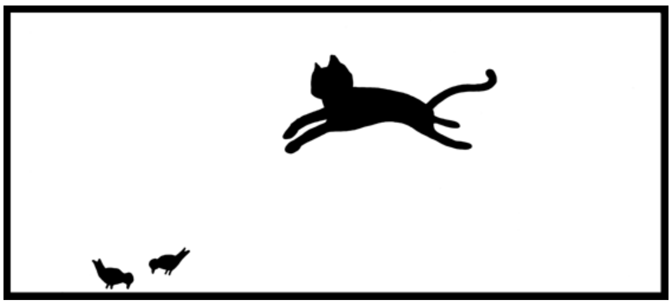
Name ____ Date _________________ Section ____________

UNIT 6: GRAVITY AND PROJECTILE MOTION

Dolphins are powerful, graceful, and intelligent animals. As this dolphin leaps out of the water in delighted play, she experiences a change in velocity that is the same as that of any other mass moving freely close to the surface of the earth. She is undergoing what physicists call projectile motion. The path she follows while above the water has the same mathematical characteristics as the path of a ball, a ballet dancer performing a grand jeté, or Michael Jordan executing a slam dunk. What is this path like? What kind of force causes projectile motion? In this unit we will use a faith in Newton's Second Law to discover the nature of the Earth's gravitational attraction for masses and to explore the mathematical behavior of free motions close to the Earth's surface.

UNIT 6: GRAVITY AND PROJECTILE MOTION



Science is a game . . . with reality. . . . In the presentation of a scientific problem, the other player is the good Lord. He has . . . devised the rules of the game—but they are not completely known, half of them are left for you to discover or deduce . . . the uncertainty is how many of the rules God himself has permanently ordained, and how many apparently are caused by your own mental inertia, while the solution generally becomes possible only through freedom from its limitations. This is perhaps the most exciting thing in the game. For here you strive against the imaginary boundary between yourself and the Godhead—a boundary that perhaps does not exist.

Erwin Schrödinger

OBJECTIVES

1. To explore the phenomenon of gravity and study the nature of motion along a vertical line near the earth's surface.

2. To use Newton's laws to invent or discover invisible forces for describing phenomena such as "gravity."

3. To learn to describe positions, velocities, and accelerations using vectors.

4. To understand the experimental and theoretical basis for describing projectile motion as the superposition of two independent motions: (1) a body falling in the vertical direction, and (2) a body moving in the horizontal direction with no forces.

5. To observe the similarity between the type of motion that results from projectile motion and that which results from tapping a rolling ball continuously.

6.1. OVERVIEW

When an object is dropped close to the surface of the earth, there is no obvious force being applied to it. Whatever is causing it to pick up speed is invisible. Most people casually refer to the cause of falling motions as the action of "gravity." What is gravity? Can we describe its effects mathematically? Can Newton's laws be interpreted in such a way that they can be used for the mathematical prediction of motions that are influenced by gravity? We will study the phenomenon of gravity for vertical motion in the first few activities in this unit.

Next you will prepare for the mathematical description of two-dimensional motion by learning about some properties of two-dimensional vectors, that can be used to describe positions, velocities, and accelerations. Finally, you will study projectile motion, in which an object accelerates in one dimension and moves at a constant velocity in the other.

VERTICAL MOTION

6.2 DESCRIBING HOW OBJECTS FALL*

Let's begin the study of the phenomenon of gravity by predicting and then observing the motion of a couple of falling objects. For these activities you will need:

- 1 small rubber ball
- 1 flat-bottomed coffee filter

Recommended Group Size:	All	Interactive Demo OK?:	Y

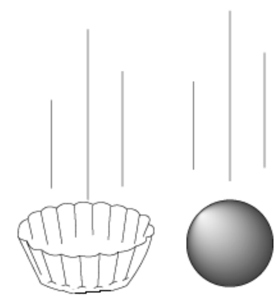


Fig. 6.1.

6.2.1. Activity: Predicting Falling Motions

a. Predict how the ball will fall in as much detail as possible. For example, will it speed up quickly to a constant falling velocity and then fall at that velocity? If not, what will it do? Explain the reasons for your prediction.

b. Predict how the coffee filter will fall in as much detail as possible. For example, will it speed up quickly to a constant velocity of fall? If not, what will it do? Do you think the filter will fall more slowly or more rapidly than the rubber ball? Explain the reasons for your prediction.

c. Now predict how the coffee filter will fall *if it is crumpled up into a little ball*. For example, will it speed up quickly to a constant velocity of fall? If not, what will it do? Do you think the crumpled filter will fall more slowly or more rapidly than the rubber ball? Explain the reasons for your prediction.

*This activity was suggested by Professor Richard Hake of the University of Indiana.

At this point you should observe what actually happens. Someone in your class should stand on a table or chair and drop the rubber ball and the coffee filter at the same time. Then the observation should be repeated with the rubber ball and the crumpled coffee filter falling at the same time.

6.2.2. Activity: Predicting Falling Motions

Describe your observations. How did they compare with your predictions. If your predictions and observations were not the same in each case, develop a new explanation for your observations.

