CAPACITORS & RC CIRCUITS

OBJECTIVES

- Define capacitance and learn to measure it with a digital multimeter.
- Explore how the capacitance of conducting parallel plates is related to the area of the plates and the separation between them.
- Explore the effect of connecting a capacitor in a circuit in series with a resistor or bulb and a voltage source.
- Explore how the charge on a capacitor and the current through it change with time in a circuit containing a capacitor, a resistor and a voltage source.

OVERVIEW

Capacitors are widely used in electronic circuits where it is important to store charge and/or energy or to trigger a timed electrical event. For example, circuits with capacitors are designed to do such diverse things as setting the flashing rate of Christmas lights, selecting what station a radio picks up, and storing electrical energy to run an electronic flash unit. Any pair of conductors that can be charged electrically so that one conductor has positive charge and the other conductor has an equal amount of negative charge on it is called a capacitor.

A capacitor can be made up of two arbitrarily shaped blobs of metal or it can have any number of regular symmetric shapes such as one hollow metal sphere inside another, or a metal rod inside a hollow metal cylinder.

Figure 1-1: Some different capacitor geometries
The type of capacitor that is the easiest to analyze is the parallel plate capacitor. We will focus exclusively on these.

Although many of the most interesting properties of capacitors come in the operation of AC (alternating current) circuits (where current is first moves in one direction and then in the other), we will limit our present study to the behavior of capacitors in DC (direct current) circuits.

The circuit symbol for a capacitor is a simple pair of lines as shown in Figure 1-2. Note that it is similar to the symbol for a battery, except that both parallel lines are the same length for the capacitor.

In Investigation 1 we will measure the dependence of capacitance on area and separation distance. In Investigation 2 we shall learn how capacitances react when charge builds up on their two surfaces. We will investigate what happens to this charge when the voltage source is removed and taken out of the circuit.

**INVESTIGATION 1: CAPACITANCE, AREA AND SEPARATION**

The usual method for transferring equal and opposite charges to the plates of a capacitor is to use a battery or power supply to produce a potential difference between the two conductors. Electrons will then flow from one conductor (leaving a net positive charge) and to the other (making its net charge negative) until the potential difference produced between the two conductors is equal to that of the battery. (See Figure 1-3.)

In general, the amount of charge needed to produce a potential difference equal to that of the battery will depend on the size, shape, location of the conductors relative to each other, and the properties of the material between the conductors. The capacitance, or the ability to store electric charge, of a given capacitor is defined as the ratio of the magnitude of the charge, \(q\) (on either one of the conductors) to the voltage (potential difference), \(V\), applied across the two conductors. Thus,

\[
C = \frac{q}{V} \quad (1)
\]
Capacitance is defined as a measure of the amount of net or excess charge on either one of the conductors per unit potential difference. Thus the more charge a capacitor can store at a given voltage, the larger the capacitance.

![Diagram of a parallel plate capacitor with a voltage V across it.](image)

\[ V = \text{voltage} \]
\[ A = \text{area} \]
\[ d = \text{separation} \]

**Figure 1-3:** A parallel plate capacitor with a voltage V across it.

**Activity 1-1: Predicting the Dependence of Capacitance on Area and Separation.**

Consider two identical metal plates of area \( A \) that are separated by a distance \( d \). The space between the plates is filled with a non-conducting material (air, for instance). Suppose each plate is connected to one of the terminals of a battery.

**Prediction 1-1:** Suppose you now double the area of each plate. Does the voltage between the plates change (recall that the plates are still connected to the battery)? Does the amount of charge on each plate change? Since \( C = q/V \), how must the capacitance change?

**Prediction 1-2:** Now return to the original capacitor. The easiest way to find the dependence of capacitance on separation distance is to charge the plates first and then disconnect the battery. After we do that, the separation distance is doubled. Can the charge on the plates change? Does the electric field between the plates change (assume ideal conditions: plates large compared to separation distance)? How does the voltage between the plates change? Since \( C = q/V \), how must the capacitance change?
The unit of capacitance is the farad, F, named after Michael Faraday. One farad is equal to one coulomb/volt. As you should be able to demonstrate to yourself shortly, the farad is a very large capacitance. Thus, actual capacitances are often expressed in smaller units with alternate notation as shown below:

- micro farad: 1 mF = 1 µF = 1 UF = 10^{-6} F
- nano farad: 1 nF = 1000 mmF = 1000 µµF = 1000 UUF = 10^{-9} F
- pico farad: 1 pF = 1 mmF = 1 µµF = 1 UUF = 10^{-12} F

(Note that m, µ, and U when written on a capacitor all stand for a multiplier of 10^{-6}.)

There are several types of capacitors typically used in electronic circuits including disk capacitors, foil capacitors, electrolytic capacitors and so on. You should examine some typical capacitors that should be on your table.

