Optical Spectra with a Diffraction Grating
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OPTICAL SPECTRA WITH A DIFFRACTION GRATING

INTRODUCTION

Light is an electromagnetic wave, its color is determined by its wavelength. The red light from a He-Ne laser has a wavelength of 0.6328 Å. The yellow light from a sodium lamp has a wavelength of 0.589 Å, etc. White light, such as sunlight, is a mixture of light waves with many different wavelengths. Light is emitted whenever electrons are accelerated rapidly. Such rapid acceleration can be due to random thermal motion, as in the hot filament of an incandescent light bulb, or it can be due to the transition of an electron from one quantum state in an atom to another. In the latter case the light wave will have a well-defined frequency

\[ \nu = \frac{\Delta E}{h}, \]  

(1)

where \( \Delta E \) is the energy difference between the two quantum states, \( \nu \) is the frequency of the light, and \( h = 6.63 \times 10^{-34} \) J·s is a universal constant called Planck’s constant. Since

\[ c = \frac{\nu}{\lambda} \]  

(2)

we can solve Eq. (1) for the wavelength and get

\[ \lambda = \frac{hc}{\Delta E} \]  

(3)

for the wavelength of the light emitted during such quantum jumps. The atoms of different elements have different energy levels (quantum states) and emit light of different wavelengths. These wavelengths can be measured with extraordinary precision and are characteristic of the atom that has emitted the light. It is thus possible to determine the elemental composition of any substance by measuring the wavelengths of the light emitted by it. This technique is known as spectroscopic analysis.

The electrons in an atom are normally in the state with the lowest possible energy, the ground state. In order to make them emit their characteristic light they must be excited to states of higher energy from which they can go to states of lower energy by emitting light. This excitation can be achieved in various ways:

- If you place a crystal of sodium chloride, table salt, in the flame of a Bunsen burner you will notice that the flame will emit the intense yellow light characteristic of sodium. In this case the sodium atoms are stimulated to emission by thermal excitation.

- If you draw a small electrical spark from a copper wire you will see the green light
characteristic of copper. Here the excitation is both thermal and electrical.

- In this experiment, you will study the light that is emitted when gas atoms are subjected to an electric discharge. You will find that these atoms emit light of several different wavelengths (colors) that are, together, characteristic of the atoms that have emitted them. These colors are often called “spectral lines” from their appearance in spectrometers such as the ones that you will be using. By measuring many of the different spectral lines emitted by a particular element one can obtain detailed information about the energy levels of its atoms. A thorough study of the spectrum of the simplest element, hydrogen, led physicists to the development of quantum mechanics, the theory that takes the place of Newtonian mechanics in the submicroscopic realm and which led in turn to a complete understanding of the atomic structure of all elements.

Even with your simple spectrometer you will be able to measure the wavelengths of spectral lines with an accuracy of 3 significant figures. With modern research equipment one can measure wavelengths to six or more significant figures. Quantum mechanics can account for these experimental results down to the last decimal place. It is this kind of agreement between theory and experiment that makes us confident that quantum mechanics provides an accurate description of nature at the most fundamental level.

An excellent instrument to measure the wavelengths of light is the diffraction grating, a metal mirror into which a large number of closely spaced parallel lines have been inscribed. Light is specularly reflected only by the smooth metal surface and scattered at random by the grooves. A grating is therefore equivalent to a large number of narrow parallel slits. The grating you will be using is actually a film replica of such a grating to be used with transmitted light. In order to understand how a grating works, we
recall\footnote{For a more detailed description of the double slit experiment you might want to have another look at the chapter on the Diffraction and Interference of light in this manual.} the double slit experiment of Thomas Young: If one shines monochromatic light with a wavelength $\lambda$ on two very narrow slits that are a distance $d$ apart (see Fig. 1), constructive interference will result at an angle $\Theta$ at which

$$\sin \Theta = \frac{\lambda}{d} = \frac{y}{D}. \quad (4)$$

Now consider a grating consisting of very many narrow equidistant slits. For each adjacent pair of slits, constructive interference will result when Eq. (4) is satisfied. In other words a grating in which the individual “slits” are a distance $d$ apart will act like a double slit with the same slit to slit distance $d$. Well, almost like a double slit: While the “spectral lines”, i.e. the regions of highest intensity, will appear at the same angles in either case, the lines produced by the grating will be the brighter and narrower the more slits it has. This can easily be explained quantitatively, but we will just give a handwaving argument.