6.3. DESCRIBING HOW OBJECTS RISE AND FALL

Let's continue the study of the phenomenon of gravity by predicting the nature of the motion of an object, such as a small rubber ball, when it is tossed up and then allowed to fall vertically near the surface of the earth. This is not easy since the motion happens pretty fast! To help you with this prediction, toss a ball in the laboratory several times and see what you think is going on. After making the prediction, you will be asked to use a technique for studying the motion in more detail than you can by using direct observation. You can either analyze a video of the motion of a ball toss or use a motion sensor with a computer-based laboratory system to record position vs. time for a toss. In order to make the prediction and observations, you will need:

- 1 small rubber ball
- 1 video analysis system w/VideoPoint software
- 1 meter stick

—or—

- 1 basketball
- 1 computer-based laboratory system
- 1 ultrasonic motion sensor
- 1 spreadsheet software

Recommended Group Size:	3	Interactive Demo OK?:	Y

6.3.1. Activity: Predicting the Motion of a Tossed Ball

a. Toss a ball straight up a couple of times and then describe how you think it might be moving when it is moving *upward*. Some possibilities include: (1) rising at a constant velocity; (2) rising with an increasing acceleration; (3) rising with a decreasing acceleration; or (4) rising at a constant acceleration. What do you think?

b. Explain the basis for your prediction.

c. Now describe how you think the ball might be moving when it is moving *downward*. Some possibilities include: (1) falling at a constant velocity; (2) falling with an increasing acceleration; (3) falling with a decreasing acceleration; or (4) falling at a constant acceleration. What do you think?

d. Explain the basis for your prediction.

e. Do you expect the acceleration when the ball is rising to be different in some way than the acceleration when the ball is falling? Why or why not?

f. What do you think the acceleration will be at the moment when the ball is at its highest point? Why?

The motion of a tossed ball is too fast to observe carefully by eye without the aid of special instruments. We recommend that you use a video camera to film the ball at a rate of 30 frames per second; you can then replay the film a single frame at a time. **Note:** If you don't have a video analysis system available, you may analyze a ready-made digitized movie of a ball toss using video analysis software. Alternatively, you can use a computer-based laboratory system with a motion detector to track the motion of a basketball.

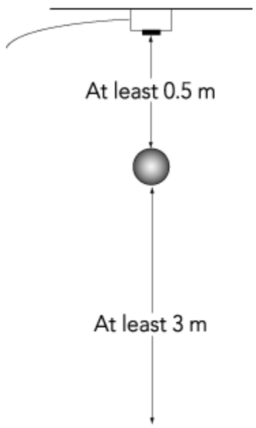


Fig. 6.2.

To Use a Computer-Based Lab

1. Tape the motion detector to a light fixture or something else as high above the floor as possible, with the detector looking downward, as shown on the right.
2. Since the fall is quite rapid, set up the motion software to record position vs. time at 30 points/second.
3. If you want to use a conventional coordinate system in which down is negative, you should set the motion software to designate distance from the motion sensor as negative.
4. It is the data describing a bounce that has the rise and fall in it. Thus, as soon as you start recording data, drop the basketball and allow it to bounce once. Transfer just the bounce data to your spreadsheet.

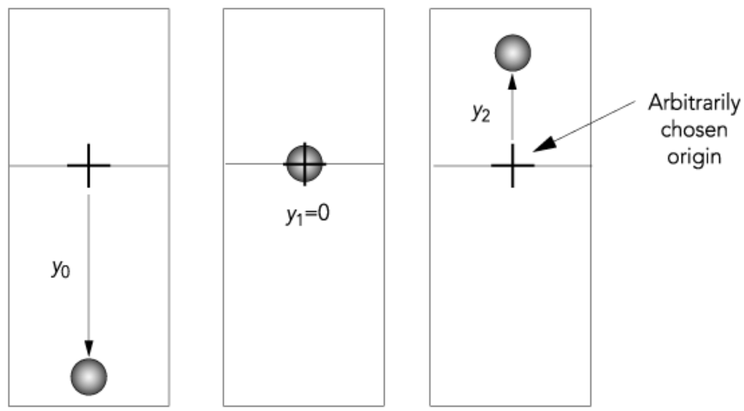


Fig. 6.3. Three "snapshots" of a ball rising showing suggested notation for keeping track of its positions as recorded by a video analysis system. Which *y*-values are positive and which are negative? The location of the origin is chosen arbitrarily.

Fig. 6.4.

6.3.2. Activity: Observing Motion of a Ball

a. Follow the directions to find times and corresponding distances from an origin of your choice as the object rises and falls. Record up to 36 data points in the table below or attach a computer printout of your data table.

	t(s)	*y*(m)
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		

	t(s)	*y*(m)
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		
21.		
22.		
23.		
24.		

	t(s)	*y*(m)
25.		
26.		
27.		
28.		
29.		
30.		
31.		
32.		
33.		
34.		
35.		
36.		

b. Place the data in a spreadsheet and use a computer graphing routine to plot a graph of y vs. t. Affix a copy of the graph in the space below.

c. Use a sketch to describe where you decided to place your origin. What is the initial value of y (usually denoted y_0) in the coordinate system you chose?

d. By examining your data table, calculate the *approximate* value of the initial velocity of the ball in the y-direction. Include the sign of the velocity and its units. (Use the convention that on the y-axis up is positive and down is negative.)

