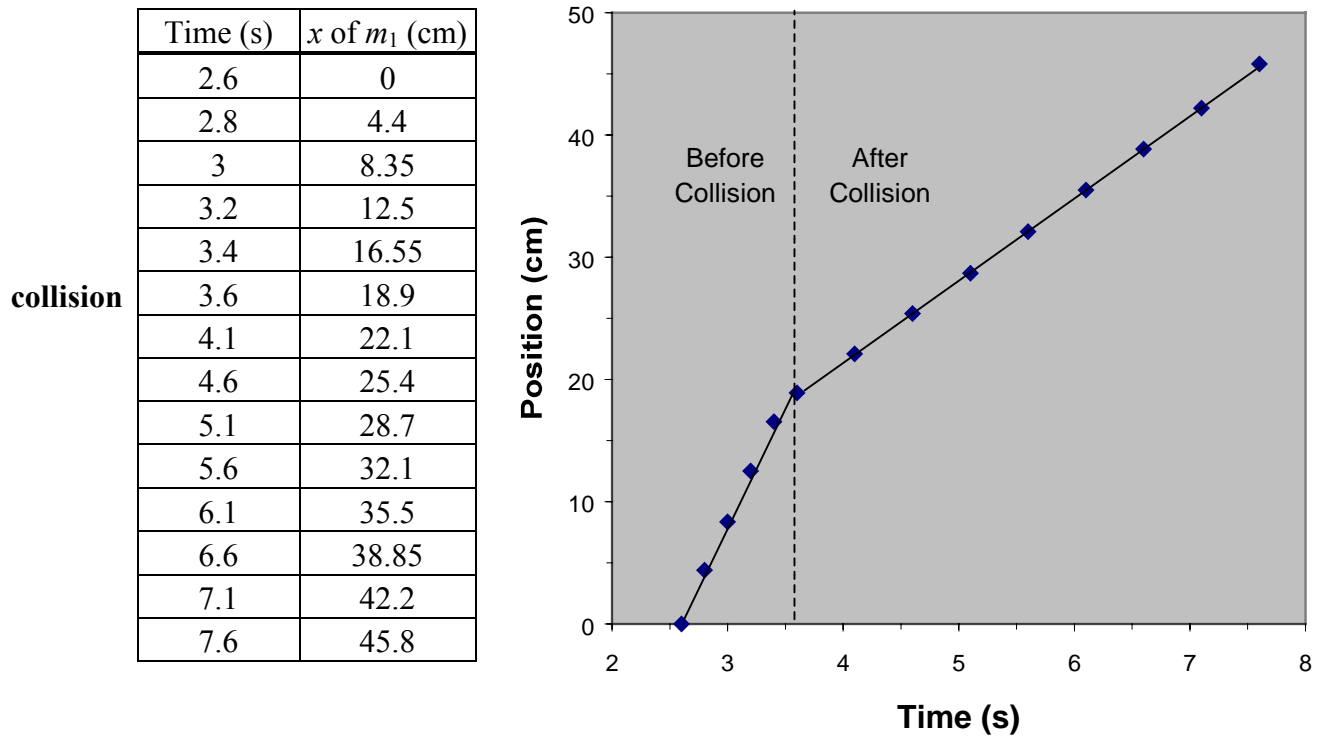
In this collision demonstration, the two cars on the air track have unequal masses, $m_1 = 0.183 \text{ kg}$; the second one has zero initial velocity. We can obtain the velocities from the plot of position versus time.

To solve for the final velocities, we can only use conservation of momentum (a vector) as follows:

$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find the position versus time for the two masses:



From the graph we can determine the values of the velocities from the slopes of the lines as:

$$v_{1i} = \frac{16.7 \text{ cm} - 0.1 \text{ cm}}{3.4 \text{ s} - 2.6 \text{ s}} = 20.75 \text{ cm/s} \quad \text{and} \quad v_{1f} = \frac{45.6 \text{ cm} - 18.5 \text{ cm}}{7.6 \text{ s} - 3.6 \text{ s}} = 6.78 \text{ cm/s}$$

The first value, when used with the equation of momentum conservation, yields:

$$v_f = 6.70 \text{ cm/s, an error of 1.2\%}$$

Note: In the inelastic collisions I have measured the position by using the left hand side of the left mass. This is easier to do, particularly after the collision.