To complete the next few activities you will need to construct a parallel plate capacitor and use a multimeter to measure capacitance. You'll need the following items:

- 2 sheets of aluminum foil 20 cm x 20 cm (8"x 8")
- A "fat" UVa phone book directory
- one or several massive objects (textbooks)
- digital multimeter with a capacitance mode and clip leads
- ruler with a centimeter scale
- micrometer
- a collection of old capacitors

You can construct a parallel plate capacitor out of two rectangular sheets of aluminum foil separated by pieces of paper. Pages in the UVa directory work quite well as the separator for the foil sheets. You can slip the two foil sheets on either side of paper sheets, and weigh the book down with something heavy like some textbooks. The digital multimeter can be used to measure the capacitance of your capacitor.

Activity 1-2: Measuring How Capacitance Depends on Area or on Separation

Be sure that you understand how to use the multimeter to measure capacitance and how to connect a capacitor to it. If you are sitting at an even numbered table, then you will devise a way to measure how the capacitance depends on the foil area. If sitting at an odd
numbered table then you will devise a way to measure how the capacitance depends on the separation between foils. Of course, you must keep the other variable (separation or area) constant.

When you measure the capacitance of your "parallel plates", be sure that the aluminum foil pieces are pressed together as uniformly as possible (mash them hard!) and that they don't make electrical contact with each other. We suggest you cut the aluminum foil so it does not stick out past the pages except where you make the connections as shown in Fig. 1-4. Notice the connection tabs are offset.

![Figure 1-4](Shapes of aluminum foil for capacitors.)

**Hint**: To accurately determine the separation distance, simply count the number of pages and multiply by the nominal thickness of a single page. To determine the nominal page thickness, use the micrometer to measure the thickness of 100 or more pages and divide by the number of pages.

**If you are keeping the separation constant**, a good separation to use is about ½ mm. The area may be varied by using different size sheets of aluminum foil. One convenient way to do it is to simply fold the aluminum foil each time to make the area smaller. Or you may just cut it. The aluminum foil must precisely overlap, so you may not want to divide it exactly in half each time or the final area may be very small.

**If you are keeping the area constant**, use a fairly large area – almost as large as the telephone/directory book you are given. A good range of pages to use for the separation is 0.1 mm to 1.75 mm.

**Important**: When you measure $C$ with the multimeter, be sure to subtract the capacitance of the leads (the reading just before you clip the leads onto the aluminum sheets).

1. Take five data points in either case. Record your data in Table 1-1.
Table 1-1

<table>
<thead>
<tr>
<th>Number of Sheets</th>
<th>Thickness (mm)</th>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Area (m²)</th>
<th>Capacitance (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$C_{leads}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$C_{foil}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$C_{final}$</td>
</tr>
</tbody>
</table>

2. After you have collected all of your data, open the experiment file Dependence of $C$. Enter your data for capacitance and either separation or area from Table 1-1 into the table in the software. Be sure there is no "zero" entry in the case of $C$ vs. separation distance. Graph capacitance vs. either separation or area.

3. If your graph looks like a straight line, use the fit routine in the software to find its equation. If not, you should try other functional relationships until you find the best fit.

4. Print one set of graphs for your group.

**Question 1-3:** What is the function that best describes the relationship between separation and capacitance or between area and capacitance? How do your results compare with your prediction based on physical reasoning?

**Question 1-4:** What difficulties did you encounter in making accurate measurements?
The actual mathematical expression for the capacitance of a parallel plate capacitor of plate area \( A \) and plate separation \( d \) is derived in your textbook. The result is

\[
C = \kappa \varepsilon_0 \frac{A}{d}
\]

(2)

where

\[
\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}
\]

and \( \kappa \) is the dielectric constant.

Question 1-5: Do your predictions and/or observations on the variation of capacitance with plate area and separation seem to agree qualitatively with this result?

Question 1-6: Use one of your actual areas and separations to calculate a value of \( \kappa \) using this equation. Show your calculations. What value of the dielectric constant of paper do you determine? (The actual dielectric constant varies considerably depending on what is in the paper and how it was processed.) Typical values range from 1.5 to 6.

\( \kappa \) __________________________

INVESTIGATION 2: CHARGE BUILDUP AND DECAY IN CAPACITORS

Capacitors can be connected with other circuit elements. When they are connected in circuits with resistors, some interesting things happen. In this investigation you will explore what happens to the voltage across a capacitor when it is placed in series with a resistor in a direct current circuit.

You will need:

- one current and one voltage probe
- 6 V battery
- #133 flashlight bulb and socket
- capacitor (~23,000 µF)
- 6 alligator clip wires
- single pole, double throw switch
- RLC circuit board

You can first use a bulb in series with one of the amazing new ultra capacitors with a large capacitance. These will allow you to see what happens. Then later on, to obtain more quantitative results, the bulb will be replaced by a resistor.