The constructive interference from all the “slits” in the grating will occur at the same angle and make the lines that much brighter. However, the destructive interference will also be more effective. Assume that we observe a double slit from an angle at which the path length difference is not 1.00 but 1.01 $\lambda$. Clearly that will mean almost complete constructive interference. If we view a grating from the same angle, there will be another “slit” 50 $d$ away whose light will be half a wavelength out of step and will lead to complete destructive interference with the light from the first “slit”, etc., etc.

Just as the double slit produces lines of higher order when the path length difference is an integer multiple of one wavelength, so will the grating. In the case of the grating, however, it is possible by cutting the grooves to a certain shape, a process called blazing, to concentrate most of the light intensity in the first order spectrum.

You will measure the wavelengths of spectral lines in the visible parts of the spectra of several gases. Hydrogen, being the simplest element, one proton and one electron, has the simplest spectrum. We can interpret the lines we see in the hydrogen spectrum and measure one of the most fundamental numbers of atomic physics, the Rydberg. This is the amount of energy required to remove the electron completely from the proton (divided by $hc$ where $h$ is Planck’s constant and $c$ is the velocity of light).

The three most prominent lines in the hydrogen spectrum are a red line at a wavelength of 656.3 nm, a blue-green line at 486.1 nm, and a purple line at 434.1 nm. These are members of the famous Balmer series. You may also be able to see the fourth line of the series at 410.2 nm in the deep purple. The wavelengths of the hydrogen spectral lines are given by

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad (5)$$
where \( m \) and \( n \), \((m < n)\), are integers; \( n \) represents the initial, higher, energy state from which the atom makes a **transition** to the final, lower, energy state represented by \( m \). In the case of the Balmer series, \( m = 2 \) while \( n \) is equal to 3 for the red line, 4 for the blue-green line, and 5 for the purple line, as shown in Fig. 2. In other words the three lines that you see are caused by the electrons falling from the third, fourth, and fifth levels into the second. In each case the energy difference between the two levels is converted into light energy according to

\[
E_m - E_n = \frac{hc}{\lambda}.
\]

The ionization energy of hydrogen is the energy needed to remove the electron from the ground state \((m = 1)\) to an *unbound* state \((n = \infty)\). This energy is just \( E = hcR \).

**REFERENCES**

Read the section on the hydrogen spectrum in your textbook.

**APPARATUS**

Spectrometer, diffraction grating, spectrum tube power-supply, spectrum tubes, sodium arc lamp.

**WARNING:**

PLEASE DO NOT TOUCH THE SURFACES OF THE DIFFRACTION GRATING. FINGERPRINTS WILL DESTROY IT. PLEASE DO NOT TOUCH THE ENGRAVED SCALES OF THE SPECTROMETER. FINGERPRINTS WILL CORRODE THEM.

**WHAT TO DO**

A picture of the spectrometer can be found in Fig. 3. Refer to the figure as necessary.

**DO NOT UNSCREW OR DISASSEMBLE THE APPARATUS IN ANY WAY!!!**

1. Plug the Sodium lamp in when you start the experiment. *It needs 20 minutes to warm up.*
2. Remove the spectrometer table holding the grating. Make sure that the eyepiece is fully inserted into the telescope tube. Focus the eyepiece until the cross hairs are in sharp focus; then focus the telescope for parallel light by focusing on some distant object. Bricks or shingles of distant buildings make excellent targets. The focus adjustment is the knurled knob on the side of the telescope.