Now that you have collected and graphed data for the rise and fall of a ball, in the next two activities you will consider the types of motion separately for the rise and the fall. In particular, you will explore whether the rise and fall accelerations are the same or different.

6.3.3. Activity: How a Ball Actually Rises

a. Examine the portion of the graph in Activity 6.3.2b that represents the upward motion of the ball. What is the nature of this *upward* motion? Constant velocity, constant acceleration, an increasing or decreasing acceleration? How does your observation compare with the prediction you made in Activity 6.3.1?

b. Using the convention that on the y-axis up is positive and down is negative, is the *acceleration* of the tossed object positive or negative as it rises (i.e., in what direction is the magnitude of the velocity increasing)?

c. If you think the object is undergoing a *constant* acceleration, use the modeling technique you used in Unit 4, Activity 4.10.1, to find an equation that describes y as a function of t as the ball rises. **Hints:** You might try to model the system with kinematic equation number 1. Since you are dealing with the second dimension (i.e., the vertical dimension), you should replace x with y and x_0 with y_0. Write the equation of motion in the space below. Then use coefficients of time to find the values of a, v_0, and y_0 with the appropriate units. **Note:** Since the acceleration is caused by gravity, our notation for it will be a_g rather than just a.

1. The equation of motion with proper units is:

$y =$

2. The acceleration with proper sign and units is:

$a_g =$

3. The initial velocity with proper sign and units is:

$v_0 =$

4. The initial position with proper sign and units is:

$y_0 =$

Now let's consider the motion of the ball as it falls.

6.3.4. Activity: How a Ball Actually Falls

a. Examine the portion of the graph in Activity 6.3.2b that represents the *downward* motion of the ball. What does the nature of this downward motion look like? Constant velocity, constant acceleration, an increasing or decreasing acceleration? How does your observation compare with the prediction you made in Activity 6.3.1?

b. Using the convention that on the y-axis up is positive and down is negative, is the acceleration of the tossed object positive or negative as it falls (i.e., in what direction is the magnitude of the velocity increasing)?

c. If you think the object is undergoing a constant acceleration, use the modeling technique you used in Unit 4, Activity 4.10.1, to find an equation that describes y as a function of t as the ball falls. **Hints:** You might try to model the system with kinematic equation number 1. Since you are dealing with the second dimension (i.e., the vertical dimension), you should replace x with y and x_0, with y_0. Write the equation of motion in the space below. Then use coefficients of time to find the values of a, v_0 and y_0 with the appropriate units. **Note:** Since the acceleration is caused by gravity, our notation for it will be a_g rather than just a.

1. The equation of motion with proper units is:

$$y =$$

2. The acceleration with proper sign and units is:

$$a_g =$$

3. The initial velocity with proper sign and units is:

$$v_0 =$$

4. The initial position with proper sign and units is:

$$y_0 =$$

Now that you've analyzed the rising and falling motions of the ball, let's put it all together.

6.3.5. Activity: The Acceleration of a Tossed Ball

a. Is the ball's acceleration as it rises the same as or different from its acceleration as it falls? How does this compare to your prediction in Activity 6.3.1? What do you conclude about the acceleration of a tossed ball?

b. Many people are interested in what happens when the ball "turns around" at the top of its trajectory. Some students argue that its acceleration at the top is zero; others think not. What do you think happens to the acceleration at this point?

c. Explain your answer to part b. on the basis of your data, graph, and analysis. **Hint:** Suppose you use the modeling technique on all of your data, instead of separating the data into "rising" and "falling" sections. Does anything special happen at the top of the trajectory?

6.4. WHAT IS GRAVITY?

Hey look, no hands! The object that was tossed experienced an acceleration without the aid of a visible applied force. But if Newton's second law holds, then the net force in the y-direction should equal the mass of the object times its acceleration.

$$\Sigma \vec{F}_y = ma_g$$

where m is the mass of the object. Maybe a belief in Newton's second law can help us explain the nature of gravity mathematically.

In order to do the next two activities about the nature of gravity you will need the following items:

- 1 balance or electronic scale
- 1 rubber ball
- 1 steel ball
- 1 spring scale, 10 N

Recommended Group Size:	2	Interactive Demo OK?:	N

The Net Force Needed for Your Observed Acceleration

First, you should describe the nature of the force that could cause the acceleration you observed.

6.4.1. Activity: Describing Gravity

a. Use a balance or electronic scale (but not the spring scale) to determine the mass of the object you used in the last set of activities in kg and write it in the space below.

m = _______kg

b. Suppose your object was floating in outer space (away from the gravity of the earth, friction, or any other influence) and that Newton's second law holds. Calculate the force in newtons that you would have to apply to your object so it would accelerate as much as the acceleration that you observed in Activity 6.3.2b.

c. If Newton's second law is to be used in the situation where you tossed the object with no visible applied force on it, what force do you need to invent* to make Newton's second law valid? Is the force constant or varying during the time the ball is tossed? What is its magnitude? Its direction? In particular, what is the net force on the object when it is on its way up? At the top of its path? When it is on its way down?

