

Name \_\_\_\_\_ Date \_\_\_\_\_ Partners \_\_\_\_\_

## GAUSS' LAW

On all questions, work together as a group.

1. The statement of Gauss' Law:

(a) in words:

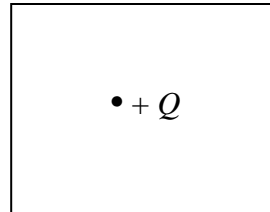
**The electric flux through a closed surface is equal to the total charge enclosed by the surface divided by  $\epsilon_0$ .**

(b) in symbols:

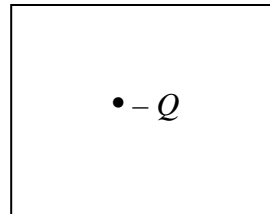
$$\oint_{\text{closed surface } S} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

2. The next few questions involve point charges.

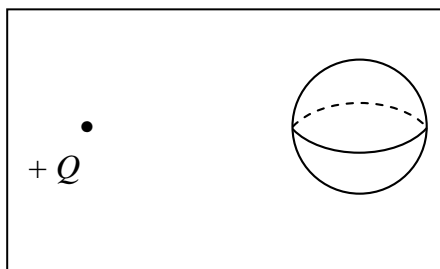
(a) Draw the electric field lines in the vicinity of a positive charge  $+Q$ .



(b) Do the same for a negative charge  $-Q$ .



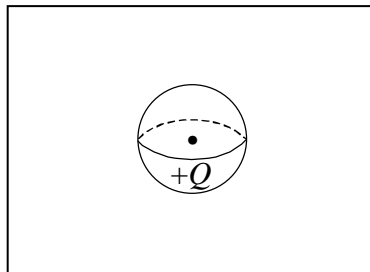
3. Consider a "Gaussian sphere," outside of which a charge  $+Q$  lies. Remember, a Gaussian surface is just a mathematical construct to help us calculate electric fields. Nothing is actually there to interfere with any electric charges or electric fields.



- (a) Draw several electric field lines from  $+Q$ , but only ones that intersect the sphere. (For this question, omit field lines that don't intersect the sphere. This is to keep the drawing looking neat.)
- (b) How much charge is enclosed by the sphere? Applying Gauss' Law, what is the total electric field flux through the sphere? Justify your answer.
- (c) Looking back at your drawing, field lines impinge on the spherical surface from the outside heading inward (this is defined as negative flux) and eventually impinge on another part of the surface from the inside heading outward (positive flux). Does it seem reasonable that the total flux through the sphere is exactly zero? Explain why the total flux through the sphere is exactly zero when the field lines exit through a larger area.
- (d) Suppose we replaced the sphere with a cube. Would the total flux still be zero? Explain.

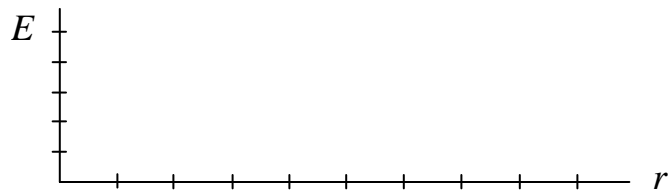
Note that it would be difficult to actually calculate the electric field flux through these surfaces (although you certainly could do it), because the electric field strength and angle of intersection vary over the surface. Applying Gauss' Law, however, made it easy.

4. Now consider a Gaussian sphere centered on  $+Q$ .



- (a) Draw some electric field lines. Make them long enough to intersect the sphere.

- (b) Is the total electric field flux through the sphere positive or negative? Does this make sense, considering the charge enclosed?
- (c) In symbols, what is the total flux through the sphere? (Use equation in step 1b.)
- (d) Notice that in this case the electric field lines intersect the sphere perpendicular to its surface and that the electric field strength is uniform over the surface. How do we know that the electric field strength does not vary over the surface?
- (e) Because of the simplifying conditions discussed in part (d), we can apply Gauss' Law to find the electric field due to  $+Q$ . Find the electric field for a point charge  $+Q$ . (Recall that the surface area of a sphere is  $4\pi r^2$ .)
- (f) Graph  $E$  (the magnitude of  $\mathbf{E}$ ) versus  $r$ , using the axes given. Both axes are linear.