#16. Inelastic Collision: Dart and Cart

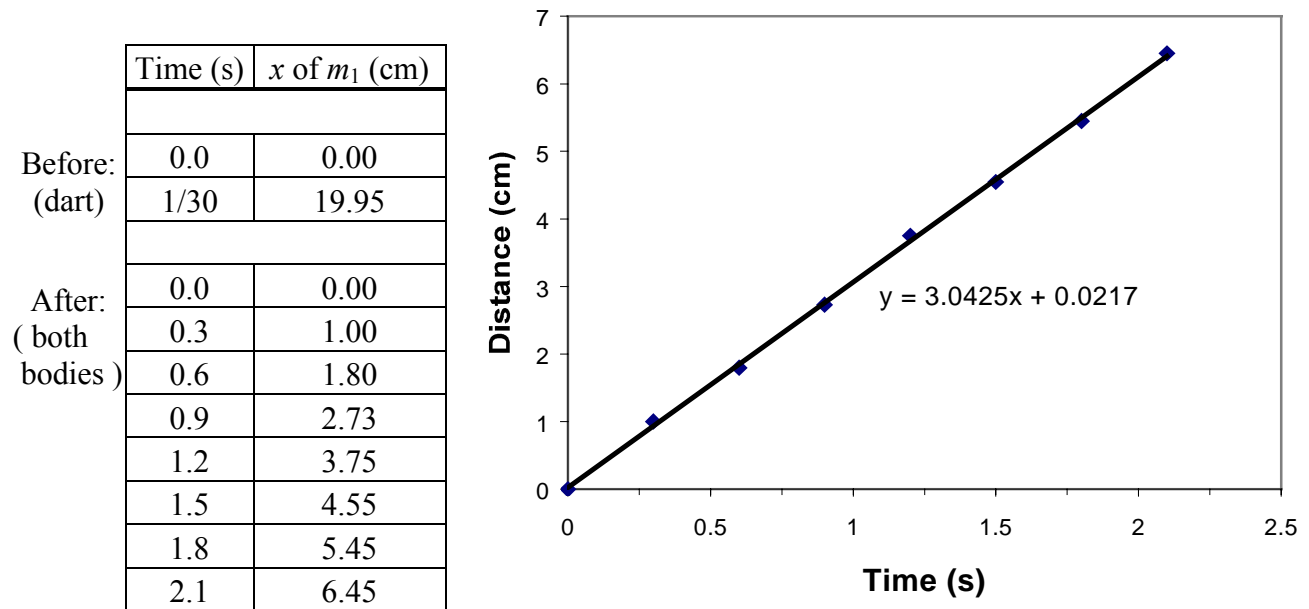
In this collision demonstration, a dart is fired at a cart on an air track. Due to the limit of camera resolution, we could only get two positions of the dart before the collisions ($\Delta t = 1/30$ s). We then look at the combined dart and cart body as it travels down the air track: an interval of $\Delta t = 0.3$ s seems to yield good results.

To solve for the final velocities, we can only use conservation of momentum (a vector) as follows:

$$(1) \quad m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

where "i" stands for initial and "f" stands for final values.

From the television screen we can find values for the positions versus time:



From the two data points we can determine the initial velocity of the dart:

$$v_i = \frac{19.95 \text{ cm} - 0 \text{ cm}}{1/30 \text{ s} - 0 \text{ s}} = 598.5 \text{ cm/s}$$

The final velocity of the dart and cart is 3.04 cm/s, given by the slope above. The first value when used with the equation of momentum conservation, and the values of the masses of the dart (2.1 g) and the cart (442 g) yields:

$$v_f = 2.83 \text{ cm/s}$$

This yields an error of 7.4% when comparing the experimental to theoretical values.