Mass, Acceleration, and Gravitational Influences

So far you have studied the motion of just one object under the influence of the gravitational force you invented or discovered. You should have observed that the acceleration is constant so that the gravitational force is constant. This doesn't tell the whole story. How does the mass of the falling object affect its acceleration? Is the gravitational force constant independent of the mass of the falling object, just the way a horizontal push of your hand might be on a cart that moves in a horizontal direction?

6.4.2. Activity: F_g when Different Masses Fall

a. If you were to drop a massive steel ball and a not very massive rubber ball at the same time, will they fall with the same acceleration? Explain the reasons for your prediction.

b. Release the two objects at the same time. What do you observe? Does it match with your prediction?

c. Use a balance or an electronic scale to determine the mass of the two objects in kg. **Note:** Do not use the spring balance yet.

m_{rubber} = ________kg

m_{steel} = ________kg

*If you already believe Newton's Second Law is an inherent property of nature, then you might prefer to say you discovered the gravitational force. If you feel that you and Newton have been constructing this law on the basis of some interplay between your minds and nature's rules, then you could say you are inventing the idea of gravitational force.

d. Although you only made a casual qualitative observation of the objects you dropped, it turns out that in the absence of air resistance or other sources of friction all objects accelerate at the rate of $a_g = 9.8$ m/s^2 close to the surface of the earth. There are small variations from place to place and, of course, uncertainties in measurements. If both objects accelerate at the same standard rate, calculate the magnitude of the gravitational force exerted on each one.

$F_{g,\text{ rubber}}$ = ____________N

$F_{g,\text{ steel}}$ = _____________ N

e. If you have any object of mass m accelerating at a constant rate given by a_g what is the equation that you should use to determine the gravitational force F_g on it? F_g is often referred to as the weight of an object (see below).

f. Check out some weights using a spring scale which has been calibrated in newtons by filling in the chart below.

Object	With balance or electronic scale Mass(kg)	Calculated using Newton's Second Law F_g(N)	Measured by a spring scale Weight (N)
1. Rubber ball			
2. Steel ball			

Fig. 6.5.

6.5. FORCE AND WEIGHT

The Concept of Force—A Review

Force can be defined in several ways which, happily, seem to turn out to be consistent: (1) It can be defined as a push or pull and measured in terms of the stretch of a rubber band or spring; (2) Alternately, the net or combined force on an object can be defined as the cause of motion; (3) Finally, force can be defined in terms of the pull exerted on a mass by the earth as determined by the stretch of a spring when a mass is hanging from it.

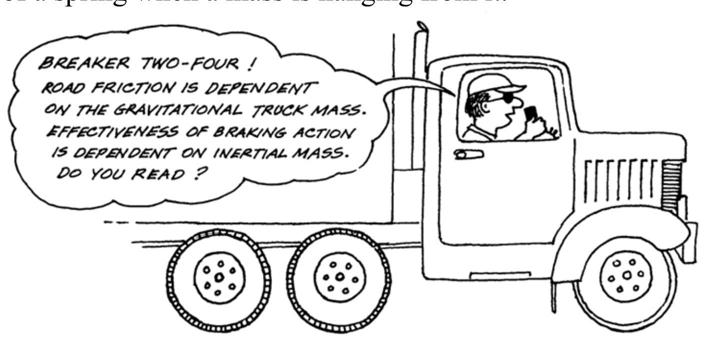


Fig. 6.6.

Is There a Difference Between Mass and Weight?

Weight is a measure of the gravitational force, F_g, on a mass m. Mass represents the resistance to motion. Many individuals confuse the concepts of mass and weight. Now that you understand Newton's laws, you should know the difference. Test your understanding by answering the questions posed in the next Activity. Take the fact that gravitational forces on the moon are about 1/6th of those on the Earth.

6.5.1. Activity: Mass and Weight

a. If mass is a measure of the amount of "stuff" in an object, does an astronaut's mass change on the moon? How can astronauts jump so high?

b. Does the astronaut's weight change on the moon? Explain.

c. If weight is a force, what is pushing or pulling? How is weight related to the acceleration of gravity?

d. When did astronauts experience weightlessness? Could they ever experience masslessness?

2D VECTORS AND PROJECTILE MOTION

6.6. EMULATING FALLING MOTIONS BY WHACKING A BALL

Can you learn how to whack a ball so that you can recreate the type of acceleration you have observed for a ball rolling down a ramp or falling freely? If so, we can use the similarity between a falling ball and a whacked ball to study *projectile motion*, in which an object falls vertically and moves horizontally at the same time.