3. Position the lamp in front of the slit (as shown in Fig. 4) and make the slit narrow, without closing it completely. Align the telescope with the collimator: Swing the telescope arm until, looking through the telescope, you can see the collimator slit. Focus the collimator by turning the knurled knob on the side of the collimator until the slit is in sharp focus. (You may need to make fine adjustments to the telescope’s focus as well.) Rotate the eyepiece until the cross hairs (“crossline graticule” in Fig. 3) in the telescope look like this: $\times$. If you now adjust the size of the slit, how does the image in the telescope change? Therefore, when making measurements of the angle, where should you place the intersection of the crosshairs on the spectral lines? Adjust the slit width to an appropriate size. If the image of the slit is too high or too low in the field of the telescope ask your instructor to adjust it. Lock the telescope with the knob under it.

4. A digital readout and display has been added to the spectrometer. Turn on the display. (If it
Fig. 4. Placing the light source, aligning the telescope, and measuring the angle of diffraction.

does not work, i.e. the instructor cannot make it work, you will need to measure angles using the vernier; follow the instructions in the brackets below and refer to the instructions the instructor will give you on how to read the vernier. You may want to use the vernier at first to verify that the digital readout is working properly.) One can set the angle on the digital display to zero by pushing the PRESET button. It is best to measure a spectral line on the left first (set the display angle to zero) followed by the same line on the right. Note that the display will read a number equal to twice the angle of interest.

BE CAREFUL NOT TO BUMP OR DISCONNECT THE CONNECTOR FOR THE DIGITAL METER ON THE TURNTABLE.

[If the digital display is not working correctly: Turn the spectrometer table until the 0° mark on the table is aligned with the 0° mark on the vernier. Lock the table in that position.]

CAUTION: THE SPECTRUM TUBES REQUIRE A HIGH OPERATING VOLTAGE. MAKE SURE THE POWER SUPPLY IS TURNED OFF WHENEVER YOU INSERT OR TAKE OUT A SPECTRUM TUBE.

5. Replace the spectrometer table (which holds the diffraction grating) on the spectrometer and align it by eye so that it is perpendicular to the collimator tube and in the center of the table.
6. Put the hydrogen spectrum tube into the spring-loaded holders of the power supply, and **then** turn it on. Set the tube as close as possible to the collimator slit. Look at the slit through the telescope and position the spectrometer with respect to the spectrum tube for maximum light intensity.

7. Unlock the telescope, swing it out to the left, and look for the first order spectral lines of hydrogen’s Balmer series. Make sure you can see at least 3 of them on each side, one violet, one blue-green, and one red. (If you cover the spectrometer with the blackout cloth and look carefully, you may be able to see the fourth deep violet line.) You may have to open the slit somewhat to see them clearly, but the wider the slit is, the less accurately you will be able to resolve closely spaced lines. Find a reasonable compromise. If you cannot make the necessary adjustments to the apparatus to see sharp lines, call your instructor.

8. Carefully position the cross hairs appropriately on one of the first order lines. Set the display angle to zero and then swing the telescope over to the right and position the cross hairs on the same first order line. (If the grating is not level, the spectral lines on both sides will not appear at the same height; call your instructor.) Take the angle on the display and divide by two. Repeat this procedure for the other spectral lines. Use the results and Eq. (4) to calculate the wavelengths of the lines you have measured. (The spacing \( d \) of the grating lines is printed on the grating.) Turn off the spectrum tube power supply.

Compare your measured wavelengths to the expected values.

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**CAUTION: THE SPECTRUM TUBES WILL BE VERY HOT AFTER USE. ALLOW THEM TO COOL BEFORE HANDLING THEM!**

9. Think of a way to plot your results so that you get a straight line whose slope is equal to the Rydberg constant. (Let Eq. (5) suggest a way.) How does it compare to the theoretical value? Using the value of the Rydberg constant that you have just measured, calculate the ionization potential of hydrogen, i.e. the amount of energy (in electron volts, eV) that it takes to remove the electron completely from the proton. Useful numbers for this are \( c = 2.998 \times 10^8 \) m/s, \( h = 6.626 \times 10^{-34} \) J \cdot s, 1 eV = \( 1.602 \times 10^{-19} \) J, 1 \( \mu \)m = \( 10^{-6} \) meters.