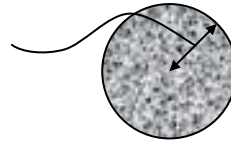
- (g) If we had picked a cube as our Gaussian surface instead of a sphere, would it still have been easy to determine the total flux through the surface? What about calculating the electric field strength? Explain.

5. The previous problem was mathematically fairly simple. Here's another problem requiring Gauss' Law, but this time you will have to do a bit of integration.

**NOTE:** Keep your results in symbolic form and only substitute in numbers when asked for a numerical result. Also, pay careful attention to the distinction between the radius of the sphere,  $R$ , and the distance,  $r$ , from the center of the sphere at which you are evaluating  $\mathbf{E}$ .

Consider a small sphere (an actual sphere, not a Gaussian surface) of radius  $R = 0.1$  m that is charged throughout its interior, but not uniformly so. The charge density (in  $\text{C}/\text{m}^3$ ) is  $\rho = Br$ , where  $r$  is the distance from the center, and  $B = 10^{-4} \text{ C}/\text{m}^4$  is a constant. Of course, for  $r$  greater than  $R$ , the charge density is zero.

$$R = 0.1 \text{ m}$$



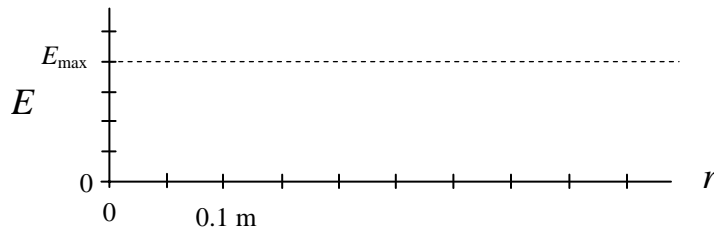
- (a) What is the charge density at the center? Does it increase or decrease as we move toward the surface?
- (b) What is  $\mathbf{E}$  as the center is approached? How do you know? How does this compare to the  $\mathbf{E}$  of a point charge?
- (c) Apply Gauss' Law to find an expression for  $E$  when  $r$  is less than  $R$ . [Hint: the volume of a thin spherical shell is  $dV = 4\pi r^2 dr$ .]

(d) What is  $E$  at  $r = 0.05$  m? [Your answer should be in N/C.] Use  $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$ .

(e) Apply Gauss' Law to find an expression for  $E$  when  $r$  is greater than  $R$ .

(f) What is  $E$  at  $r = 5.0$  m?

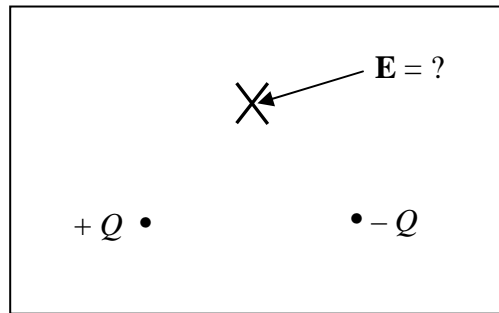
(g) Make an approximate graph of  $E$  versus  $r$  on the axes shown.



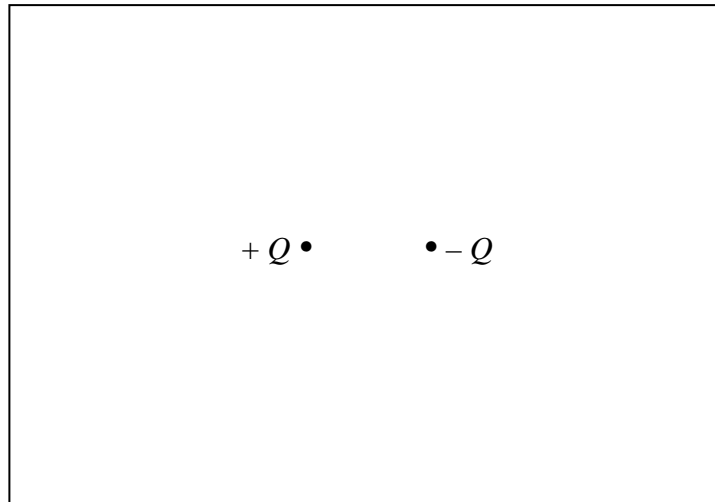
(h) Determine the total charge inside the sphere. Outside the sphere, how does  $\mathbf{E}$  compare to  $\mathbf{E}$  of a point charge of that magnitude? Verify for  $r = 5.0$  m.