Let's start with one-dimensional measurements. For the measurements described below you will be using a twirling baton with a rubber tip to tap gently on a duck pin ball, which is rather like a small bowling ball. You and a partner should gather the following equipment to study the motion of the ball:

- 1 duck pin ball (or 1 bowling ball)
- 1 twirling baton
- 1 digital stopwatch
- 1 tape measure, 15 m
- 10 bean bags (put dried beans in baby socks)

Recommended Group Size:	3	Interactive Demo OK?:	N

Find a stretch of fairly smooth, level floor that is about 10 meters long. A hallway is a good bet for this series of measurements. You are to record data for position vs. time for your ball for three different situations: (1) a *briskly* rolling ball receiving no whacks, (2) a ball starting at rest and receiving regular light taps, (3) a ball that has an *initial* velocity but is tapped lightly and regularly in the direction opposite to its initial velocity. Before taking data, you and your partner should practice techniques for making these measurements. **Hints:** You should concentrate on making predictions and then taking and recording data for all three types of measurements. Practice coordinating the tapping, timing, and position marking several times before attempting to take data. Data that is not taken carefully will have too much variation to be interpreted easily. *You should obtain at least six values of position*. You may need to spend time after class analyzing the data using spreadsheets and computer graphing.

6.6.1. Activity: The Motion of a Freely Rolling Ball

a. Assume that the ball is rolling freely without friction so there is no net force on it. What do you expect the graphs of position vs. time and of velocity vs. time will look like? Sketch the predicted graphs below. What do you predict the nature of the acceleration of the ball will be? For instance, will it increase, decrease, remain constant, be zero, etc.?

b. Decide how to take and analyze data for position vs. time for a briskly rolling ball that receives no whacks, so as to determine the acceleration of the ball. For the time being, just ignore the friction that causes the ball to stop eventually. Take, analyze, and display your data and findings. Affix a computer printout of your graph in the space below.

c. What is the nature of the acceleration of the ball? Is it increasing, decreasing, constant, zero, etc.? Is that what you predicted? What experimental results are you basing your conclusion on?

6.6.2. Activity: 1D Tapping of a Ball Starting from Rest

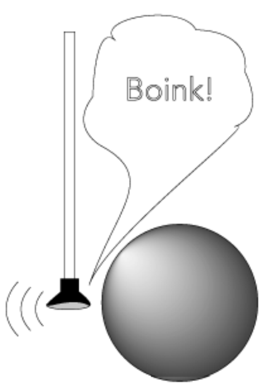


Fig. 6.7.

a. What do you expect the velocity vs. time graph of a ball that starts from rest and receives a series of steady light taps will look like? Sketch the predicted graph below. What do you predict the nature of the acceleration of the ball will be? Will it increase, decrease, remain constant, be zero, etc.?

b. Decide how to take and analyze data for position in meters vs. time in seconds for a ball that is initially at rest and then receives a series of light whacks. Display your data in the following table. Use a spreadsheet to calculate the average velocity of the ball as a function of time, and affix a printout of your spreadsheet in the space below. (You may cover the data table if needed.)

	t(s)	x(m)
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		

	t(s)	x(m)
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		

c. Transfer the spreadsheet data to your graphing routine and create a graph of the average velocity in each time interval as a function of time. Use the slope of the graph to determine the average acceleration of the ball as it experiences continuous tapping. Enclose a printout of the graph in the space below.

d. How did your actual graph of velocity vs. time compare with your predicted graph?

e. What is the nature of the acceleration of the ball? Is it increasing, decreasing, constant, zero, etc.? Is that what you predicted? What experimental results are you basing your conclusion on?

Next, you should set up a situation that is similar to tossing a ball up in the air. To do this, you will need to give the ball an initial velocity in a direction that is opposite to the direction of the force you exert on it.

6.6.3. Activity: 1D Tapping of a Ball with Initial Motion

a. What do you expect the velocity vs. time graph will look like for a ball that is initially rolling toward you as you give it a series of steady light taps in a direction opposite to its initial motion? Sketch the predicted graph below. What do you predict the nature of the acceleration of the ball will be? Will it increase, decrease, remain constant, be zero, etc.? Ignore any slight decreases in velocity that result from small bumps on the floor.

b. Decide how to take and analyze data for position vs. time for a ball that is initially rolling toward you as you give it a series of light whacks that causes it eventually to stop and then turn around. Display your data in the table below. Use a spreadsheet to calculate the average velocity of the ball as a function of time, and affix a printout of it in the space below. (Cover the data table if needed.)

	t(s)	x(m)
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		

	t(s)	x(m)
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		

c. Transfer the spreadsheet data to your graphing routine and create a graph of the average velocity in each time interval as a function of time. Use the slope of the graph to determine the average acceleration of the ball as it experiences continuous tapping. Affix a printout of the graph in the space below.

d. What is the nature of the acceleration of the ball? Is it increasing, decreasing, constant, zero, etc.? Is that what you predicted? What experimental results are you basing your conclusion on? Ignore slight decreases in velocity.

6.7. FITTING EQUATIONS TO YOUR DATA

Now you will further analyze the data you took on the one-dimensional motion of the tapped ball. You should review the modeling techniques described in Unit 4.

If the ball was rolling freely (i.e., experiencing no force) or if you applied a constant force to your ball in the x-direction, the x vs. t graph ought to have a predictable shape.