The energy levels of an atom depend not only on the electrostatic force between its nucleus and its
electrons but also (very slightly) on effects due to the intrinsic magnetic moment of the electrons. Consider the sodium atom, whose outer most electron orbits the nucleus and other electrons. From this electron’s point of view, the rest of the atom (which is positively charged) orbits around this electron. This moving charge gives rise to a current, which in turn gives rise to a magnetic field \( B \). It is with this magnetic field that the electron’s magnetic moment interacts. If the electron’s moment \( \mu \) is aligned with the field, its energy state is slightly lower than if \( \mu \) is aligned opposite to the field. The difference in this energy \( \Delta \epsilon \) can be calculated and is found to be \( \Delta \epsilon = \mu \cdot B \), where \( \mu = \frac{e\hbar}{2mc} \). (Note that \( h = \frac{\hbar}{2\pi} \). You can find the values of these constants in Appendix A.)

This effect can be most easily seen in the bright yellow line from sodium, called the sodium D line. This ‘line’ (\( \lambda \approx 589.3 \) nm) is actually two closely spaced lines that are due to transitions of outer electrons (of different atoms) from the two states discussed above. (Hence, the D stands for doublet.) By careful adjustment of the spectroscope you should be able to separate the lines belonging to these two energy levels.

10. Measure the wavelength \( \lambda \) of the sodium D lines in the first order and second order spectrum. In the second order spectrum estimate the angle between the two lines. If you only see one line, you may need to reduce the size of your slit as well as adjust the focus of the telescope. Use the angle between the lines to estimate the difference in wavelength. Optionally, you can calculate the effective magnetic field \( B \) that the outer electron of a sodium atom “sees”.

Look at the spectra of the gases in the other spectrum tubes: Helium has two electrons that do not only interact with the He nucleus but also with each other. This makes the spectrum a great deal more complicated than the hydrogen spectrum and, in general, too complicated for you to unravel. Neon has a very rich spectrum, dominated by red lines that are responsible for the beautiful red glow of neon tubes. The nitrogen spectrum is much more complex than any of the others because nitrogen consists of diatomic molecules: The light is emitted by the atomic electrons but while this happens the two atoms vibrate against each other while orbiting around one another. The most intense lines in the mercury spectrum are in the blue and green regions, which give the light from mercury lamps its characteristic color.

11. Describe the spectrum of each tube and relate the spectrum to the color of the light emitted by the tube (as seen by the naked eye).

Despite the complexity of the helium atom, there is one He line whose wavelength you ought to be
able to calculate. Often, one of the two electrons is completely removed, leaving a He ion which is hydrogen-like, i.e. it consists of a central nucleus and one electron. The only difference between such an ion and a genuine hydrogen atom is that its nucleus has a nuclear charge $Ze = 2e$. The charge $Z$ enters as the square in Eq. (5), which thus becomes

$$\frac{1}{\lambda} = Z^2 R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) = 4R \left( \frac{1}{m^2} - \frac{1}{n^2} \right), \quad (7)$$

The strong blue line that you see in the He spectrum comes from singly ionized helium.

12. Are your eyes more sensitive to green or to red light? Hint: look at the spectrum from the white of the desk lamp.

13. Describe the difference between incandescent (desk lamp) and fluorescent light. Observe the spectrum of the fluorescent lamp and identify which gas is used as the starter for the phosphor coating the bulb. Hint: refer back to your descriptions in step 11.

If time permits:

14. Measure the wavelength of the blue He line and determine the final ($m$) and initial ($n$) states of the underlying transition.

15. Measure the wavelengths for the four bright lines in the Hg spectrum. What color is missing?

**PLEASE CLEAN UP YOUR LAB AREA. TURN OFF THE LAMPS, THE DIGITAL READOUT, AND THE SPECTRUM TUBE POWER SUPPLY.**