(i) On your graph in part (g) above, use a dotted line to represent the electric field if the sphere shrank to a point charge, but still contained the same total charge.

6. Consider two point charges of opposite sign as shown:



- (a) In what direction is the electric field at the point indicated? Draw a vector on the sketch above at the **X** to represent **E**. [Hint: Use superposition.]
- (b) On the sketch below, draw several (at least ten) electric field lines for the configuration above. This should be enough to indicate **E** in much of the vicinity of the two charges.



- (c) On your drawing above, add a spherical Gaussian surface that encloses both charges, centered on the point midway between the charges. Make it fairly large, but be sure several of the electric field lines penetrate the Gaussian sphere. Add more field lines as needed.
- (d) Look carefully at your drawing above. In what direction does **E** point (inward? outward? tangent?) at various locations on the Gaussian surface? Redraw the Gaussian surface below and draw short arrows on the surface indicating the direction of **E** on the Gaussian surface.

- (e) Consider the following argument from a student who is trying to determine  $\mathbf{E}$  somewhere on the previous Gaussian surface:

"The total charge enclosed by the surface is zero. According to Gauss' law this means the total electric field flux through the surface is zero. Therefore, the electric field is zero everywhere on the surface."

Which, if any, of the three sentences are correct?

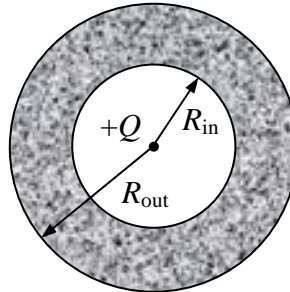
Explain how the student came to an incorrect conclusion.

- (f) Refer back to 6a. If you were asked to calculate the electric field at that point, would you attempt to apply Gauss' Law or would you use another method? Explain.

Gauss' law is always true, no matter how complicated the distribution of electric charges. In fact it's even true when the charges are in motion. However, it's rare to find a situation with enough symmetry that applying Gauss' law becomes a convenient method to calculate the electric field.

**Electric fields in the vicinity of a conductor**

A point charge  $+Q$  lies at the center of an uncharged, hollow, conducting spherical shell of inner radius  $R_{\text{in}}$  and outer radius  $R_{\text{out}}$  as shown. Your ultimate goal is to find the electric field at all locations for this arrangement.



7. Why is it important to know whether the shell is a conductor or insulator? Is there any real difference?
  
  
  
  
  
  
  
  
  
  
8. What does the charge distribution within the shell itself (between  $R_{\text{in}}$  and  $R_{\text{out}}$ ) look like
  - (a) before the point charge  $+Q$  is inserted? (draw a picture)
  
  
  
  
  
  
  
  
  
  
  - (b) with the point charge  $+Q$  in place? (draw a picture)
  
  
  
  
  
  
  
  
  
  
  - (c) Are the charge distributions the same? Why or why not?



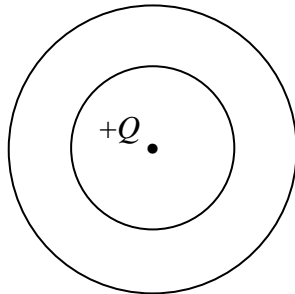
9. What is the electric field within the shell itself (between  $R_{\text{in}}$  and  $R_{\text{out}}$ )

(a) before the point charge  $+Q$  is inserted?

(b) after the point charge  $+Q$  is inserted?

(c) How did the properties of a conductor help you answer the two previous questions?

10. We are still considering the conducting shell. Draw electric field lines in the region  $0 < r < R_{\text{in}}$  ( $r$  is the distance from the point charge) and for the region  $r > R_{\text{out}}$ . What are the field lines between  $R_{\text{in}}$  and  $R_{\text{out}}$ ? Also show the relative amount of charge that has moved to the inner and outer surfaces of the conductor.



11. Use Gauss' law to find the electric field as a function of  $r$  for all three regions, and then graph it below.



12. Compare the electric field for  $r$  just inside  $R_{\text{in}}$  and just outside  $R_{\text{out}}$ . Are the magnitudes equal? Explain.
13. In specifying the electric field, do you need to give any other information besides the distance from the point charge (for instance, whether you're on the right side or the left side of the point charge, or something like that)? Explain.