6.7.1. Activity: Finding Equations to Describe *x* vs. *t*

a. Refer to the data you collected in Activity 6.6.1 for the freely rolling ball. Use the modeling techniques you have employed in previous units to fit a line or curve to your data. Affix an overlay graph of your original data and the fitted line or curve in the space below. Also give the equation that best describes how x varies with time.

b. Repeat part a. for the data on the ball tapped from rest that you collected in Activity 6.6.2.

Predicted curve:

Best fit equation:

Printout of graph:

c. Repeat part a. for the data on a ball tapped with initial motion that you collected in Activity 6.6.3.

Predicted curve:

Best fit equation:

Printout of graph:

d. Now look at the equations you obtained in parts a, b, and c. Comment on how the equations are similar and how they differ. Is one of them more different than the others, are all three alike, or are all three quite different? Explain.

6.8. PROJECTILE MOTION—SOME OBSERVATIONS

So far we have been separately dealing with horizontal motion along a straight line and vertical motion along a straight line. In this unit and the next we would like to study motions in a plane such as the motion of a cannon ball or the circular orbit of the planet Venus.

If a rock is hurled off a cliff with an initial velocity in the *x*-direction and *y*-direction, it will continue to move forward in the *x*-direction and at the same time fall in the *y*-direction as a result of the attraction between the earth and the ball. The two-dimensional motion that results is known as *projectile motion*.

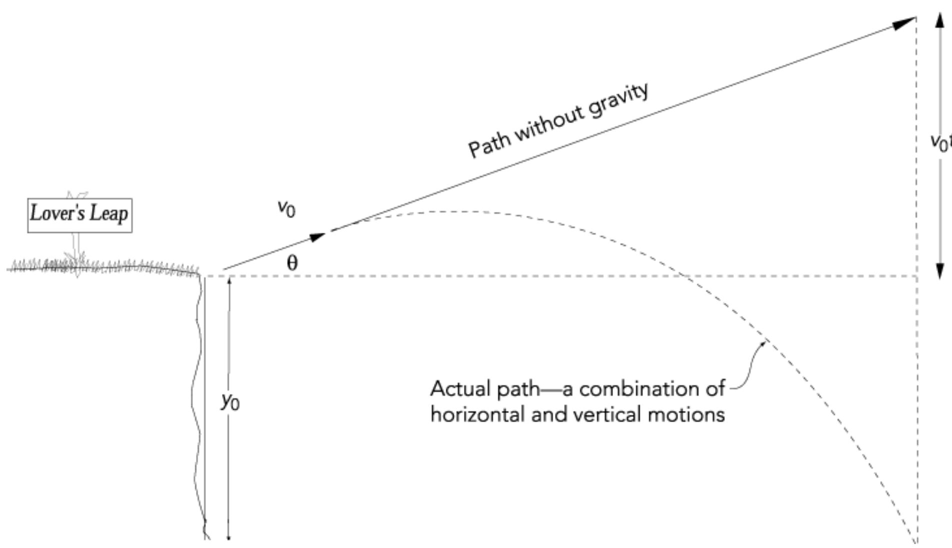
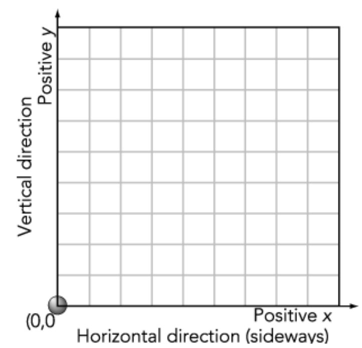


Fig. 6.8. Diagram of a motion with and without gravitational forces present.

In this session you are going to simulate projectile motion by combining the two kinds of one-dimensional motion of the bowling ball that you have already observed. The first motion was that of a ball rolling along with no external forces on it. The second motion was that of a ball receiving a series of rapid light whacks. Once you have set up the motion, you can do a series of quantitative measurements to record the shape of the path described by the ball and measure its position, velocity, and acceleration in the x- and y-directions as the ball progresses. Let's start with some predictions.

6.8.1. Activity: Predicting the Nature of Projectile Motion in a Vertical Plane with F_g Acting

a. Suppose you were to toss a ball at a 45° angle with respect to the horizontal direction. Can you guess what the resulting y vs. x graph of its two-dimensional motion would look like? You can sketch the predicted motion in the graph below. Please use your previous observations of vertical and falling motion in 1D and of horizontal motion in 1D to make an intelligent prediction of the path followed by the ball.



b. Explain the basis for your prediction.

Demonstration of the Independence of Vertical and Horizontal Motion for a Projectile

Suppose someone riding on a moving cart with negligible friction in the wheel bearings tosses a ball straight up in the air. Will the ball fall behind him or will he be able to catch the ball later before it lands?

Fig. 6.9. Diagram of a person tossing or launching a ball straight up while moving at a constant velocity horizontally.

For this demonstration you will need:

- 1 Kinesthetic cart
- 1 spring-loaded projectile launcher with a ball

Recommended Group Size:	All	Interactive Demo OK?:	Y

6.8.2. Activity: Predicting the Path of a Ball Tossed from a Moving Cart

a. Describe what you think will happen when a ball is tossed straight up from a moving cart by sketching the path an observer at rest in the laboratory will see.

b. Explain the reasons for your prediction

c. Observe the demonstration and describe what happens.