**Activity 2-1: Observations with a Capacitor, Battery and Bulb**

1. Set up the circuit shown in Figure 2-1 using the 23,000 µF capacitor. Be sure that the positive and negative terminals of the capacitor are connected correctly (because of the dielectric, electrolytic capacitors have a definite polarity).

![Circuit Diagram](image)

Figure 2-1: Circuit to examine the charging and discharging of a capacitor through a light bulb.

**Question 2-1:** Sketch the complete circuit when the switch is in position 1, and when it is in position 2. For clarity, don’t draw components or wires that aren’t contributing to the function of the circuit.

position 1  
position 2

2. Move the switch to position 2. After several seconds, switch it to position 1, and describe what happens to the brightness of the bulb.
Question 2-2: Draw a sketch on the axes below the approximate brightness of the bulb as a function of time for the above case of moving the switch to position 1 after it has been in position 2. Let $t = 0$ be the time when the switch was moved to position 1.

![Graph](image)

3. Now move the switch back to position 2. Describe what happens to the bulb. Did the bulb light again without the battery in the circuit?

Question 2-3: Draw a sketch on the axes below the approximate brightness of the bulb as a function of time when it is placed across a charged capacitor without the battery present, i.e. when the switch is moved to position 2 after being in position 1 for several seconds. Let $t = 0$ be when the switch is moved to position 2.

![Graph](image)

Question 2-4: Can you explain why the bulb behaves in this way? Is there charge on the capacitor after the switch is in position 1 for a while? What happens to this charge when the switch is moved back to position 2?

4. Open the experiment file **Capacitor Decay**, and display $V_{PB}$ and $C_{PA}$ versus time.
5. Connect the probes in the circuit as in Figure 2-2 to measure the current through the light bulb and the potential difference across the capacitor.

![Diagram](attachment:figure22.png)

Figure 2-2: Current and voltage probes connected to Figure 5-4 circuit.

6. Move the switch to position 2. Wait about ten seconds and then **begin graphing**. When the graph lines appear, move the switch to position 1. When the current and voltage stop changing, move the switch back to position 2.

7. **Print** out one set of graphs for your group.

8. Indicate on the graphs the times when the switch was moved from position 2 to position 1, and when it was moved back to position 2 again.

**Question 2-5:** Does the actual behavior over time observed on the current graph agree with your sketches in Questions 2-2 and 2-3? How does the brightness of the bulb depend on the **direction** and **magnitude** of the current through it?

**Question 2-6:** Based on the graph of potential difference across the capacitor, explain why the bulb lights when the switch is moved from position 1 to position 2 (when the bulb is connected to the capacitor with no battery in the circuit)?

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**Activity 2-2: The Rise of Voltage in an RC Circuit**

1. Open the experiment file **RC Circuit**. This will take data at a much higher rate than before, and will allow us to graph the charging of the capacitor, using a smaller C which we can measure with the multimeter.
2. Replace the light bulb in your circuit (Figure 2-2) with a 100 Ω resistor, and the large capacitor with one in the 80 µF to 120 µF range. (Use the RLC circuit board.) Move the switch to position 2. **Begin graphing** and immediately move the switch to position 1. Data taking will cease automatically.

3. You should see an exponential curve which, for the charging of the capacitor, is:

   \[ V(t) = V_f \left(1 - e^{-t/RC}\right) \]  

   Use the **Smart Tool** to determine from your graph the *time constant* (the time for the voltage across the capacitor to reach 63% [actually \(1 - 1/e\)] of its final value--after the switch is moved to position 1). Record your data below.

   - Final voltage:________
   - 63% of final voltage:________
   - Time constant:________

4. Convince yourself from the equation above that the time constant must equal exactly \(RC\). Measure \(R\) and \(C\) with the multimeter, calculate the time constant, and compare to your data above.

   - \(R\):________ Ω
   - \(C\):________ µF
   - \(RC\):________ s
   - Percentage difference:________

5. Now you can use the software to fit an "Inverse Exponent Fit" to your data. The data box should already be present on your voltage graph. Double click on it, choose manual, type in the expected parameters, and click "accept." Look at the mean square error. It should be much less than 1.0. If not, perhaps you need to look at the fit of the data for \(t < 0\). Choose the range of times that you want to fit.

   Equation of curve fit:

   Parameters for a good fit:
6. **Print** one set of graphs for your group.

**Question 2-7:** What is the significance of the parameter “B” in your fit? What should its value be?

**Question 2-8:** Which of the parameters in the function that you fit allows you to find the time constant? Calculate the time constant from your fit, and compare to what you found from your measured values of \( R \) and \( C \). How good is the agreement?

Parameter(s): ______________________

Calculation of time constant from parameter(s):

Calculated time constant (from step 4): ________________

Agreement (%): ________________