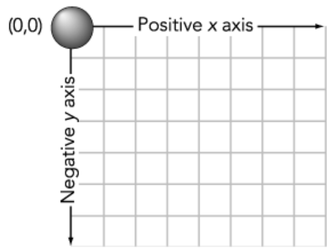
d. Suppose the cart is moving at 3 m/s. What is the initial horizontal velocity of the ball according to an observer in the laboratory? What is its horizontal velocity a few moments later when it lands on the floor?

e. What happens to the vertical velocity of the ball according to an observer in the laboratory?

f. Do the horizontal and vertical motions seem to be independent?

6.8.3. Activity: Predicting Horizontal Projectile Motion Using an Applied Force

a. Suppose you were to roll the ball briskly in one direction and then proceeded to tap on it at right angles to its original direction. Can you guess what the resulting graph of its two-dimensional motion would look like? You can sketch the predicted motion in the graph below. *Please use your previous observations of vertical and falling motion in 1D and of horizontal motion in 1D to make an intelligent prediction of the path followed by the ball.*



b. Explain the basis for your prediction.

6.9. USING VECTORS FOR THE ANALYSIS OF 2D MOTION

Your next task will be to describe projectile motion mathematically. In order to do this, let's take a break from physics and learn about the mathematics of some abstract entities that mathematicians call *vectors*.

The world is full of phenomena that we know directly through our senses—objects moving, pushes and pulls, sights and sounds, winds and waterfalls. A vector is an abstract, mathematical entity that obeys certain predefined rules. It is a mere figment of the mathematician's imagination. But vectors can be used to *describe* aspects of "real" phenomena such as positions, velocities, accelerations, and forces. In the following figure, the position of an object relative to a coordinate system we have chosen can be represented by the vector $\vec{r}$.

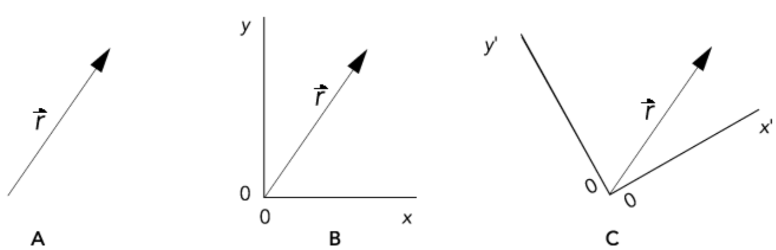


Fig. 6.10. A position vector without a coordinate system and the same vector described by two different coordinate systems with the same origin.

A vector has two key attributes represented by an arrow pointing in space—*magnitude* and *direction*. The *magnitude* of a vector is represented by the length of the arrow and its *direction* is represented by angles between the arrow and the coordinate axes chosen to describe the vector. To answer questions about the magnitude and direction of vector $\vec{r}$ shown in Figure 6.10, you will need:

- 1 ruler
- 1 protractor

Recommended Group Size:	2	Interactive Demo OK?:	N

6.9.1. Activity: Vector Magnitude and Direction

a. What is the magnitude of the position vector in the *x*, *y* coordinate system shown in Figure 6.10B?

b. What angle does the vector make with the *x*-axis in the same coordinate system?

c. What is the magnitude of the vector in the *x*', *y*' coordinate system shown in Figure 6.10C? How does this compare to the magnitude of the vector in the *x*, *y* coordinate system?

d. What angle does the vector make with the *x*'-axis? Is this the same as the angle with respect to the *x*-axis?

Vector Notation

One of the most common ways to indicate that a quantity is a vector is to represent it as a letter with an arrow over it. Thus, in this activity guide symbols such as $\vec{r}$, $\vec{v}$, $\vec{a}$, and $\vec{F}$ all represent quantities which have magnitude *and* direction while *r*, *v*, *a*, and *F* represent only the magnitudes of those vectors. **Note:** You should *always* place an arrow over vector quantities.

There are some alternate ways to represent vectors. For example, the diagram below illustrates the representation of a two-dimensional vector that lies in the *x*-*y* plane using unit vector notation.

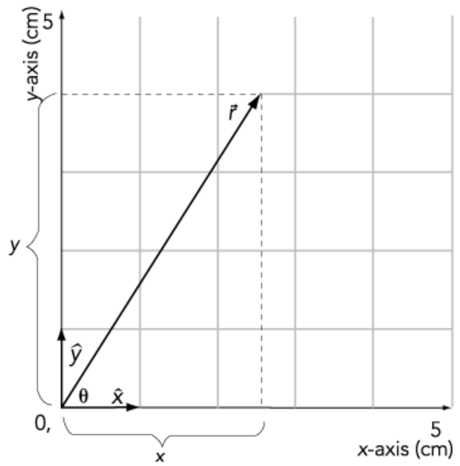


Fig 6.11. Vector components for a position vector $\vec{r} = x\hat{x} + y\hat{y}$ where $\hat{x}$ and $\hat{y}$ are unit vectors and *x* and *y* are vector components.

Reminder: $\hat{x}$ and $\hat{y}$ represent *unit vectors* pointing along the *x*- and *y*-axes, respectively. Many texts use $\hat{i}$ and $\hat{j}$ for these quantities.

6.9.2. Activity: 2D Vector Components

a. Use the Pythagorean Theorem to find an equation that relates the magnitude of the vector $\vec{r}$ to the value of its *x*-component and its *y*-component, *x* and *y*.

b. Using Figure 6.11, measure the value of the vector magnitude, *r*, and of the vector components, *x* and *y*. Don't forget to specify units!

$r_{meas} =$

$x_{meas} =$

$y_{meas} =$

c. Use the measured values of *x* and *y* to calculate the magnitude, *r*, of the vector $\vec{r}$. How does this calculated value compare to the measured value you obtained in part a?

d. Use the definition of sine and cosine in Appendix J to show that if the angle θ is known, then the values of x and y can be calculated from the equations $x = r \cos \theta$ and $y = r \sin \theta$.

e. Measure the value of θ and combine it with the previously measured magnitude of $\vec{r}$ to calculate the values of x and y. How do these compare with the values you measured directly for x and y?

f. Alternatively, show that if the vector components x and y are known, then θ can be determined from the equation $\theta = \arctan (y/x)$.

g. How does the calculated value of θ compare with the measured values?

ANALYZING PROJECTILE MOTION

6.10. DETERMINING THE PATH OF A BALL IN TWO DIMENSIONS

In this activity you will make measurements of "projectile" motion. Then you will use vectors to analyze these measurements.

To make the two-dimensional measurements described below you will be using the twirling baton and duck pin or bowling ball. You and a partner should gather the following equipment to study the motion of the ball:

- 1 duck pin ball (or 1 bowling ball)
- 1 twirling baton
- 1 digital stopwatch
- 1 tape measure, 15 m
- 10 bean bags (put dried beans in baby socks)

Recommended Group Size:	3	Interactive Demo OK?:	N

Find a stretch of fairly smooth level floor that is about 10 meters on a side. A gym floor is a good bet for this series of measurements. You are to record data for position vs. time for a ball that has a brisk initial velocity but is tapped lightly and regularly in the direction perpendicular (i.e., at right angles) to its *initial* velocity. Before taking data, you and your partner should practice techniques for making these measurements. *Try to hit the ball with lots of* *relatively rapid even taps. Always hit at right angles to the original direction of motion.*

6.10.1. Activity: 2D Tapping of a Big Ball w/ Initial Motion

Record the x and y position of the ball as a function of time and fill in the data table in the space below. *Be sure to describe the procedures you used to take the data*.

No.	t(s)	x(m)	y(m)
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			

No.	t(s)	x(m)	y(m)
9.			
10.			
11.			
12.			
13.			
14.			
15.			
16.			

6.11. FITTING EQUATIONS TO YOUR DATA

The main task in this session is to analyze the data you took on the two-dimensional motion of the tapped ball. To do this, you might want to review the section on vectors included in the last session. You should also review the modeling techniques described in Unit 4.

If you applied a constant force to your ball in the y-direction and no force in the x-direction, then the x vs. t and y vs. t graphs ought to have predictable shapes.

6.11.1. Activity: What Equation Describes *x* vs. *t* ?

a. Refer to the data you entered in Activity 6.10.1. Transfer your data into a spreadsheet with the following columns in it. Affix it below (you can cover up the list if you like!). *Save your spreadsheet since you will need it in subsequent activities*.

1. Time data
2. Measured x-value
3. Measured y-value (you'll need this in the next activity)

b. Assuming that there is no force in the x-direction, predict the shape of the x vs. t graph and sketch it below.

c. Now graph x vs. t. Does it have the shape you expected?

d. Use the modeling techniques you have used in previous units to fit a line or curve to your data. Affix an overlay graph of your original data and the modeled line or curve in the space below. Also give the equation that best describes how x varies with time.

Note: If there is a (more or less) constant force on the ball in the y-direction, the mathematical relationship between y and t should be a bit more complicated than the one between x and t.

6.11.2. Activity: What Equation Describes *y* vs. *t* ?

a. Return to the spreadsheet you created in Activity 6.11.1. Assuming that there is a constant force in the y-direction, predict the shape of the y vs. t graph and sketch it below.

b. Now graph y vs. t. Does it have the shape you expected?

c. Use the modeling techniques you have used in previous units to fit a line or curve to your y vs. t data. Affix an overlay graph of your original data and the modeled line or curve in the space below. Also give the equation that best describes how y varies with time.

Now, let's combine the equations from Activities 6.11.1d and 6.11.2c to find an equation that describes how y varies with x.

6.11.3. Activity: What Equation Describes *y* vs. *x* ?

a. Find an equation for y vs. x by combining the equations for x vs. t and y vs. t and eliminating t. Write the equation below.

b. Create a table for actual measured values of y vs. x in your spreadsheet from Activity 6.11.1. Then add a column using your equation from part a. to calculate a *theoretical* value of y for each *measured* value of x. Create an overlay graph showing both measured y vs. x and theoretical y vs. x. Affix your table and graph in the space below.